2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Noise-assisted detection in sensor network with suboptimal fusion of optimal detections

Shin Mizutani†, Kenichi Arai†, Peter Davis†, Naoki Wakamiya‡ and Masayuki Murata‡

†NTT Communication Science Laboratories, NTT Corporation
2-4 Hikaridai, Seika-cho, Soraku-gun, Kyoto, 619-0237 Japan
Email: †shin@cslab.kecl.ntt.co.jp, ken@cslab.kecl.ntt.co.jp, davis@cslab.kecl.ntt.co.jp
‡Graduate School of Information Science and Technology, Osaka University
1-5 Yamadaoka, Suita, Osaka, 565-0871 Japan
Email: ‡wakamiya@ist.osaka-u.ac.jp, murata@ist.osaka-u.ac.jp

Abstract—We analyze the performance of a distributed sensor network which fuses the detection results from multiple binary detectors at a single data fusion center in the presence of noise. We show the property of noise-assisted detection, whereby the detection correctness probability can be improved by adding noise. We point out that this property can be observed when the data fusion is not optimal, even when each detector has an optimal threshold. We also show the nonmonotonic behavior of noise-assisted detection for increasing added noise. Here we consider a simple model of distributed detection (DD) in which the detection result depends on the outputs of multiple identical signal detectors with common signal and independent noise. DD decides its output based on the rule of data fusion executed at the data fusion center which receives outputs from all detectors. Noise-assisted detection means that the correctness probability can be larger with noise compared to without noise. It also means that it may be possible to optimize correctness probability by adjusting the intensity of noise. Noise-assisted detection is known to occur for suboptimal detectors, and so can also be expected to occur for DD with non-optimal detectors. However, we show that noise-assisted detection in DD can occur even with optimal detectors if the rule of data fusion is suboptimal. This result is significant from the point of view of optimizing the whole system including noise levels to optimize detection.

1. Introduction

Noise-assisted effects in nonlinear systems have recently received considerable attention. In particular, stochastic resonance (SR) has been studied in various systems [1]. SR means that the resonance response of a nonlinear system to a subthreshold signal can be optimized by adding noise. It is considered that sensors in biological system may use this SR to detect signals in noisy environments. Information theoretical approaches have been used to study SR of aperiodic signals, for binary signals [2-11]. In these studies, bit error probability or mutual information is used to measure transmission between input and output.

SR in a threshold system with a signal and noise is closely related to the signal detection problem [7, 8, 10] in classical engineering studies [12]. Here we consider the signal detection problem in which the signal is binary $(0, 1)$ and noise has continuous values. Signal detection determines from the noisy input whether the signal's value is 0 or 1. Full knowledge about the signal level and noise can be used to obtain the optimal detector by calculating the optimal threshold. The optimization criterion is generally the minimal Bayesian risk or the maximal correctness probability.

An optimal detector with an optimal threshold shows monotonic decay of correctness probability with increasing noise intensity. On the other hand, when the threshold of the detector is suboptimal, increasing the noise intensity can maximize the probability of correctness detection [13]. We call this noise-assisted detection. One of the reasons why we may need to use detectors with suboptimal thresholds is that we do not have full knowledge about the signal and noise. When the threshold is estimated from the partial knowledge, the threshold may be suboptimal.

Here we consider noise-assisted detection in sensor networks and especially focus on distributed detection (DD) [14-16]. Simple DD has multiple detectors working in parallel and decides global output at a data fusion center based on the local decisions gathered from each detector. For simplicity, we assume that the local detectors are identical and each detector has a common signal and independent noise. When the local detector output is binary, the input-output rule of the data fusion becomes a Boolean function.

We can expect that there is noise-assisted DD when the detectors have suboptimal thresholds. However we can also show noise-assisted DD, even when each detector has the optimal threshold with respect to the signal and noise, when the data fusion rule is not optimal. Specifically, we show noise-assisted DD, even when each detector has the optimal threshold with respect to the signal and noise, when the data fusion rule is not defined by a monotonic function of the number of detectors, which detect the existence of signal. Such cases occur for example when receiving too many detections from local detectors is judged by the fusion center to be a malfunction state of the system.

2. Noise-assisted detection

A threshold system is a typical system exhibiting SR. Classification by threshold systems is equivalent to the decision task in signal detection [12] whether there is a dc signal or not in noise. As in Ref. [9, 10], we consider the following signal detection problem. A binary input signal has values 0 (input 0) and 1 (input 1) and prior probabilities for each input bit are defined as p_0 and $p_1(= 1 - p_0)$, respectively. Noise is Gaussian with mean 0 and variance σ^2. We define $P_0(x)$ and $P_1(x)$ as two probability distributions corresponding to the inputs 0 and 1 with added noise, respectively. Detection of the signal with noise corresponds to determining that the input signal is 0 or 1 by comparing with a threshold.

