# Modified decomposition technique for polarimetric SAR image analysis

<sup>#</sup>Toshifumi Moriyama<sup>1</sup>, Hiroaki Matsushita<sup>2</sup>

<sup>1</sup> Faculty of Engineering, Nagasaki University

1-14 Bunkyo-machi, Nagasaki-shi, 852-8521 Japan, t-moriya@nagasaki-u.ac.jp<sup>2</sup> Graduate School of Marine Science and Engineering, Nagasaki University

1-14 Bunkyo-machi, Nagasaki-shi, 852-8521 Japan

## **1. Introduction**

This paper considers a decomposition technique of polarimetric synthetic aperture radar's covariance matrix which considers the azimuth rotation of each scattering component. This type technique decomposes a covariance matrix into surface, double-bounce and volume scattering components. Although the past techniques assume that the scattering components have the reflection symmetry property, there is a possibility that the urban and mountain areas do not satisfy it due to the azimuth rotation. Thus, we propose a modified decomposition technique dealing with a compensation of the rotation. It is shown that the proposed method can remove influences of rotation from the decomposition result and the polarization ratios between HH and VV in surface and double-bounce scattering components can be estimated without an assumption that one of them becomes 1 or -1.

#### 2. Three-component decomposition

The scattering matrix and covariance matrix are defined as

$$\begin{bmatrix} S(HV) \end{bmatrix} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}, \quad \langle \begin{bmatrix} C(HV) \end{bmatrix} \rangle = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & \sqrt{2} \langle S_{HH} S_{HV}^* \rangle & \langle S_{HH} S_{VV}^* \rangle \\ \sqrt{2} \langle S_{HV} S_{HH}^* \rangle & 2 \langle |S_{HV}|^2 \rangle & \sqrt{2} \langle S_{HV} S_{VV}^* \rangle \\ \langle S_{VV} S_{HH}^* \rangle & \sqrt{2} \langle S_{VV} S_{HV}^* \rangle & \langle |S_{VV}|^2 \rangle \end{bmatrix},$$
(1), (2)

where  $\langle \rangle$  denotes the ensemble average and the superscript \* denotes the complex conjugate. Due to the backscattering, it is assumed that  $S_{HV}$  is equal to  $S_{VH}$ . Freeman and Durden proposed a three-component scattering model for POLSAR image decomposition based on covariance matrix. In case of their decomposition, the covariance matrix is decomposed as

$$\left\langle \left[ C(HV) \right] \right\rangle \approx f_s \left\lfloor C \right\rfloor_{surface}^{hv} + f_d \left\lfloor C \right\rfloor_{double}^{hv} + f_v \left\lfloor C \right\rfloor_{volume}^{hv}$$
(3)

where  $f_s$ ,  $f_d$  and  $f_v$  are the expansion coefficients. It is assumed that  $[C]^{hv}_{surface}$ ,  $[C]^{hv}_{double}$  and  $[C]^{hv}_{volume}$  does not have (1,2), (2,1), (2,3) and (3,2) components due to the reflection symmetry property.

$$\begin{bmatrix} C \end{bmatrix}_{surfce}^{hv} = \begin{bmatrix} \left| \beta \right|^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta^* & 0 & 1 \end{bmatrix}, \begin{bmatrix} C \end{bmatrix}_{double}^{hv} = \begin{bmatrix} 1 & 0 & \alpha^* \\ 0 & 0 & 0 \\ \alpha & 0 & \left| \alpha \right|^2 \end{bmatrix}, \begin{bmatrix} C \end{bmatrix}_{volume}^{hv} = \frac{1}{8} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$
(4a,b,c)

The decomposition results are obtained as

$$P_{s} = f_{s} \left( 1 + |\beta|^{2} \right), P_{d} = f_{d} \left( 1 + |\alpha|^{2} \right), P_{v} = f_{v}.$$
(5,a,b,c)

