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Image Denoising for Poisson Noise by Pixel Values Based Division and Wavelet Shrinkage

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Abstract—In this paper, we propose a method for restoration from an image degraded by Poisson noise. Poisson noise is signal-dependent so that the variance of the noise changes in proportion to pixel values of the images. In the proposed method, the pixel values based division method, which was recently addressed, is first utilized to whiten Poisson noise. Then, the wavelet shrinkage is used to remove the remaining noise in the processed image. We verify the effectiveness of the proposed method through simulation experiments.

1. Introduction

Today, we can deal with medical images by digital signal processing. Quality improvement of the degraded medical image is important, which is a current research topic [1]. The medical data represented by photon images has a tendency to be degraded by Poisson noise. The characteristics of Poisson noise are determined by photon counting statistics. Because of this, it is a difficult task to remove the noise. However, if we can remove Poisson noise effectively, we will acquire a fine image from the degraded image produced with a small amount of photons. This may lead to a reduction of the probability of not only medical exposure, but also medical errors.

To accomplish the effective reduction of Poisson noise, the techniques based on wavelet transform have been proposed recently [2][3][7]. The method in [2] gives thresholding in each wavelet domain, maintaining the features of an original signal in the noise reductioned image. In [3], Anscombe's transformation is used as the preprocessing step, which behaves to transform Poisson noise into a Gaussian white noise process. In [7], the M-transformation is used instead of Anscombe's transformation. They make the followed wavelet shrinkage effective. The idea of whitening Poisson noise is similar to that we use below.

In this papar, we propose a method to restore an image degraded by Poisson noise by combining the pixel values based division [6] with the wavelet shrinkage. The division method comes from a motivation that the difference of the variance of additive Poisson noise is small for similar levels of brightness. Poisson noise can be approximated to white Gaussian noise by the pixel values based division method. As a result, the wavelet shrinkage works effectively. In [6], an Iterative Spectal Subtraction (ISS) method is used after the pixel values based division. The computational complexity of the ISS method is, however, very high unfortunately. On the other hand, the computation of the wavelet shrinkage is very efficient. From this point of view, we verify the effectiveness of the proposed method through simulation experiments.

The organization of this paper is as follows. We explain the characteristics of Poisson noise and the wavelet shrinkage in Sections 2 and 3, respectively. In Sections 4 and 5, we describe the proposed method in ideal environments and in real environments. Section 6 is devoted to drawing a conclusion.

2. Poisson Noise

Poisson noise is signal-dependent, that is often seen in photon images. The variance of the noise is proportional to the original image values. The noise model is described as

$$d(m,n) \sim \frac{1}{\lambda}\text{Poisson}\{\lambda o(m,n)\} \tag{1}$$

where $o(m,n)$ and $d(m,n)$ mean the pixel values in the original and degraded images, respectively. The degraded image is generated by multiplying the original pixel values by λ and by using these as the input to a random number generator which returns Poisson-distributed values. The amount of noise depends on λ. An example of the original image and degraded image with $\lambda = 0.5$ is shown in Figure 1.

3. Wavelet Shrinkage

There exists the wavelet shrinkage as one of the techniques to remove the noise included in the image. The wavelet shrinkage is implemented in the wavelet domain, and the wavelet coefficients which include noise are discarded by thresholding. The thresholding value is given by [4];

$$T = P\sigma\sqrt{2\log N} \tag{2}$$

where σ is the variance of the noise, N is the number of signal samples and P is a constant number. P is used as a weighting factor which adjusts the amount of noise we would like to remove. Usually, P is determined experimentally.

(a)Original image (b)Degraded image

Figure 1: Original image and degraded image with $\lambda = 0.5$.

The wavelet shrinkage is used on the assumption that the noise is white Gaussian and the noise variance enable to estimate. Thus, we need to estimate the noise variance. We, however, can not obtain the true noise variance directly in the most of cases. To solve this problem, in this paper, we rely on the technique of frequency band division processing [5].

4. Proposed Method

The proposed method is a combination of the pixel values based division method with the wavelet shrinkage. First, the division method is utilized to whiten Poisson noise. Then, the wavelet shrinkage is used to remove the approximated white noise in the processed image.

