

Synchronization in coupled Morris-Lecar neurons with class I and class II excitability

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Abstract—We investigate mutual synchronization of the coupled Morris-Lecar neurons with class I and class II excitabilities. Many researchers have showed that class II neurons easily achieve synchronization. However, we find that the parameter region of synchronous firing for class I neurons is larger than that for class II neurons.

1. Introduction

In recent neuroscience, many researchers have claimed that synchronization of coupled neurons plays a very important role to code information in the brain. Neurons are classified into two types, class I and class II, by the difference of the onsets of the firing. These two types are confirmed by the physiologic experiments and it is very important to know how these differences affect synchronization.

For mutual synchronization of two neurons, it is clarified by using the phase resetting curve (PRC) that class II neurons easily achieve synchronization [1]-[3]. Also for a large number of neurons with random connections, class II neurons present a good level of synchronization regardless of the connection topology [4]. Also for forced synchronization, class II neurons have advantage of acquiring synchronization [5]. However, Tsuji et al. showed that class I neurons have wider parameter regions of synchronous firing than those for class II neurons by detailed bifurcation analysis [6].

In this paper, we compare the parameter regions of in-phase synchronization for the coupled Morris-Lecar(ML) neurons by a chemical synapse, when the values of the synaptic delay and the synaptic conductance are changed. The ML neuron model can be switched between class I and class II excitabilities by changing the value of one parameter [7]. Thus, in this paper, we use the ML model and compare the bifurcation structure for class I and class II by using the method [8] for analyzing the system with the synaptic delay.

2. Coupled ML Equation

The ML neuron model [2, 3], proposed as a model for describing a variety of oscillatory voltage patterns of Bar-

nacle muscle fibers, is described by

$$C \frac{dV}{dt} = g_{Ca} M_{\infty} (E_{Ca} - V) + g_K N (E_K - V) + g_L (E_L - V) + I_{ext} \quad (1)$$

$$\frac{dN}{dt} = \frac{N_{\infty} - N}{\tau_N} \quad (2)$$

where V is the membrane potential, $N \in [0, 1]$ is the activation variable for K^+ , I_{ext} is the external current and t denotes the time measured in milliseconds. The system parameters E_{Ca} , E_K and E_L represent equilibrium potential for Ca^{2+} , K^+ and leak currents, respectively, g_{Ca} , g_K and g_L denote the maximal conductances of corresponding ionic currents. The V -dependent functions, M_{∞} , N_{∞} and τ_N are given by

$$\begin{aligned} M_{\infty} &= 0.5[1 + \tanh(V - V_a)/V_b] \\ N_{\infty} &= 0.5[1 + \tanh(V - V_c)/V_d] \\ \tau_N &= 1.0/[\phi \cosh(V - V_c/2V_d)] \end{aligned} \quad (3)$$

where V_a and V_c are the midpoint potential at which the calcium current and the potassium current is halfactivated, V_b is a constant corresponding to the steepness of voltage dependence of activation, V_d denotes the slope factor of potassium activation and ϕ is the temperature-like time scale factor.

In this paper, we consider a system of chemically mutual-coupled two ML neurons. The system equation is described by

$$C \frac{dV_i}{dt} = g_{Ca} M_{\infty i} (E_{Ca} - V_i) + g_K N (E_K - V_i) + g_L (E_L - V_i) + I_{ext} + g_{syn} a_{i+1} (V_{syn} - V_i) \quad (4)$$

$$\frac{dN_i}{dt} = \frac{N_{\infty i} - N_i}{\tau_N} \quad (5)$$

$$\frac{da_i}{dt} = \frac{b_i}{\tau} \quad (6)$$

$$\frac{db_i}{dt} = -2 \frac{b_i}{\tau} - \frac{a_i}{\tau} \quad (i = 1, 2, a_3 \equiv a_1). \quad (7)$$

Note that the solution a_i in Eqs.(6) and (7) with initial condition $(a_i, b_i) = (0, 1)$ at $t = 0$ represents the α -function or $a_i(t) = (t/\tau)e^{-t/\tau}$, which is a model for describing the

time-dependent conductance of the synapse. Each vector (a_i, b_i) jumps to the constant (0, 1) at $t = t_0^i + \tau_d$ where t_0^i is the time when V_i changes to $V_i > 0$. Namely, the firing information of the neuron transforms to connected neurons with the time delay τ_d , which is mainly caused by the length of synapses.

