2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

An adaptive observer design for biological neural network identification

Yu Mao†, Wallace Tang† and Ljupco Kocarev‡

†Department of Electronic Engineering
City University of Hong Kong, Hong Kong SAR, P.R. China

‡Institute for Nonlinear Science
University of California San Diego, San Diego, USA
Macedonian Academy of Sciences and Arts, Skopje, Macedonia
Email:yumao2@student.cityu.edu.hk,kstang@ee.cityu.edu.hk, lkocarev@UCSD.Edu

Abstract– In this paper, an adaptive observer approach is proposed to identify and monitor the topology of a biological neural network with synaptic coupling. By observing a single state of each neuron, it is proved that the topology of the entire network can be identified accurately, while all the unknown states of the neurons can also be estimated at the same time. It is also demonstrated that the design can serve as a monitoring system of the network so that any changes can be captured and reported.

1. Introduction

The biological neural network is long to be an active and challenging research topic. It is commonly considered as a complex network of coupling neurons, for which the complexity is governed by various elements, including the topological structure, neuron model, dynamical evolution, and so on.

Due to the nature of neural network, which is usually nonlinear, complex and high dimensional, it is difficult to identify its topological structure correctly. The difficulty further escalates when the accessibility of the network is very limited.

In this paper, an adaptive observer is proposed to identify and monitor the topology of a biological neural network. Adaptive observer is a classical design in control theory that simultaneously estimates the states and unknown parameters of a targeted system [2,7]. Recently, this approach has been successfully adopted to identify the topological structure of a neuron system with electrical coupling, and some encouraging results have been reported [6]. In this paper, with a different design of adaptive observer and a proof of stability, the concept is further extended to biological neural network with synaptic coupling, for which a more common situation is managed.

The organization of this paper is as follows. In Sect. 2, the mechanism of transmitting signals between neurons is briefly described. In our design, Hindmarsh-Rose neuron model is used and the overall biological neuron network is hence formulated. In Sect. 3, an adaptive observer is designed to estimate the states and the topology of a targeted neural network based on the accessible state of the neurons, while its stability is proved by the Lyapunov stability theorem. The design is then demonstrated with simulation results presented in Sect. 4. Finally, some conclusion remarks are drawn in Sect. 5.

2. System Model of Biological Neural Network

Neuron is a basic unit in a biological neural network, used for processing and transmitting information. It has specialized projections, namely dendrites and axons, for bringing in and sending away information from the cell body, respectively. In most of the neuron cells, the communication is via chemical synapses, yet some neurons may communicate via electrical synapses with rarer cases.

To transmit a signal (impulses) via chemical synapses, a chemical, called neurotransmitter, is released across a gap called synaptic cleft, between the axon terminal and the receptor site of the dendrite, and binds to the receptor site. A potential change is then triggered on the cell membrane, which will then propagate along the axon. After that, a depolarizing process will be undergone to restore the original state.

Models of neuron have been extensively studied and a comprehensive summary can be referred to [4]. In this paper, the Hindmarsh-Rose (HR) model [3] is adopted, which is governed by the following dynamical system:

$$\begin{aligned} \dot{x}(t) &= \alpha x^2 - x^3 - y - z \\ \dot{y}(t) &= (a+\alpha)x^2 - y \\ \dot{z}(t) &= \mu(bx + c - z) \end{aligned} \tag{1}$$

where $x(t)$ is the membrane potential, $y(t)$ and $z(t)$ are the recovery variables with respect to the fast and slow currents, respectively.

The HR model gains its recognition for having the ability to model different kinds of electrophysiological characteristics of neurons. For example, a tonic spiking can be observed (representing that a constantly active neuron) with (1) using the parameters: a=2.8, b=9, c=5, α =1.6 and μ =0.001.

