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Synchronization in Chaotic Spiking Neural Networks Using An Extended CSM-Scheme

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Abstract—We report on numerical studies of synchronization phenomena in spiking neural networks with chaotic behavior. We use the 3D Hindmarsh-Rose (HR) model as a typical spiking neuron. The HR models are coupled by a simple extension of the standard CSM scheme, where *the standard CSM scheme* is known as a bio-inspired powerful synchronization scheme for limit cycles but not effective for chaotic oscillations. In this paper we investigate the coupled HR-models with *the extended CSM scheme* and show this spiking neural network with different model parameters can achieve synchronization for not only limit cycles but also for chaotic oscillations under some conditions of couplings. An analog electronic neuron is also presented for realizing the spiking neural network with the extended CSM couplings and the performance was proved by SPICE.

1. Introduction

In the past years, there has been a growing interest in the engineering community for biologically inspired problems. Among these problems, the individual and collective behaviors of neurons in nervous system have got considerable attention. Within this line of research interesting phenomena arise, from chaotic spiking bursts to their synchronization in the neural networks. Evidence is emerging that some natural systems might exploit the versatility of chaos to provide adaptability in their behavior. Some researchers suggest applications where the rich dynamics of a chaotic behavior might offer a competitive advantage over a periodic state. Synchronization of periodic and chaotic oscillations plays an important role for adjusting their signals in the spiking neural networks.

The original work of Hodgkin and Huxley provide us with a phenomenological model of action potentials of neurons. This model is however rather complex, since it involves a prescription for the dynamics of the ionic conductances underlying the voltage changes. The three-variables model established by Hindmarsh and Rose [1] is one of the most used mathematical representation of the widespread phenomenon of oscillatory burst discharges that occur in real neuronal cells. We need a powerful synchronization scheme for the coupled HR models, because the 3D-HR model is capable of behaving various types of neural activities, from periodic to chaotic bursts of spikes.

Recently a novel and biologically inspired synchronization scheme was presented [3][4]. This scheme, which was derived from a mathematical model of *the cellular slime mold Dictyostelium discoideum*, is powerful and robust for synchronization in limit cycle oscillations (we call this *standard CSM scheme* here) : It can synchronize large number of limit cycles with broad different natural frequencies and mechanically quite different types of oscillators [4]. The standard CSM scheme was simplified up to the point that there was only one coupling parameter between limit cycle oscillators. This scheme is however not applicable for synchronization in chaotic oscillations. Instead of the uniform coupling parameter in the standard CSM scheme, the coupling parameters should be allowed to take different signs and values for chaotic synchronization [7],[8]. We call this *Extended CSM (ECSM) scheme*.

In this paper using Simulink (one of Matlab tool boxes), we investigate coupled HR-models with the extended CSM scheme and show this spiking neural network with different model parameters can achieve synchronization for not only limit cycles but also chaotic oscillations under some conditions of couplings. An analog electronic neuron is also presented for realizing the spiking neural network with the extended CSM couplings using the state Controlled CNN (SC-CNN) configuration[5] and the performance was proved by SPICE.

2. The standard CSM scheme and its extension

The standard CSM scheme[4] was derived from the similarity of biological receptors and limit cycle oscillators in the system of the cellular slime mold Dictyostelium discoideum cells, and described by the coupled equations

$$\begin{aligned}\frac{dx_j}{dt} &= X_j(x_j, y_j)\\ \frac{dy_j}{dt} &= Y_j(x_j + \gamma\sum_{l=1}^{N} x_l, y_j).\end{aligned} \qquad (1)$$

where $X_j,\ Y_j,\ ...(j = 1, 2, ..., N)$ are in general nonlinear functions of state variables $x_j,\ y_j, ...(j = 1, 2, ..., N)$ and γ is an arbitrary positive number (uniform for all j) that is chosen to correspond to the sensitivity of a biological receptor. In the case of $\gamma = 0$ the system (1) stands for the isolated limit cycles.

The standard CSM scheme was found to be very robust for global synchronization of limit cycle oscillators[4]. It is hard to find the general theorem of global synchronization in the system (1), because of nonlinearities in $X_j,\ Y_j$. The qualitative explanation accounts for the robustness of this scheme may be like this: since the standard CSM scheme is a mutual synchronization method described by Eq.(1), from the equations we find that the outer signals act to destroy own regular rhythms while each cell tends to keep its characteristic as a limit cycle oscillator until they achieve synchronization adjusting each other.