### 3. Modified three-component decomposition

If the azimuth rotation which means a rotation around the radar line of sight (LOS) is considered to the covariance matrix, it is changed as

$$\left\langle \left[ C(HV(\theta)) \right] \right\rangle = \left[ U_{\theta} \right] \left\langle \left[ C(HV) \right] \right\rangle \left[ U_{\theta} \right]^{*T}, \left[ U_{\theta} \right] = \begin{bmatrix} \cos^{2}\theta & \sqrt{2}\sin\theta\cos\theta & \sin^{2}\theta \\ -\sqrt{2}\sin\theta\cos\theta & \cos2\theta & \sqrt{2}\sin\theta\cos\theta \\ \sin^{2}\theta & -\sqrt{2}\sin\theta\cos\theta & \cos^{2}\theta \end{bmatrix}$$
(6a,b)

where the superscript *T* denotes the transposition. The azimuth rotation affects the reflection symmetry property. For example,  $\langle S_{HH}S_{HV}^* \rangle$  and  $\langle S_{HV}S_{VV}^* \rangle$  in (4a) and (4b) are varied from zero as follows

a) Surface scattering case

$$\left\langle S_{HH}S_{HV}^{*}\left(\theta\right)\right\rangle = \sin\theta\cos\theta\left\{\sin^{2}\theta - \left|\beta\right|^{2}\cos^{2}\theta + \beta\cos^{2}\theta - \beta^{*}\sin^{2}\theta\right\}$$

$$\left\langle S_{HV}S_{VV}^{*}\left(\theta\right)\right\rangle = \sin\theta\cos\theta\left\{\cos^{2}\theta - \left|\beta\right|^{2}\sin^{2}\theta + \beta\sin^{2}\theta - \beta^{*}\cos^{2}\theta\right\}$$

$$(7a,b)$$

b) Double-bounce scattering case

$$\left\langle S_{HH}S_{HV}^{*}(\theta)\right\rangle = \sin\theta\cos\theta\left\{ \left|\alpha\right|^{2}\sin^{2}\theta - \cos^{2}\theta + \alpha^{*}\cos^{2}\theta - \alpha\sin^{2}\theta\right\}$$

$$\left\langle S_{HV}S_{VV}^{*}(\theta)\right\rangle = \sin\theta\cos\theta\left\{ \left|\alpha\right|^{2}\cos^{2}\theta - \sin^{2}\theta + \alpha^{*}\sin^{2}\theta - \alpha\cos^{2}\theta\right\}$$
(8a,b)

These influences appear in urban and mountain areas. For example, if a street pattern in urban area is not parallel to a direction of radar platform's orbit, a ground-wall structure is regarded to be rotated in the projection plane as shown in Fig. 1(a). Moreover, a slope area on mountain is considered that a flat area is tilted as shown in Fig. 1(b). Thus, three component scattering model proposed by Freeman and Durden can not be applied to these areas where the reflection symmetry is not satisfied. A rotation angle  $\theta$  in the projection plane that is perpendicular to range direction can be estimated. The scattering matrix rotated by  $\theta$  is expressed as

$$\begin{bmatrix} S(HV(\theta)) \end{bmatrix} = \begin{bmatrix} \tilde{S}_{HH} & \tilde{S}_{HV} \\ \tilde{S}_{HV} & \tilde{S}_{VV} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$
(9)

The elements of scattering matrix in circular polarization basis (LR) are derived the following transformation.

$$S_{LL} = \frac{1}{2} \left( \tilde{S}_{HH} - \tilde{S}_{VV} + j2\tilde{S}_{HV} \right), \quad S_{LL} = \frac{1}{2} \left( \tilde{S}_{VV} - \tilde{S}_{HH} + j2\tilde{S}_{HV} \right)$$
(10a,b)

If  $S_{HV}$  in eq. (9) is assumed to be zero,  $Arg(-S_{LL}S_{RR}^*)$  provides the rotation angle  $\theta$ .

$$Arg(-S_{LL}S_{RR}^{*})\Big|_{S_{HV}=0} = \tan^{-1}\left(-\frac{4\operatorname{Re}\left\{\left(S_{HH}-S_{VV}\right)S_{HV}^{*}\right\}}{\left|S_{HH}-S_{VV}\right|^{2}-4\left|S_{HV}\right|^{2}}\right) - 4\theta\Big|_{S_{HV}=0} = -4\theta \qquad (11)$$