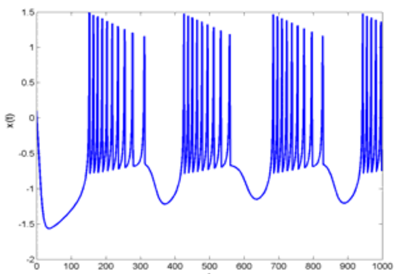

Fig. 1. Tonic spiking generated by a HR neuron model

The overall biological neural network can then be considered as the evolution of connected neurons, where the evolution of *i*-th neuron is governed by:

$$\begin{aligned}\dot{x}_i(t) &= \alpha_i x_i^2 - x_i^3 - y_i - z_i + \sum_{j=1}^{N} g_{ij}\sigma_{V_s}(x_i)\gamma_{v,\theta_s}(x_j)\\ \dot{y}_i(t) &= (a_i+\alpha_i)x_i^2 - y_i \\ \dot{z}_i(t) &= \mu_i(b_i x_i + c_i - z_i)\end{aligned} \quad (2)$$

where $\sigma_{V_s}(x_i) = -(x_i - V_s)$, $\gamma_{v,\theta_s}(x_j) = \frac{1}{1+e^{-v(x_j-\theta_s)}}$ for synaptic coupling, $a_i, b_i, c_i, \alpha_i, \mu_i$ are constants specifying the HR model, θ_S, v and V_s are some free parameters to control the neuron's synaptic coupling. The connection $g_{ij} > 0$ if neurons i and j are connected to each other, and $g_{ij} = 0$ otherwise. It is also assumed that $g_{ij} = g_{ji}$ and $g_{ii} = 0$.

3. Adaptive Observer Design

For a biological neural network consisting of N neurons, (2) can be rewritten as follows:

$$\dot{\boldsymbol{\xi}} = \mathbf{F}(\boldsymbol{\xi}, \mathbf{g}) \quad (3)$$

where $\boldsymbol{\xi} \in R^{3N}$ defines as

$$\begin{aligned}\boldsymbol{\xi} &= \begin{bmatrix}\xi_1 & \xi_2 & \dots & \xi_{3N}\end{bmatrix}^T \\ &= \begin{bmatrix}x_1 & \dots & x_N & y_1 & \dots & y_N & z_1 & \dots & z_N\end{bmatrix}^T\end{aligned}$$

and $g = \{g_{ij}\}$ specifies the topology of the network.

Problem: Assuming that the membrane potential x_i of the neurons is observable, it is to design an adaptive observer scheme such that the topology $g = \{g_{ij}\}$ can be identified. The following assumptions are also given:

1. Similar to all the estimation processes, the situation of persistent excitation [5] is always assumed and the following hypothesis is true.

Hypothesis H1: Let $\boldsymbol{\xi}(t)$ be a solution of (3), and

$$\sum_{j=1}^{N} r_{ij} f_{ij}(\boldsymbol{\xi}(t)) = 0, \quad i = 1,2,\cdots 3N \quad (4)$$

implies r_{ij} =0.

2. Let $\bar{\boldsymbol{\xi}} = (\xi_1, \xi_2, \cdots \xi_N)$, i.e. $\bar{\xi}_i = \xi_i$, $i = 1,2,\cdots N$, be the observable output of (3), there exists an observer design expressed as follows:

$$\dot{\hat{\boldsymbol{\xi}}} = \mathbf{F}(\hat{\boldsymbol{\xi}}, \mathbf{g}) + \mathbf{u}(\hat{\boldsymbol{\xi}}, \bar{\boldsymbol{\xi}}) \quad (5)$$

which can synchronize (3) based on the control signal **u**, having a positive definite Lyapunov function $V_1(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T\mathbf{e}$, where $\mathbf{e} = \hat{\boldsymbol{\xi}} - \boldsymbol{\xi}$, that evaluated along the solution of error dynamics, and its derivative is negative definite.