The standard CSM scheme is robust and powerful for limit cycles but not for chaotic oscillations. In order to achieve synchronization of chaotic oscillations, we must allow coupling parameter γ be nonuniform for each cell, i.e. γ_j [7],[8]. In this case Eq.(1) is changed into

$$\begin{aligned} \frac{dx_j}{dt} &= X_j(x_j, y_j) \\ \frac{dy_j}{dt} &= Y_j(x_j + \sum_l \gamma_l x_l, y_j) \end{aligned} \qquad (2)$$

where γ_l can take both signs and different values depending on each cell. We call this *Extended CSM (ECSM) scheme*.

In this paper applying the ECSM to coupled chaotic Hindmarsh-Rose models, we investigate the capability of achieving synchronization and robustness for different values of model parameters and coupling parameters γ_j of the scheme.

3. Application of the extended CSM Scheme to chaotic coupled H-R Models

In this paper we apply the ECSM scheme to chaotic coupled Hindmarsh-Rose models.

3.1. The Hindmarsh-Rose model

Hindmarsh and Rose have proposed the following model of spiking-bursting neural behavior [1]

$$\begin{cases} \dot{x} = -ax^3 + bx^2 + y - z + I \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r(s(x - x_A) - z). \end{cases} \qquad (3)$$

where x represents the membrane potential, y an internal, or recovery, variable, z a slowly varying current, and I the external current. Unlike classical approaches, such as the Hodgkin Huxley model, the HR model is based on global behavior rather than any considerations of electrophysiological process. The 3D-HR model is able to exhibit chaotic oscillations as well as the other various types of neural activities that occur in real neuronal cells. Curves of bifurcation points in the parameter space (I, r) of the HR model for the other parameters fixed have been investigated in detail [2]. Figure 1 shows the various types of oscillations in the Hindmarsh-Rose model.

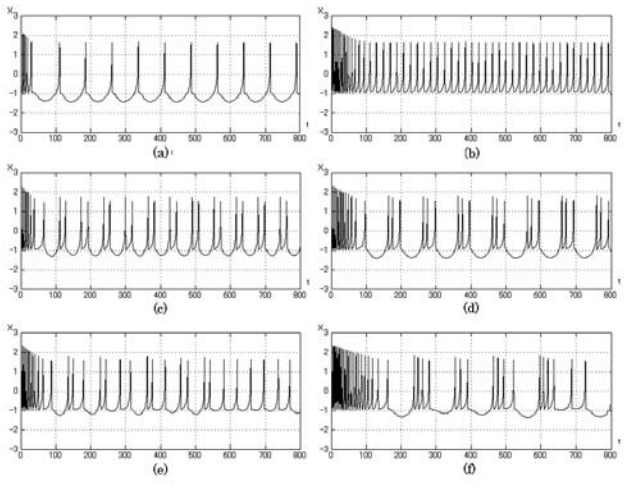


Figure 1: Various types of oscillations in the Hindmarsh-Rose model. (a)-(d): the periodic bursts of spikes, (e)(f): the chaotic bursts of spikes where (a)$I = 2, r = 0.015$ with 1 burst, (b)$I = 4, r = 0.015$ with 1 burst, (c)$I = 3, r = 0.02$ with 2 bursts, (d)$I = 2.8, r = 0.01$ with 3 bursts, (e)$I = 3.2, r = 0.014$, (f)$I = 3.1, r = 0.0065$.

3.2. Installation of the ECSM scheme in the coupled HR-models

We begin with the two coupled HR-models using the ECSM scheme described by

$$\begin{cases} \dot{x_1} = -ax_1^3 + bx_1^2 + y_1 + (\gamma_{1y_1}y_1 + \gamma_{1y_2}y_2) - z_1 - \\ \qquad (\gamma_{1z_1}z_1 + \gamma_{1z_2}z_2) + I \\ \dot{y_1} = c - dx_1^2 - y_1 \\ \dot{z_1} = r(s(x_1 - x_A) - z_1) \\ \dot{x_2} = -ax_2^3 + bx_2^2 + y_2 + (\gamma_{2y_2}y_2 + \gamma_{2y_1}y_1) - z_2 - \\ \qquad (\gamma_{2z_2}z_2 + \gamma_{2z_1}z_1) + I \\ \dot{y_2} = c - dx_2^2 - y_2 \\ \dot{z_2} = r(s(x_2 - x_A) - z_2) \end{cases} \qquad (4)$$

where the second and third variables y and z in the right hand side of the first equation in each cell are modified by adding the linear combinations of $\{y_l\}$ and $\{z_l\}$ respectively.