Thus, an approximated rotation angle  $\theta$  in pixel of image can be estimated from the measured POLSAR data and inverse rotation (- $\theta$ ) can be done by eq. (6). The measured data which has a rotation angle is expressed as  $\langle [C(HV(\theta))] \rangle$ . The data after turning  $\langle [C(HV(\theta))] \rangle$  to  $-\theta$  is denoted as  $\langle [C(HV(0))] \rangle$ . Then a difference between  $\langle [C(HV(\theta))] \rangle$  and  $\langle [C(HV(0))] \rangle$  is calculated. In the case where the double-bounce scattering component is mainly rotated, the difference is derived as follows

$$\begin{split} \left\{ \begin{bmatrix} C(\theta) \end{bmatrix}^{h\nu} - \begin{bmatrix} C(0) \end{bmatrix}^{h\nu} \right\}_{double} &: \\ \Delta C_{11} &= f_d \left\{ (\cos^4 \theta - 1) + 2\operatorname{Re}(\alpha) \sin^2 \theta \cos^2 \theta + |\alpha|^2 \sin^4 \theta \right\} \\ &- f_s \left\{ |\beta|^2 (\cos^4 \theta - 1) + 2\operatorname{Re}(\beta) \sin^2 \theta \cos^2 \theta + \sin^4 \theta \right\} \\ \Delta C_{33} &= f_d \left\{ \sin^4 \theta + 2\operatorname{Re}(\alpha) \sin^2 \theta \cos^2 \theta + |\alpha|^2 (\cos^4 \theta - 1) \right\} \\ &- f_s \left\{ |\beta|^2 \sin^4 \theta + 2\operatorname{Re}(\beta) \sin^2 \theta \cos^2 \theta + (\cos^4 \theta - 1) \right\} \\ \Delta C_{22} &= 2f_d \left( 1 + |\alpha|^2 - 2\operatorname{Re}(\alpha) \right) \sin^2 \theta \cos^2 \theta - 2f_s \left( 1 + |\beta|^2 - 2\operatorname{Re}(\beta) ) \sin^2 \theta \cos^2 \theta \right] \end{split}$$

$$\Delta C_{13} = f_d \left\{ |\alpha|^2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta + \alpha^* (\cos^4 \theta - 1) + \alpha \sin^4 \theta \right\} - f_d \left\{ |\beta|^2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta + \beta (\cos^4 \theta - 1) + \beta^* \sin^4 \theta \right\} \Delta C_{12} = \sqrt{2} f_d \sin \theta \cos \theta \left\{ |\alpha|^2 \sin^2 \theta - \cos^2 + \alpha^* \cos^2 \theta - \alpha \sin^2 \theta \right\} + \sqrt{2} f_s \sin \theta \cos \theta \left\{ \sin^2 \theta - |\beta|^2 \cos^2 + \beta \cos^2 \theta - \beta^* \sin^2 \theta \right\} \Delta C_{23} = \sqrt{2} f_d \sin \theta \cos \theta \left\{ |\alpha|^2 \cos^2 \theta - \sin^2 + \alpha^* \sin^2 \theta - \alpha \cos^2 \theta \right\} + \sqrt{2} f_s \sin \theta \cos \theta \left\{ \cos^2 \theta - |\beta|^2 \sin^2 + \beta \sin^2 \theta - \beta^* \cos^2 \theta \right\}.$$
(12)

Similarly, we can derive the difference in the case where the surface scattering component is mainly rotated. The unknown parameters in eq.(12) are  $\alpha$ ,  $\beta$ ,  $f_s$ , and  $f_d$ . If these parameters are estimated, the power contributions of surface, double-bounce, and volume scatterings are derived as

$$P_{s} = f_{s} \left( 1 + \left| \beta \right|^{2} \right), \quad P_{d} = f_{d} \left( 1 + \left| \alpha \right|^{2} \right), \quad P_{v} = Total \ Power - P_{s} - P_{d} ,$$
  

$$Total \ Power = \left\langle \left| S_{HH} \right|^{2} \right\rangle + 2 \left\langle \left| S_{HV} \right|^{2} \right\rangle + \left\langle \left| S_{VV} \right|^{2} \right\rangle.$$
(13a,b,c,d)