Proof:

From (2), it can be observed that **F** depends linearly on **g**, and hence we have

$$\dot{\xi}_i = c_i(\boldsymbol{\xi}) + \sum_{j=1}^{N} g_{ij} f_{ij}(\boldsymbol{\xi}) \equiv F_i(\boldsymbol{\xi}, \mathbf{g}) \quad (6)$$

where F_i are smooth functions and

$$f_{ij}(\boldsymbol{\xi}) = \begin{cases}\sigma_{V_s}(\xi_i)\gamma_{v,\theta_s}(\xi_j) & \text{if } 1 \le \text{i} \le \text{N} \\ 0 & \text{if } \text{N} < \text{i} \le 3\text{N}\end{cases}$$

with $\sigma_{V_s}(\xi_i) = -(\xi_i - V_s)$ and $\gamma_{v,\theta_s}(\xi_j) = \frac{1}{1+e^{-v(\xi_j-\theta_s)}}$.

Consider a design of adaptive observer given as below:

$$\begin{aligned}\dot{\hat{\xi}}_i &= c_i(\hat{\boldsymbol{\xi}}) + \sum_{j=1}^{N} q_{ij} f_{ij}(\hat{\boldsymbol{\xi}}) + \mathbf{u}(\hat{\boldsymbol{\xi}}, \bar{\boldsymbol{\xi}}), \\ \dot{q}_{ij} &= -\delta_{ij}(\hat{\xi}_i - \xi_i) f_{ij}(\hat{\boldsymbol{\xi}}), i = 1,\dots N\end{aligned} \quad (7)$$

where $\delta_{ij} > 0$ are positive constants.

The followings prove that $\hat{\boldsymbol{\xi}} \to \boldsymbol{\xi}$, $\mathbf{q} \to \mathbf{g}$ as $t \to \infty$ for all $\hat{\boldsymbol{\xi}} \in \Re^{3N}$ and $\mathbf{q} = \{q_{ij}\} \in \Re^{N\times N}$, where q_{ij} is the estimates of g_{ij}.

Consider a positive definite Lyapunov function,

$$V = \frac{1}{2}\sum_{i=1}^{3N} e_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\delta_{ij}} r_{ij}^2,$$

where $e_i = \hat{\xi}_i - \xi_i$ and $r_{ij} = q_{ij} - g_{ij}$, we have

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{3N} e_i(\dot{\hat{\xi}}_i - \dot{\xi}_i) - \sum_{i=1}^{N} e_i \sum_{j=1}^{N} r_{ij} f_{ij}(\hat{\boldsymbol{\xi}}) \\ &= \sum_{i=1}^{3N} e_i \left[c_i(\hat{\boldsymbol{\xi}}) + \sum_{j=1}^{N} (g_{ij} + r_{ij}) f_{ij}(\hat{\boldsymbol{\xi}}) + \mathbf{u}(\hat{\boldsymbol{\xi}}, \bar{\boldsymbol{\xi}}) \right] \\ &\quad - \sum_{i=1}^{3N} e_i \left[c_i(\boldsymbol{\xi}) + \sum_{j=1}^{N} g_{ij} f_{ij}(\boldsymbol{\xi}) \right] - \sum_{i=1}^{N} e_i \sum_{j=1}^{N} r_{ij} f_{ij}(\hat{\boldsymbol{\xi}}) \\ &= \sum_{i=1}^{3N} e_i \left[F_i(\hat{\boldsymbol{\xi}}, \mathbf{g}) + \mathbf{u} - F_i(\boldsymbol{\xi}, \mathbf{g}) \right] + \sum_{i=1}^{N} e_i \sum_{j=1}^{N} r_{ij} f_{ij}(\hat{\boldsymbol{\xi}}) \\ &\qquad - \sum_{i=1}^{N} e_i \sum_{j=1}^{N} r_{ij} f_{ij}(\hat{\boldsymbol{\xi}}) \\ &= \dot{V}_1\end{aligned}$$

From the equation for the error dynamics, since $e_i \to 0$, it follows that

$$\sum_{i=1}^{3N}\sum_{j=1}^{N} r_{ij} f_{ij}(\boldsymbol{\xi}(t)) = 0. \quad (8)$$

Therefore, based on H1, $r_{ij} \to 0$ for $i = 1,2,\cdots 3N$, $j = 1,2,\cdots N$ as well.