The system (4) can be easily extended to N-couples HR-models with the ECSM scheme where each neuronal cell has three state variables (x_j, y_j, z_j) $(j = 1, 2, ..., N)$ and y_j and z_j in the the right hand side of the first equation in each cell are modified by adding the linear combinations terms$\sum_l \gamma_l y_l$ and $\sum_l \gamma_l z_l$ respectively. In this paper for simplicity we assume $\gamma_l = \gamma_a$ for $l = j$ and $\gamma_l = \gamma_b$ for $l \neq j$ where j corresponds to the j th cell and $\gamma_a,\ \gamma_b$ are some constants.

3.3. Simulations by Simulink

In the simulations we used the following constants in Eq.(3)

$$a = 1, b = 3, c = 1, d = 5, s = 4, x_A = -1.6, I = 3.1. \qquad (5)$$

The values of the other parameters, $r,\ \gamma_a,\ \gamma_b$ were changed in each simulation to investigate feasibility of synchronization. Figure 2 shows a Simulink block diagram for one HR-model with the ECSM couplings. Figure 3(a) shows the subsystem block to the model shown in Figure 2 with input ports $y2, z2$ and output ports $x1, y1, z1$.

Figure 3(b) shows the Simulink block diagram for simulating the two coupled HR-models described by Equations (4). The time scales of the slow adaptation currents, $r_1 = 0.013,\ r_2 = 0.0065$ both were set in the chaotic regions of the parameter space [2]. We examined the properties of synchronization by the ECSM scheme for several combinations of coupling parameters as shown in Table 1. In the case of i in the table, a complete synchronization in chaotic mode was achieved. Figure 4 shows the time domain response x_1 and phase portrait x_2 vs. x_1 for this case. In the cases of ii and iii, synchronization was achieved but in the mode of limit cycle. In the cases of iv and v, synchronization was not achieved in both limit cycles and chaos. From the cases i-iii, we find that the following condition is necessary for achieving complete synchronization in the chaotic or limit cycle mode

$$1 + \gamma_a = \gamma_b. \qquad (6)$$

Table 1: Coupling parameters

case	γ_a	γ_b
i	-0.5	0.5
ii	-0.505	0.495
iii	-0.495	0.505
iv	-0.505	0.505
v	-0.495	0.495

Under this condition, the condition for achieving complete chaotic synchronization is $-\gamma_a = \gamma_b = 0.5$ (case i) for two coupled case. It is observed that in the complete synchronization state these conditions give each coupling term = average of $\{y_j\}$ and $\{z_j\}$ respectively: e.g. in Eq.(4) the term $y_1 + (\gamma_{1y1}y_1 + \gamma_{1y2}y_2) = (y_1 + y_2)/2$ and $z_1 + (\gamma_{1z1}z_1 + \gamma_{1z2}z_2) = (z_1 + z_2)/2$. Numerical observations show that these characteristics of complete synchronization in chaotic oscillations can be generalized to N-coupled HR-models, where the conditions are

$$1 + \gamma_a = \gamma_b \ \text{ and } \ \gamma_a = -(N-1)\gamma_b. \qquad (7)$$

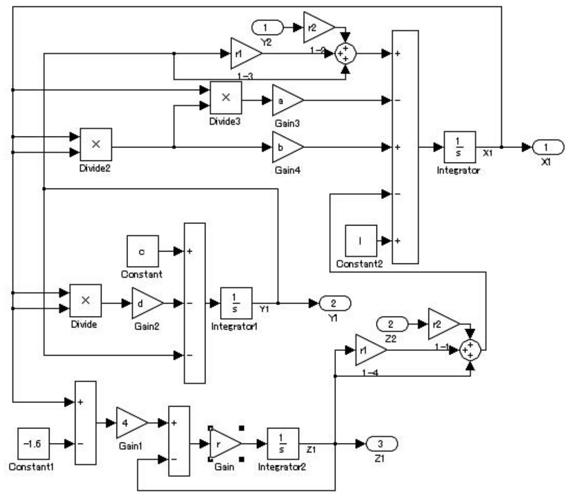
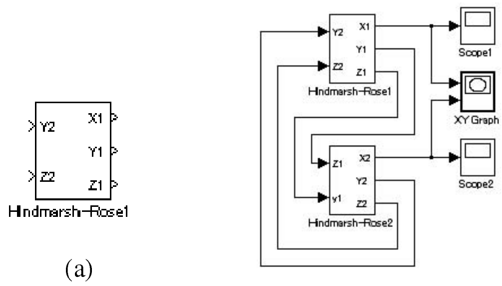