In order to estimate the unknown parameters, we use a particle swarm optimization (PSO). PSO is used to find an approximated global optimal solution to an optimization problem and has been shown to be useful for optimization about a multidimensional problem in various applications. A swarm is modelled by particles in multidimensional search space. These particles have a position and a velocity and move in the search space due to two essential reasoning capabilities which are related to their own best position and the best position in the swarm. Particles can communicate best positions to each other and adjust their own position and velocity based on these good positions. The velocity v and position x of nth particle are defined as:

$$v_{k+1}^{n} = \omega_{k} v_{k}^{n} + C_{1} r_{1} \left( p_{k}^{n} - x_{k}^{n} \right) + C_{2} r_{2} \left( g_{k} - x_{k}^{n} \right)$$
  
$$x_{k+1}^{n} = x_{k}^{n} + v_{k+1}^{n}$$
(14a,b)

where  $\omega$  is an inertial weight,  $C_1$  and  $C_2$  are an acceleration coefficient,  $r_1$  and  $r_2$  are a random variable. p is the best position in each particle and g is the best position of the swarm. k is an iteration index. In the optimization, the position and velocity of each particle is adjusted to minimize or maximize a fitness of objective function.

#### 4. Experimental results

In order to confirm the proposed decomposition method, the phased array type L-band synthetic aperture radar (PALSAR) data was used. PALSAR is one of the sensors loaded on the advanced land observing satellite (ALOS). The area selected for analysis is Nagasaki. A sample of covariance matrix in urban area from the measured data is shown as

	91015910000	-31067040000 + j 11393350000	58554320000 + j 18219650000	(15)
$\langle [C(HV)] \rangle =$	-31067040000 - j 11393350000	79988990000	15411910000 - j 14286680000	.(15)
	58554320000 - j 18219650000	15411910000 + j 14286680000	78731890000	

The data was made by 16 × 4 spatial ensemble averaging. The rotation angle of eq. (15) was estimated to be 32.3 degrees by eq. (11). To provide initial swarm, each unknown parameter was chosen as  $0 \le |\alpha| \le 2$ ,  $90^{\circ} \le \operatorname{Arg}(\alpha) \le 270^{\circ}$ ,  $0 \le |\beta| \le 1$ ,  $-90^{\circ} \le \operatorname{Arg}(\beta) \le 90^{\circ}$ . The estimated result is shown in Table 1. Thus, it was confirmed that the effect of rotation is compensated and a volume scattering is not dominant scattering mechanism in urban area. Since a radar cross section (RCS) of man-made target is approximated by a sum of single, double and triple bounce scattering contributions using Physical optics, it can be supposed that the dominant scattering mechanism in urban areas are surface and double-bounce scatterings. The result of proposed decomposition method corresponds to the scattering mechanism based on RCS calculation. Moreover,  $\alpha$  and  $\beta$  can be estimated without an assumption that one of them becomes 1 or -1.

## **5.** Conclusion

We proposed a decomposition technique dealing with a compensation of rotation in this paper. The results showed that the influences of rotation can be removed from the decomposition results and the polarization ratios between HH and VV in surface and double-bounce scattering components can be estimated without an assumption that one of them becomes 1 or -1.

## Acknowledgments

This work was supported by Grant-in-Aid for Young Scientists (B) (22760301).



#### (b) Mountain area

Figure 1: Effect of rotation in urban area and mountain area

Estimated	Estimated method			
parameters	Freeman & Durden	Proposed method		
Ps	0.0	1.47e+11		
Pd	0.0	9.28e+10		
Pv	2.49e+11	1.03e+10		
α	0.0	0.76∠209.4°		
β	0.0	1.00∠7.9°		

Table 1: D	ecomposition	results by	v two method	S
	•••••••••••••••		,	~

## References

- A. Freeman and S. L. Durden, "A three-component scattering model for polarimetric SAR data," IEEE Trans. Geoscience Remote Sensing, vol.36, no.3, pp.963-973, 1998.
- [2] Y. Yamaguchi, T. Moriyama, M. Ishido, and H. Yamada, "Four-component scattering model for polarimetric SAR image decomposition," IEEE Trans. Geosci. Remote Sens., vol. 43, no. 8, pp. 1699–1706, Aug. 2005.
- [3] P Rocca, M Benedetti, M donelli, D Franceschini and A Massa, "Evolutionary optimizatin as applied to inverse scattering problems", Inverse Problems, vol.25, Nov. 2009.