4.1. Pixel Values Based Division Method

Here, we describe the pixel values based division method, which was originally proposed by kawanaka et al. [6]. The degraded image $d(m,n)$ ($d(m,n)$ means pixel values at the (m,n)th pixel location) is divided into N images as $d_k(m,n)(1 \leq k \leq N)$, which are given at each pixel(m,n) by

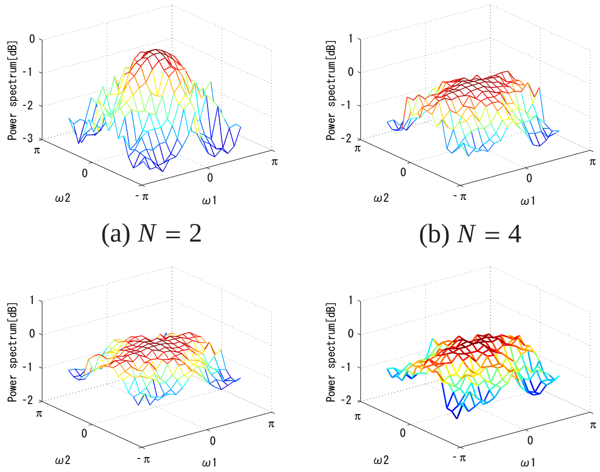
$$d_k(m,n) = \begin{cases} B_1 & \text{if } o(m,n) < B_1 \\ d(m,n) & \text{if } B_1 \leq o(m,n) < B_2 \\ B_2 - 1 & \text{if } B_2 \leq o(m,n) \end{cases} \quad (3)$$

$$B_1 = G/N \times (k-1) \quad (4)$$

$$B_2 = G/N \times k \quad (5)$$

where $o(m,n)$ is pixel values of the original image, B is the threshold for division, G is the number of gradation and N is the number of division. Equations (3) - (5) mean the following. If $o(m,n)$ is less than $G/N\times(k-1)$, then $d_k(m,n)$ is $G/N\times(k-1)$. If $o(m,n)$ is greater than or equal to $G/N\times k$, then $d_k(m,n)$ is $G/N \times k$. If $o(m,n)$ is greater than or equal to $G/N\times(k-1)$, and less than $G/N\times k$, then $d_k(m,n)$ adopts $d(m,n)$ pixel value itself.

For example, in the case of $N = 4$, $k = 3$ and $G = 256$, if $o(m,n)$ is less than 128, then $d_3(m,n)$ is 128. And, if $o(m,n)$ is greater than or equal to 191, then $d_3(m,n)$ is 191. The resulting $d_3(m,n)$ has pixel values consisting of 128~191.



(a) $N = 2$ (b) $N = 4$ (c) $N = 8$ (d) $N = 16$

Figure 2: Poisson noise power spectra obtained with different division numbers.

When Poisson noise is processed by the division method, the power spectrum is changed. Figure 2 shows the power spectra of the Poisson noise processed by the division method with $N = 2, 4, 8$, and 16. From Figure 2, we see that the power spectrum becomes flatter as the number of division, N, increases. This suggests that the processed Poisson noise is approximated to white noise, if a large N is used in the division method.

4.2. Simulation Results

We investigated the restoration accuracy of the proposed method by changing P in (2) on several degraded images. The images are divided by the division method with $N = 2, 4, 8$, and 16. We employ Peak-Signal-to-Noise-Ratio ($PSNR$) to evaluate the restoration accuracy. The $PSNR$ is given in decibel by

$$PSNR = 10 \log_{10} \frac{S_{MAX}^2}{MSE} \quad (6)$$

where S_{MAX} means the max of a signal and MSE stands for Mean Square Error. The $PSNR$ is a scaled measure of the quality of a reconstucted or denoised image. Higher $PSNR$ values indicate better quality of the resulting images.

Figure 3 shows the results of the restoration accuracy against the variation of P. From Figure 3, we see that the restoration accuracy increases by changing P in cases of $N = 8$ and $N = 16$. On the other hand, it is observed in cases of $N = 2$ and $N = 4$, the restoration accuracy decreases by changing P. This may be because Poisson noise is not enoughly approximated to white noise in low division number cases. Figure 3, as a result, indicates that the use of $N = 16$ is the best and should be used.