4. Simulation Results

The proposed adaptive observer design is applied for an illustrative network with eight neurons only, as depicted in Fig. 2. Each neuron is modeled by (2) based on the parameters: a=2.8, b=9, c=5, $\alpha \in [1.58, 1.66]$, μ =0.001, $v = 10$, $\theta_s = -0.25$ and $V_s = 2$, so that all neurons are excited in a mode of tonic spiking, ensuring the persistent excitation.

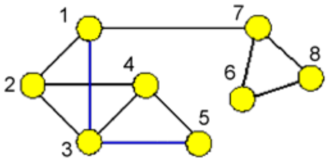

Fig. 2. A network of eight neurons

From [1], synchronization is achievable with a constant gain $g_s = 0.340$, provided that the topology is known. Let $g_{ij} = g_s c_{ij}$, we have $c_{ij} = 1$ if there is a connection between neurons i and j; otherwise, $c_{ij} = 0$. Therefore, identifying the topology of a neuron network is simply equivalent to finding c_{ij} for all *i, j*.

Now, assuming that the membrane voltage x_i is measurable, according to Sect. 3, an adaptive observer can be designed as follows:

$$\begin{aligned}
\dot{\hat{x}}_i(t) &= \alpha_i \hat{x}_i^2 - \hat{x}_i^3 - \hat{y}_i - \hat{z}_i \\
&\quad + \sum_{j=1}^{N} q_{ij} \sigma_{V_s}(\hat{x}_i) \gamma_{v,\theta_s}(\hat{x}_j) - k(\hat{x}_i - x_i) \\
\dot{\hat{y}}_i(t) &= (a_i + \alpha_i)\hat{x}_i^2 - \hat{y}_i \qquad (10) \\
\dot{\hat{z}}_i(t) &= \mu_i (b_i \hat{x}_i + c_i - \hat{z}_i) \\
\dot{q}_{ij} &= -\delta_{ij}(\hat{x}_i - x_i)\sigma_{V_s}(\hat{x}_i)\gamma_{v,\theta_s}(\hat{x}_j)
\end{aligned}$$

where *k* is some positive gain vector.

4.1 Identify the Topology of the Entire Network

Assuming that the topology is fixed during the whole identification process and the network is isolated. Figure 3 shows the dynamics of the neurons (only x_1 and x_3 are shown due to the limitation of pages. Similar tonic spiking occurs in other neurons.) in which tonic spiking is observed. However, although the membrane potentials of all the neurons are similar, they are not identically synchronized, otherwise the observer will not serve the task.

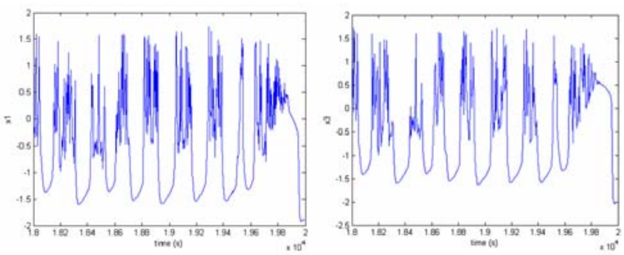

Fig. 3. The network system's state x_1 and x_3 against time *t*.

Figure 4 depicts the connections between neuron '3' and the other neurons, showing a very good estimation result. Similar cases are obtained for other connections, yet, they are not shown here due to the limitation of pages. Similar results are also noticed when the coupling between nodes are different.

The state estimation errors are shown in Fig. 5 for reference. It can be clearly observed that $\hat{x}_i \to x_i$, $\hat{y}_i \to y_i$ and $\hat{z}_i \to z_i$ as $t \to \infty$.