Here we define p_{00} as probability that input 0 is detected correctly and also define p_{10} as probability that input 0 is detected as input 1. p_{01} and p_{11} for input 1 are defined similarly. These probabilities can be calculated for signal detection case threshold θ as follows.

$$\begin{aligned} p_{00} &= \int_{-\infty}^{\theta} P_0(x)dx = 1 - p_{10}, \\ p_{10} &= \int_{\theta}^{\infty} P_0(x)dx, \\ p_{01} &= \int_{-\infty}^{\theta} P_1(x)dx, \\ p_{11} &= \int_{\theta}^{\infty} P_1(x)dx = 1 - p_{01}. \end{aligned} \quad (1)$$

We note that p_{00}, p_{10}, p_{01}, and p_{11} are functions of σ, because $P_0(x)$ and $P_1(x)$ have Gaussian distributions.

To evaluate the dependence of signal detection on noise intensity σ, we can define correctness probability as follows.

$$\begin{aligned} P_{\mathrm{cor}}(\sigma) &= P_{\mathrm{cor}}(p_0, p_1, p_{00}, p_{11}) \\ &= p_{00}p_0 + p_{11}p_1. \end{aligned} \quad (2)$$

We note that the probabilities are functions of σ.

Standard signal detection techniques assume complete knowledge concerning the values (levels, amplitudes) of the binary signals and so we can calculate the optimal threshold θ_{opt} to maximize correctness probability. However, when the values of the binary signals are unknown, we cannot know for sure the optimal threshold.

When we determine the threshold from an estimation of the binary signal values, the threshold may be suboptimal. When the value θ is larger than the true value of input 1 (or smaller than the value of input 0) in the case of $p_0 = p_1 = 1/2$, existence of noise can assist the detection. This is noise-assisted signal detection. Biological sensors that work in noisy environment can make use of this mechanism for noise-assisted detection.

We have shown an example of a system with a suboptimal threshold, in which there is a maximum of correctness probability at non-zero noise intensity σ [17]. In other words, a finite noise intensity σ can optimize the correctness probability P_{cor}. In this example, we can see that the optimal σ is non-zero when the threshold is located on a value larger than the value of input 1 or smaller than the value of input 0.

When the threshold is optimal (in the example, $\theta_{\mathrm{opt}} = 1/2$), correctness probability decreases monotonically against noise intensity σ. In other words, noise degrades the detection by an optimal threshold. Here we note that the correctness probability for suboptimal threshold never exceeds the correctness probability for the optimal threshold, as shown by the data processing inequality [18].

3. Distributed detection

DD [14, 15, 16] in sensor networks gathers local outputs of multiple detectors to detect a signal. Here we consider simple DD that has detectors working in parallel and decides global output at a data fusion center based on the local decisions from each detector as in Fig. 1. For simplicity, we assume that the local detectors are identical and each detector has a common signal and independent noise. When the local detector output is binary, the input-output

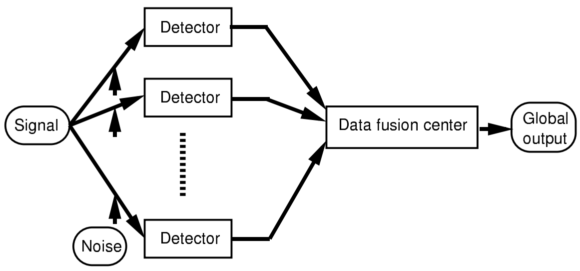

Figure 1: Distributed detection. Simple DD system is composed of multiple local detectors and a data fusion center.

rule of the data fusion becomes a Boolean function.

One of the important issues in sensor networks is optimization of the data fusion rule when the local detectors are identical [15]. This simplification is required to reduce computation and to control sensor networks easily in theory, application and implementation. Here we consider K-out-of-N fusion rule, i.e., the global decision $u_0 = 1$ if K or more local decisions are equal to 1.

$$u_0 = \begin{cases} 1, & \text{if } \sum_{i=1}^{N} u_i \geq K, \\ 0, & \text{if } \sum_{i=1}^{N} u_i < K. \end{cases} \quad (3)$$

K is an integer threshold for the global decision. For each detector, false alarm probability p_F and detection probability p_D is shown as follows.

$$p_F = p(u_i = 1|H_0) = p_{10}, \quad (4)$$
$$p_D = p(u_i = 1|H_1) = p_{11}. \quad (5)$$

Unknown hypotheses, H_0 and H_1 correspond to input 0 and 1, respectively. From false alarm and detection probability of each detector, we can calculate system false alarm probability p_F and system detection probability p_D as follows.

$$P_F = \sum_{i=K}^{N} \binom{N}{i} p_F^i (1 - p_F)^{N-i}, \quad (6)$$

$$P_D = \sum_{i=K}^{N} \binom{N}{i} p_D^i (1 - p_D)^{N-i}. \quad (7)$$

The probability of correct detection with N detectors is as follows.

$$P_{\mathrm{cor}} = (1 - P_F)p_0 + P_D p_1. \quad (8)$$

Varshney has examined K-out-of-N fusion rules and obtained the optimal K value to minimize Bayesian risk [16]. Here, for simplicity, we consider the case of a homogeneous sensor network system of identical local detectors with common signal and independent noise. In inhomogeneous sensor networks, it is necessary to decide the optimal distribution of thresholds to optimize detection [19].