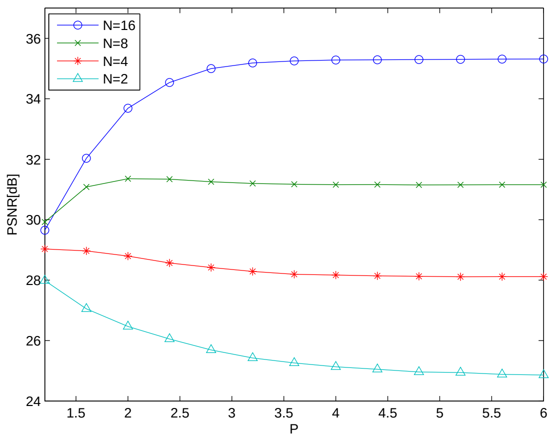


Figure 3: Changes in restoration accuracy obtained by the proposed method on Cameraman with $\lambda = 0.50$.

5. Division Method in Real Environments

In Section 4, we considered the use of the original image $o(m, n)$ in the division method. However, we can not use the original image $o(m, n)$ in real environments. Thus, we consider here a division method to use only the degraded image information $d(m, n)$ without the original image $o(m, n)$.

5.1. Division Method

Let us focus on nine pixels in the neighborhood of a remarkable pixel. Because the variance of Poisson noise is proportional to the pixel values of the original image, the variance of Poisson noise is similar to the variance of white Gaussian noise at smooth parts. Thus, we can estimate a remarkable pixel value from the average of the nine pixels. However, if there are extraordinary pixel values, which appear at edge parts basically, the pixel values estimated become inaccuracy. Hence, we need to obtain the average of the nine pixels with eliminating the extraordinary pixel values. To do this, we use $R(m, n)$ instead of $o(m, n)$ in (3). $R(m, n)$ is derived by

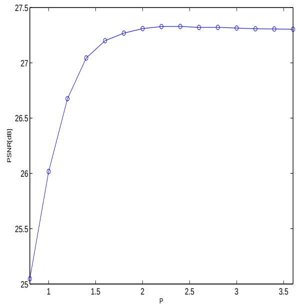
$$R(m, n) = average[\hat{d}(m, n)] \qquad (7)$$

where $\hat{d}(m, n)$ means nine pixels in the neighborhood at (m, n), which is the remarkable pixel. Here, $\hat{d}(m, n)$ is constrained by the following expression in order to cut off the extraordinary pixel values:

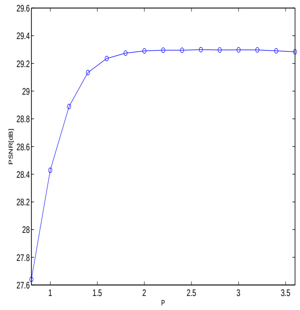
$$d(m, n) - \gamma \sqrt{d(m, n)} \leq \hat{d}(m, n) \leq d(m, n) + \gamma \sqrt{d(m, n)} \quad (8)$$

where γ is a constant number and $d(m, n)$ is the remarkable pixel value. Equation (8) is derived by the following motivation.

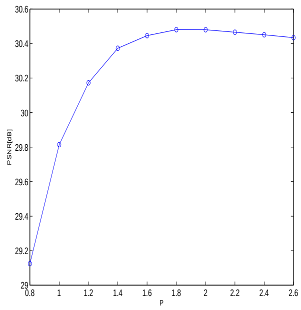
$$o(m, n) - \sqrt{o(m, n)} \leq d(m, n) \leq o(m, n) + \sqrt{o(m, n)} \quad (9)$$



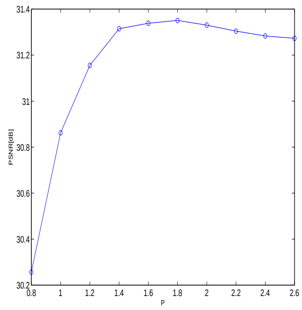
(a) $\lambda = 0.25$



(b) $\lambda = 0.50$



(c) $\lambda = 0.75$



(d) $\lambda = 1.00$

Figure 4: Changes in restoration accuracy obtained by the proposed method on Cameraman.

Equation (9) is approved when the original image is degraded by Poisson noise with $\lambda = 1.0$, because Poisson distribution has a property that the mean is equal to the variance. However, we can not use the original image $o(m, n)$ in real environments. Hence, we use $d(m, n)$ instead of $o(m, n)$. The error generated when $d(m, n)$ is used instead of $o(m, n)$ is adjusted by γ.