We set the reversal potential V_{syn} as -60 for the inhibitory synapse, or 0 for the excitatory synapse. The aim of this study is to compare parameter regions of in-phase synchronization between class I and class II. The ML model can be controlled between two classes by the value of the parameter V_c . In this paper, we choose $V_c = 12$ and $V_c = 2$ for the class I and the class II neurons, respectively. The other parameter values are fixed as shown in Tab.1.

Table 1: The values of parameter in Eq. (1)-(3).

$E_{Ca} = 120[\text{mV}]$
$E_K = -80[\text{mV}]$
$E_L = -60[\text{mV}]$
$g_{Ca} = 4.0[\text{mS}/\text{cm}^2]$
$g_K = 8.0[\text{mS}/\text{cm}^2]$
$g_L = 2.0[\text{mS}/\text{cm}^2]$
$V_a = -1.2[\text{mV}]$
$V_b = 18.0[\text{mV}]$
$V_d = 17.4[\text{mV}]$
$\phi = 1/15[\text{sec}^{-1}]$
$C = 20.0[\mu\text{F}/\text{cm}^2]$
$\tau = 1[\text{msec}]$

3. Results and Discussions

In this study, we fix the value of the external current I_{ext} as 73.67 and 78.55 [$\mu\text{A}/\text{cm}^2$] for the class I and class II neurons, respectively, to obtain similar firing frequency for the single neuron. We show regions of synchronous firing in the parameter plane (τ_d, g_{syn}) in Figs.1 and 2 obtained by using Yoshinaga's method [8]. In bifurcation diagrams, black solid, red solid and black dashed curves indicate the pitchfork, the period-doubling and the Neimark-Sacker bifurcation, respectively. In blue shaded regions, we observe stable in-phase synchronous firings.

In Figs.1(a) and 2(a) (coupled by the inhibitory synapse), we observe complicated bifurcation structure, however these period-doubling, pitchfork and Neimark-Sacker bifurcations are subcritical; we cannot observe stable solutions generated by these bifurcations. For the inhibitory synapse, the appearance of bifurcations strongly depends on both values of the synaptic delay and the synaptic conductance. On the other hand, for the excitatory synapse, the bifurcation structure is simple and mainly depends on only the value of the synaptic delay.

It is known that class II neurons easily achieve synchronization [1]-[3], however our results shown in Figs.1 and

2 are opposite to previous those studies. We claim that in some parameter region the class I neurons have advantage of acquiring synchronization.

At the points $(\tau_d, g_{syn}) = (5, 10)$ in each bifurcation diagram shown in Figs.1(a), 2(a) and 2(b), we observe a non-in-phase synchronous state shown in Figs.3, 5 and 6, respectively. We observe a non-in-phase synchronous state shown in Fig.4 at the point $(\tau_d, g_{syn}) = (5, 5)$ in the bifurcation diagram (Fig.1(b)). For the inhibitory (Figs.3 and 5) and the excitatory synapse (Figs.4 and 6) two neurons are, respectively, synchronized at almost anti-phase and in-phase. This property of the excitatory and the inhibitory synapse is a common feature for mutual coupled neurons by the delayed chemical synapse.

4. Conclusion

We investigated the bifurcation structure of the class I and the class II neurons coupled by the inhibitory synapse or the excitatory synapse. We compared it between two classes, and found that the class I neurons have wider parameter regions of in-phase synchronization. In the future works, we should clarify the reason why the class I neurons synchronizes more easily than the class II neurons. In this study, we set both the raise time and the decay time as the same. however it is found that these time constants take different values [9]. Thus studying a such system is one of our further problems.