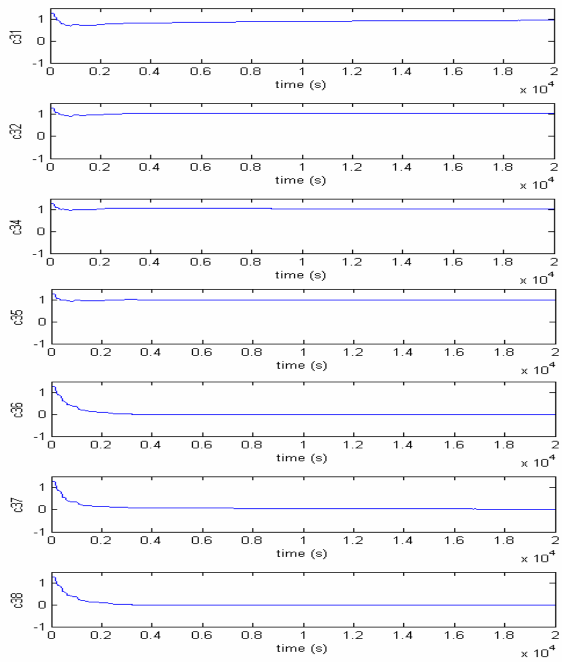

Fig. 4. Estimation of c_{3j} where $j = 1,2,4-8$.

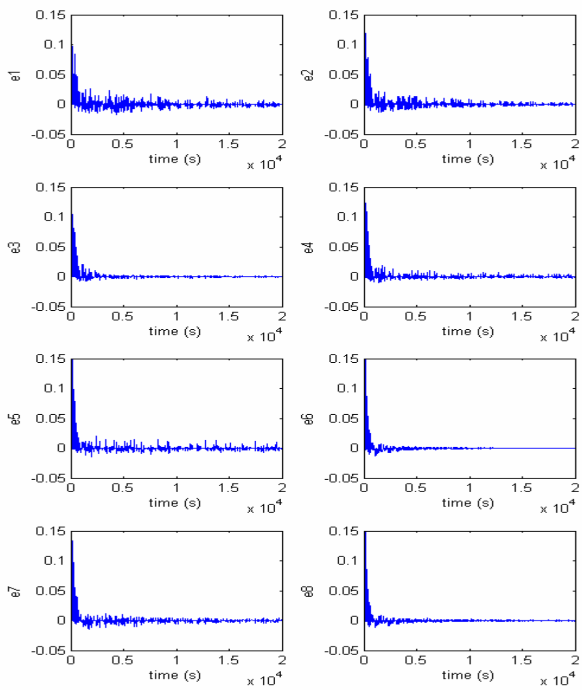

Fig. 5. Estimation errors of membrane potentials x_i.

4.2 Monitor the Changes of the Topology

The same observer can also be used to monitor the topology, which is similar to the identification process. Now assuming that the connections between neurons '1' and ‘3’, neurons ‘3’ and ‘5’ are broken at 50000s and 70000s, respectively, the occurrences of disconnections are duly captured by the observer as shown in Fig. 6.

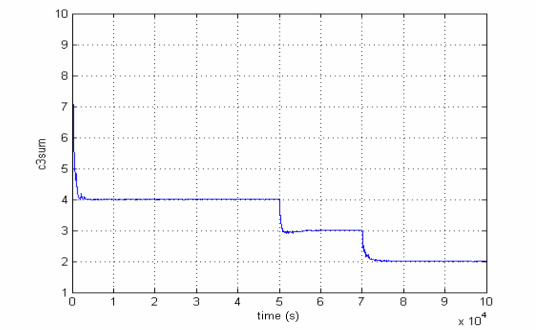

Fig. 6. The sum of c_{3j} where $j = 1,2,4-8$, with neurons ‘1’ and ‘3’, and neurons ‘3’ and ‘5’ disconnected at 50000s and 70000s , respectively.