The actual parameter γ in (8) was determined in order to provide the best performance. The resut is that if $\lambda = 0.25, 0.50, 0.75$, and 1.00, then $\gamma = 7.0, 4.0, 3.5$, and 3.0. The division method with such parameter setting makes it possible to divide correctly the degraded image without the original image.

5.2. Simulation Results

The proposed method was implemented on different three kinds of images; Cameraman, Girl and Airplane. The size of all the images is commonly 256×256. The best P in 2 for the proposed method was obtained from the results in Figure 4 where the dependency on the value of λ was investigated on Cameraman. Figure 5 shows a comparison of the restored images by the proposed method with $N = 16$, the wavelet shrinkage in [4] and the ISS method in [6]. Figure 5 suggests that the wavelet shrinkage method can not remove the noise added by high brightness, resulting in that the noise is remaining. On the other hand, the proposed method can remove the noise effectively and the restored image becomes clear. Table 1 gives the $PSNR$ values of the restored images obtained by each method. From Table 1, we see that the $PSNR$ values of the proposed method have a higher one than that of the wavelet shrinkage. This fact is also confirmed on the noisy images with different λ. Comparing the proposed method with the ISS method in Table 1, we realize that the performance of the proposed method is competitive with that of the ISS method.

6. Concluding Remarks

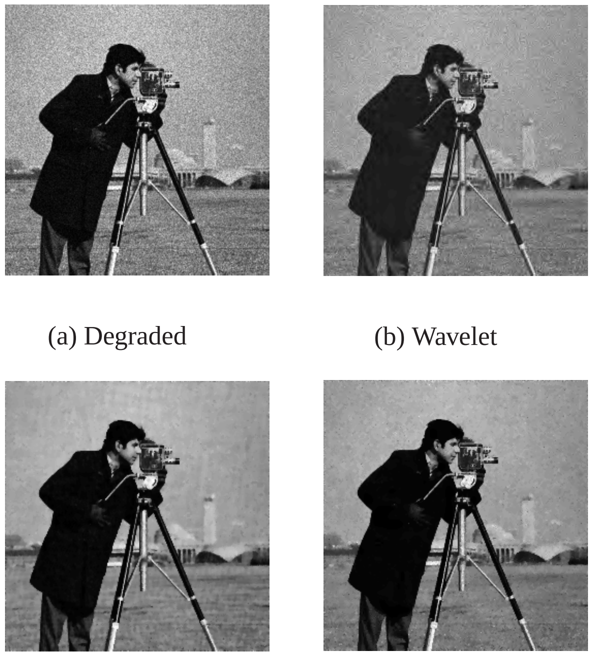
In this paper, we have studied restoration of an image degraded by Poisson noise using the pixel values based division method and the technique of the wavelet shrinkage. In ideal environments, the restoration accuracy is improved by increasing the number of division in the pixel values based division method and by utilizing an appropriate weighting factor for thresholding in the wavelet shrinkage. Experimental results in real environments show that the proposed method works effectively, relying on the averaging operation on nine pixels without the use of the original image. It is confirmed that the performance of the proposed method is better than or equal to those of the conventional methods.

For the method in [6], the spectral subtraction is iteratively utilized to remove the noise, while the wavelet shrinkage can remove the noise by only thresholding in the wavelet domain without iterative implementations. Thus, the computational complexity of the proposed method is lower and more effective.

Our future work aims at comparing our proposed method with recent works[2][3][7]. We also would like to verify the effectiveness of the proposed method on real medical images.

Image	λ	Noisy	Wave	ISS	Prop
Cameraman	0.25	21.46	25.02	27.57	27.33
	0.50	24.37	26.96	29.40	29.32
	0.75	26.16	28.22	30.36	30.50
	1.00	27.39	29.08	31.34	31.35
Girl	0.25	23.51	27.40	29.81	29.28
	0.50	26.50	29.42	31.50	30.72
	0.75	28.26	30.52	32.19	31.85
	1.00	29.44	31.25	32.95	32.50
Airplane	0.25	19.91	25.02	26.50	26.21
	0.50	22.73	26.96	28.30	28.10
	0.75	24.40	28.22	29.32	29.13
	1.00	25.66	29.01	30.33	30.04

Table 1: Comparison in $PSNR$[dB].



(a) Degraded (b) Wavelet

(c) ISS (d) Proposed

Figure 5: Degraded image with $\lambda = 0.50$ and restored images.

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