4. Noise-assisted distributed detection

We have explained that noise-assisted DD can be expected when the detectors have suboptimal thresholds with respect to the signal and noise. Next, we show that noise-assisted DD can occur even when the detectors have optimal thresholds, for some types of fusion rules. Specifically, we show an example of noise-assisted DD which can occur when the fusion rule is not a monotonic function of the number of detectors. (In practice, this case could occur, for example, when receiving too many detections from local detectors is judged by the fusion center to be an indication that local detectors are broken and outputting false detections.)

We consider the specific example of $N = 2$ and the EOR (exclusive OR) fusion rule. We assume that the input signal to each local detector has discrete binary values 0 and 1 with probability $p_0 = 0.6$ and $p_1 = 0.4$, respectively, and the input noise to local detectors has continuous values obeying a Gaussian distribution $\mathrm{N}(0, \sigma^2)$. We also assume that the detection threshold of all detectors are the same and adjusted according to the noise level, so that the threshold is optimal $\theta_{\mathrm{opt}}(\sigma)$ at each value of noise intensity σ. The i-th detector's output u_i is 0 or 1. The fusion rule takes local detector outputs u_i and then decides binary global output u_0, such that $u_0 = 0$ when $u_1 + u_2 = 0$ or 2 and $u_0 = 1$ when $u_1 + u_2 = 1$.

Figure 2 shows variation of correctness probability $P_{\mathrm{cor}}(\sigma)$ as noise intensity σ increases. There is a maximum value of correctness probability at a non-zero value of noise intensity. When there is no noise, $\sigma = 0$, the correctness probability $P_{\mathrm{cor}}(\sigma = 0)$ is just equal to the value of p_0, i.e. $p_0 = 0.6$. With increases of σ, the correctness probability $P_{\mathrm{cor}}(\sigma = 0)$ increases to a maximum and then asymptotically approaches to the value of p_0 again. This example

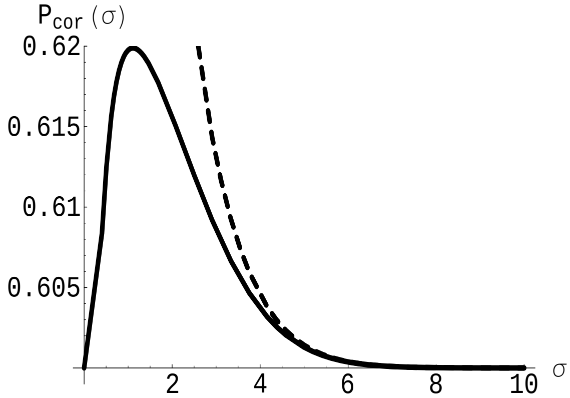

Figure 2: Dependence of correctness probability $P_{\mathrm{cor}}(\sigma)$ on noise intensity σ for a particular system of $N = 2$ detectors. The solid line shows the correctness probability obtained using an EOR (Exclusive OR) fusion rule, and the dotted line shows the correctness probability obtained using an OR rule.

shows that we can see noise-assisted DD even when the local detectors have optimal thresholds. This is not a singular condition and we can find similar behavior for any N. For other rules, such as the optimal OR rule that makes $u_0 = 0$ when $u_1 + u_2 = 0$ and $u_0 = 1$ otherwise, the correctness probability is monotonic decreasing, i.e. no noise-assisted detection is observed, as shown by the dotted line in Fig. 2. Note that the correctness probability is less for the fusion rule where noise-assisted detection is observed, than for the fusion rule where noise-assisted detection is not observed. In this sense, noise-assisted detection occurs for suboptimal fusion rule.

5. Conclusions

We analyzed the performance of distributed detection in a sensor network which fuses the detection results from multiple binary detectors at a single data fusion center. We showed the property of noise-assisted detection and pointed out that this property can be observed when the data fusion is not optimal, even when each detector has an optimal threshold. We also showed the nonmonotonic behavior of noise-assisted detection for increasing added noise, and the existence of an optimal level of noise.

We analyzed the performance of distributed detection in a sensor network which fuses the detection results from multiple binary detectors at a single data fusion center. We described the property of noise-assisted detection in this context and pointed out that this property can be observed even when each detector has an optimal threshold. We presented a particular example of a fusion rule for which the sensor network exhibits noise-assisted detection, and showed nonmonotonic behavior of noise-assisted detection for increasing added noise, and the existence of an optimal level of noise.

The results of this analysis are significant from the general point of view of showing the importance of considering the whole system including noise levels to optimize detection. The results are also significant from the particular point of view of showing how environmental noise, either natural or artificial, can assist detection of signals in distributed sensor networks.

Acknowledgments

This research was supported in part by "Special Coordination Funds for Promoting Science and Technology: Yuragi Project" of the Ministry of Education, Culture, Sports, Science and Technology, Japan.

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