References

- [1] B. Ermentrout, M. Pascal and B. Gutkin, " The effects of spike frequency adaptation and negative feedback on the synchronization of neural oscillators ", *Neuro Comput.*, 13: 1285-1310, 2001.
- [2] B. Ermentrout, " Type I membranes, phase resetting curves, and synchrony ", *Neuro Comput.*, 8: 979-1001,1996.
- [3] J. Rinzel and B. Ermentrout, " Analysis of Neural Excitability and Oscillations, in *Methods in Neuronal Modeling* ", eds.C.Koch, and I.Segev, The MIT Press, 1998.
- [4] L.F. Lago-Fernandez, F.J. Corbacho and R.Huerta, " Connection topology dependence of synchronization of neural assemblies on class 1 and 2 excitability ", *Neural Networks*, 14: 687-696,2001.
- [5] E. Ioka and H. Kitajima, " Forced synchronization of class I and class II neurons ", *Journal of Signal Processing*, vol.11, no.11, pp.111-115, Jan. 2007.
- [6] S. Tsuji, T. Ueta, H. kawakami and K. Aihara, " Bifurcation in modified BVP neurons coupled by gap-junctions ", *Proc. NCSP'04*, 495-498, Hawaii, USA, 2004.

- [7] K. Tsumoto, H. Kitajima, T. Yoshinaga, K. Aihara and H. Kawakami, " Bifurcations in Morris-Lecar neuron model ", Neurocomputing, 69(4-6): 293-316,2006.
- [8] T. Yoshinaga, Y. Sano and H. Kawakami, " A method to calculate bifurcations in synaptically coupled Hodgkin-Huxley equations ", International Journal of Bifurcation and Chaos, 9(7): 1451-1458,1999.

- [9] T. Tateno and H.P.C. Robinson, " Rate Coding and Spike-Time Variability in Cortical Neurons With Two Types of Threshold Dynamics ", J Neurophysiol 95: 2650-2663, 2006.