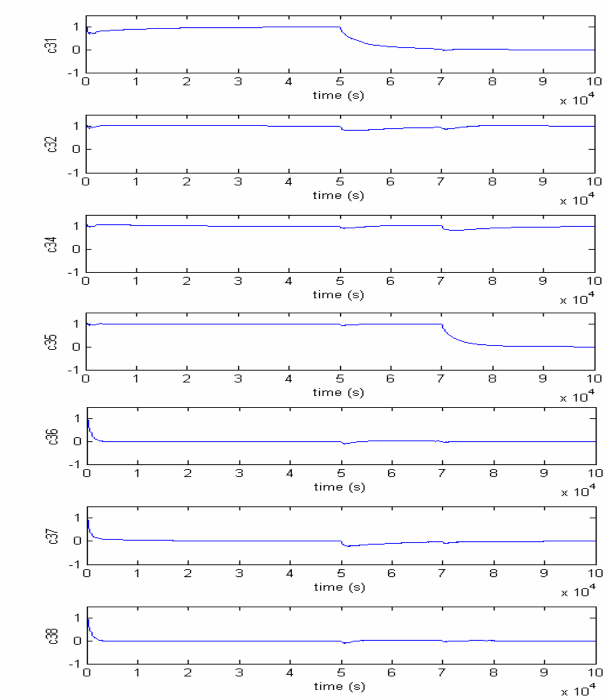

Fig.7. Estimation of c_{3j} where $j = 1,2,4-8$, with neurons ‘1’ and ‘3’, and neurons ‘3’ and ‘5’ disconnected at 50000s and 70000s , respectively.

Figure 7 shows the connections of neuron ‘3’ with the others. It clearly shows that c_{31} and c_{35} drop from 1 to 0 after 50000s and 70000s, respectively, following the changes in the actual topology. The state estimation errors are also approaching to zero as proved previously, which are shown in Fig. 8.

5. Conclusions

In this paper, an effective method based on adaptive observer design is introduced to estimate the topology of biological neural network with synaptic coupling. It is proved that the state estimation errors go to zero asymptotically while the unknown parameters, i.e. the connections of the entire network can be correctly estimated. It is also demonstrated that such design not only achieves state estimation and parameter identification simultaneously, but also is useful for monitoring any changes of the network topology.

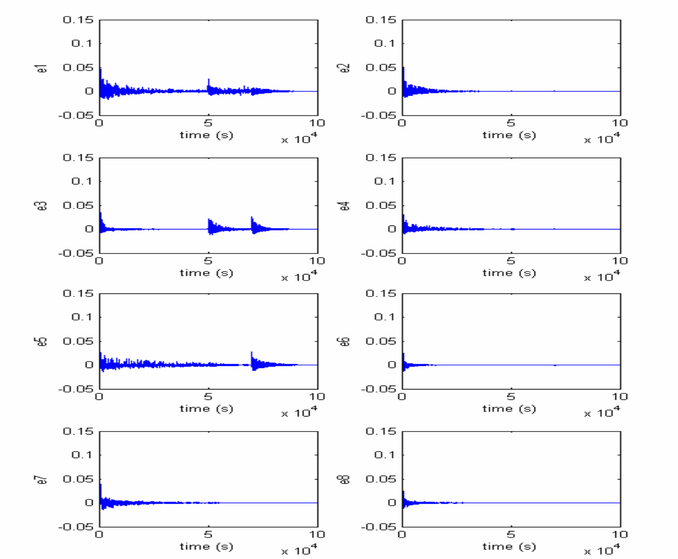

Fig. 8. Estimation errors of states x with neurons ‘1’ and ‘3’, and neurons ‘3’ and ‘5’ disconnected at 50000s and 70000s , respectively.

Acknowledgments

The work described in this paper was fully supported by a grant from the Research Grants Council of Hong Kong Special Administrative Region, China [CityU 120205].

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