Figure 2: Simulink block diagram for the HR-model with the ECSM couplings



(a) (b)

Figure 3: (a)Subsystem block to the model shown in Figure 2, (b)Simulink block diagram for simulating the two coupled HR-models with the ECSM scheme

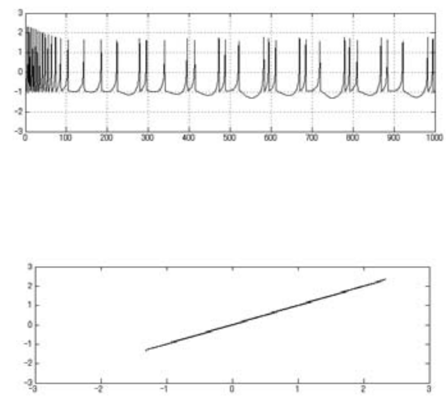


Figure 4: Time domain response x_1 and Phase portrait x_2 vs. x_1 with $\gamma_1 = -0.5$,$\gamma_2 = 0.5$

It was also observed for N=2,3,4 that under these conditions if the average values of the time scales $r_1, ..., r_N$ is in a chaotic parameter region of the isolated HR-model, then it can synchronize in a chaotic mode, and if in a limit cycle region, it can synchronize in a limit cycle mode.

In the case of Table 1 (i), $-\gamma_a = \gamma_b = 0.5$ satisfying (7) with N=2 gives complete synchronization in a chaotic oscillation. But this condition was very strict because the average of the parameter values $r_1(= 0.013)$ and $r_2(= 0.0065)$ was in a chaotic region but very near the border with limit cycles. So, if the average of r_1 and r_2 is placed not near the border, e.g. at $r_1 = 0.013$ and $r_2 = 0.015$, then $-\gamma_a = \gamma_b$ can be allowed to be 0.47 through 0.53 for achieving complete chaotic synchronization.

Figure 5 shows the four coupled HR-models for complete synchronization of chaotic oscillations with coupling parameters $\gamma_a = -3/4,\ \gamma_b = 1/4$.

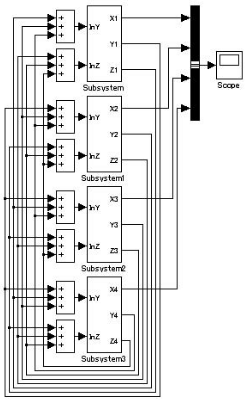


Figure 5: Simulink block diagram for simulating the four coupled HR-models with the ECSM scheme

3.4. Realization of an analog electronic neuron and its SPICE simulation

An analog electronic neuron (AEN) is presented for realizing the coupled HR-models with the ECSM scheme using the CS-CNN technique[5] and two analog multipliers AD633's (the detail will be shown in the presentation). Figure 6 shows the Spice simulation of the proposed AEN with chaotic synchronization.

4. Conclusion

We have investigated synchronization of chaotic oscillations in the coupled HR-models with the ECSM scheme. We found that the ECSM scheme can achieve synchronization of chaos as well as the limit cycles under some conditions and is robust for model parameters $\{r_l\}$ unlike the Pecora-Carroll *drive-response concept* [6].

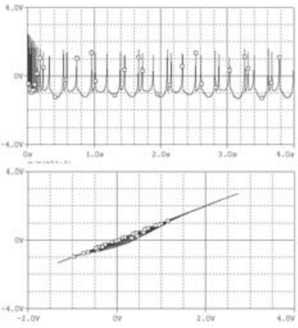


Figure 6: Time response v_1 in chaotic synchronization and Phase portrait v_2 vs. v_1 by Spice simulation of the analog electronic neuron realizing the two coupled HR-models by the ECSM scheme with $r_1 = 0.013, r2 = 0.0065, \gamma_a = -0.5, \gamma_b = 0.5$.

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