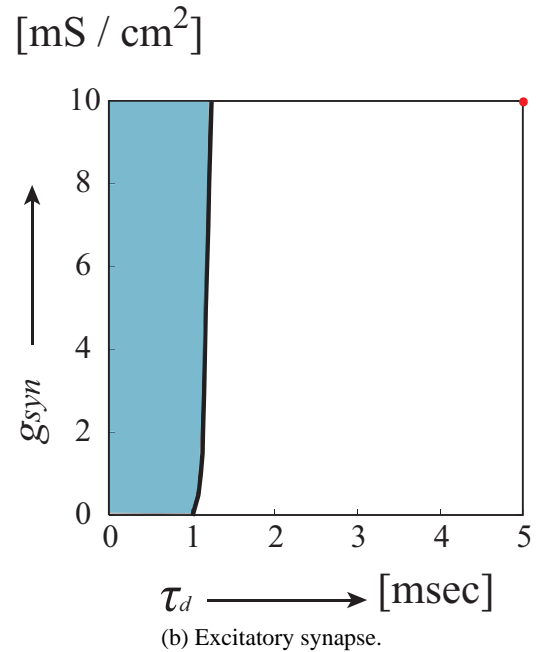
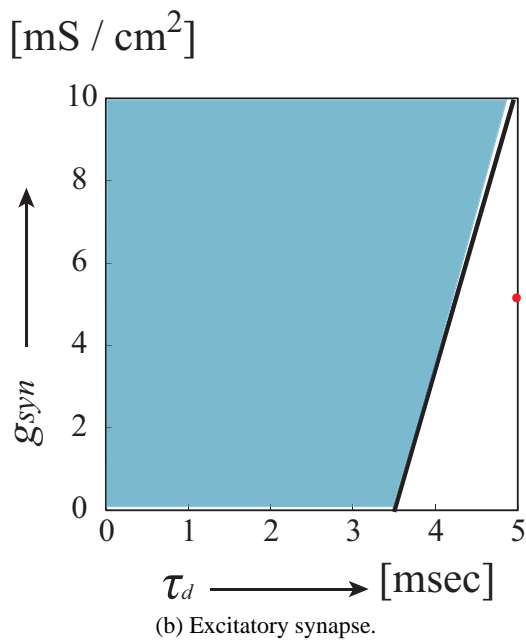
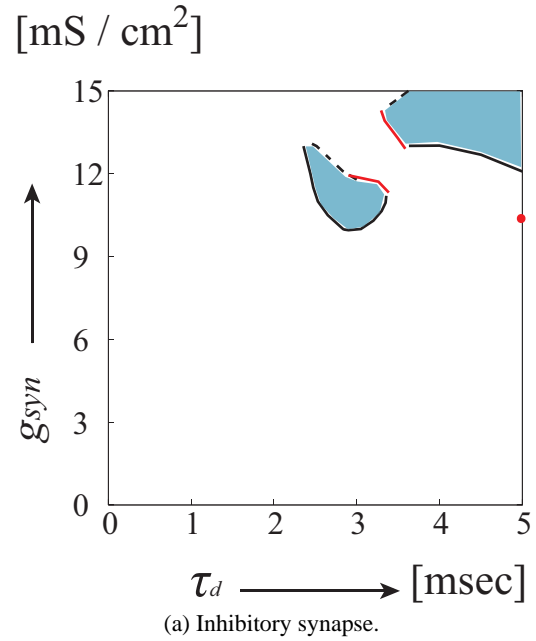
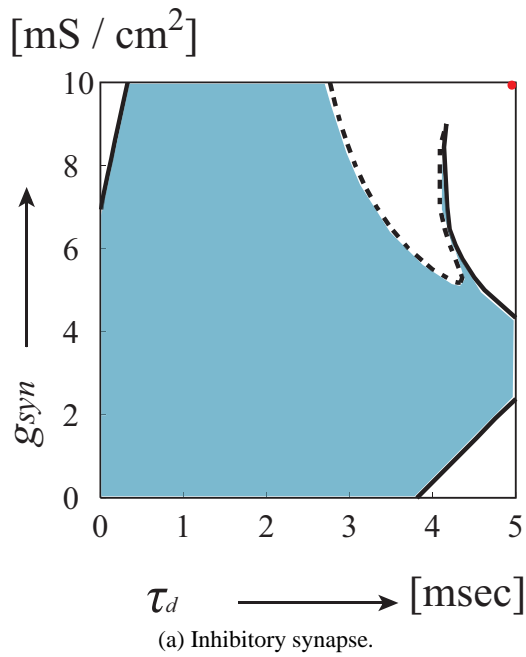


Figure 1: Bifurcation diagrams for class I.

Figure 2: Bifurcation diagrams for class II.

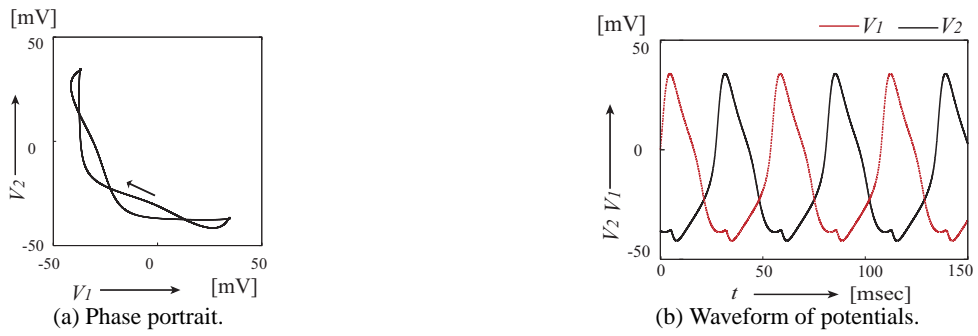


Figure 3: Stable periodic solution observed at $(\tau_d, g_{syn}) = (5, 10)$ in Fig.1(a).

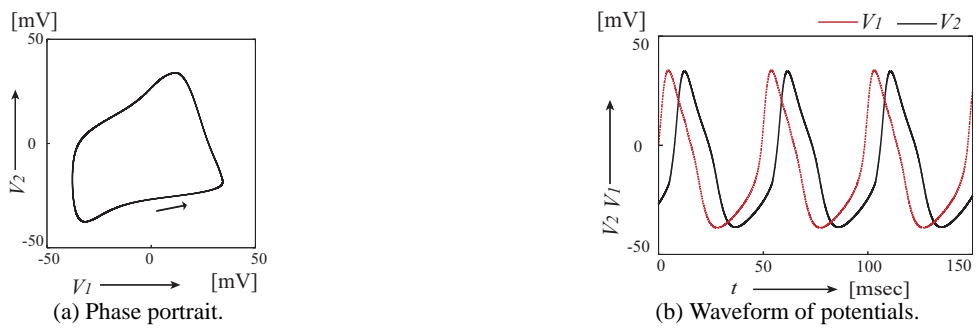


Figure 4: Stable periodic solution observed at $(\tau_d, g_{syn}) = (5, 5)$ in Fig.1(b).

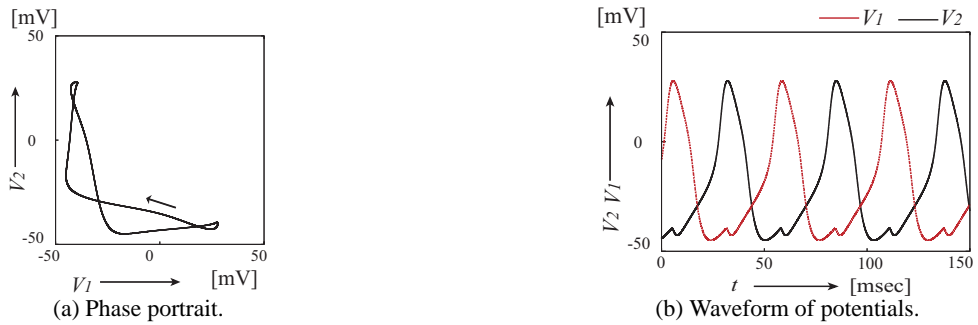


Figure 5: Stable periodic solution observed at $(\tau_d, g_{syn}) = (5, 10)$ in Fig.2(a).

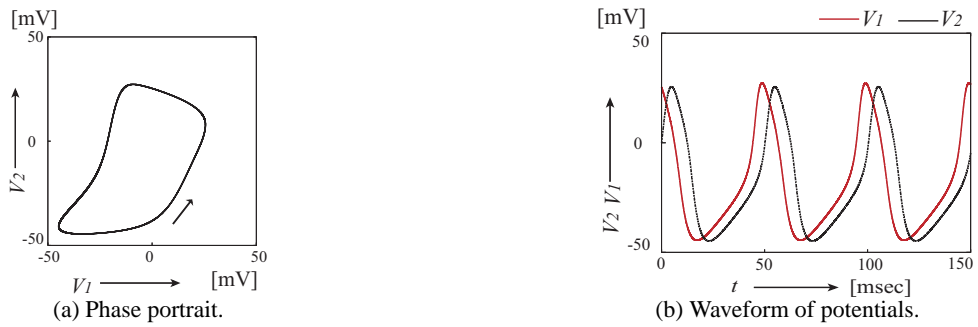


Figure 6: Stable periodic solution observed at $(\tau_d, g_{syn}) = (5, 10)$ in Fig.2(b).