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Synchronization Phenomena in Hysteresis Associative Memory

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Abstract—This article observes a synchronization phenomena in a hysteresis oscillatory associative memory that consists with a piecewise linear hysteresis neuron. The system is one of dynamical associative memories that memorizes a desired pattern as a oscillatory state. When some desired patterns are stored in the system, the system exhibits that the output wanders between some desired patterns. However, if each desired memory is orthogonal vector, the system exhibits a simple periodic output sequence.

1. Introduction

Recently, a system that can treat dynamical information, is attracted to great attention[2]-[5]. The reason why such system receives great attention, is that such dynamical information processing function can be found in biological neural networks. Especially, we think that synchronization phenomenon plays an important role for signal processing in brain.

In our previous works, we proposed Simple Hysteresis Network (abbr. SHN)[7]. The SHN has a quite simple structure, namely, all connection has uniform value. We clarified that even such simple system exhibits periodic attractors and chaotic attractors. The SHN, however, cannot generate desired output sequence. In order to control to generate a desired output sequence, we introduce a dynamical associative memory. The each information in the dynamical associative memory is represented by phase different. Our proposed system can be regarded as a coupled oscillators system. We can observe various kinds of interesting phenomena from coupled oscillators system. In order to use such rich phenomena as information, we consider the dynamical associative memory. If the rich phenomena are corresponded to the information, we can develop a novel information processing system.

2. Hysteresis Neuron

First, we consider a hysteresis neuron[8][9]. The objective hysteresis neuron is described as

$$\begin{cases} \dfrac{dx(t)}{dt} = -x(t) + e(y(t)), \\ y(t) = h(x(t)) = \begin{cases} +1 & \text{for } x > -1, \\ -1 & \text{for } x < +1, \end{cases} \end{cases} \quad (1)$$

where $x(t) \in \Re$ is a state variable, $y(t) \in \{+1, -1\}$ is an output, and $e(y(t)) \in \Re$ corresponds to a feedback parameter. $h(x(t))$ is a bipolar piecewise linear hysteresis as shown in Figure 1.

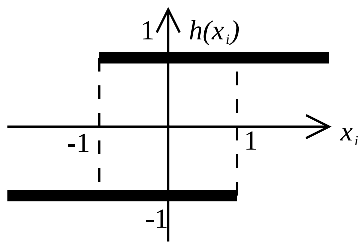


Figure 1: Normalized bipolar hysteresis

The state variable of (1) varies toward its equilibrium that denotes $e(y(t))$. Since the equilibrium point depends on its output, the equilibrium point becomes a constant while the output does not change. If the equilibrium point does not exist on the hysteresis branch, the trajectory reaches the threshold of the hysteresis before it converges to the equilibrium point. When the trajectory hits the threshold of the hysteresis, the output changes its sign, and the corresponding equilibrium point changes too. This hysteresis neuron can be regarded as a relaxation oscillator whose oscillation frequency can be controlled in the feedback parameter.

Here, we consider the case where some hysteresis neurons are coupled as

$$\tau_i \frac{dx_i(t)}{dt} = -x_i(t) + \sum_{j=1}^{N} w_{ij} y_j(t), \quad (2)$$

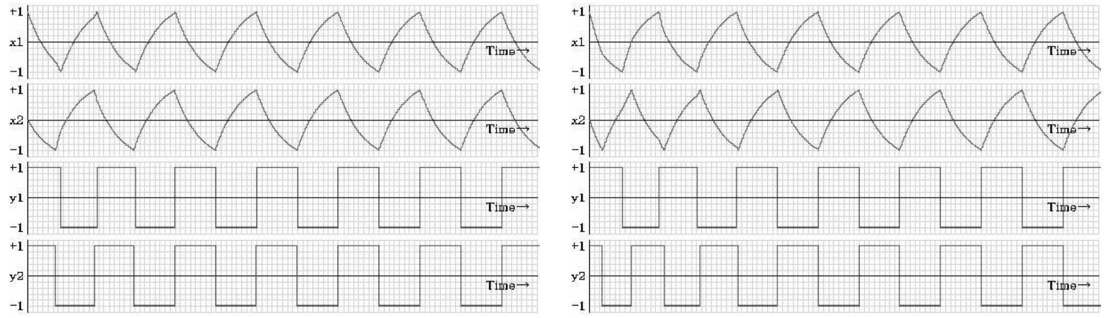
where $\tau_i > 0$ is a time constant, and N is the number of neurons. $w_{ij} \in \Re (i \neq j)$ is a coupling coefficient, w_{ii} is a self feedback parameter. For simplicity, we consider all time constants have the same value, hereafter.

The equilibrium point $p_i(t)$ is given from Eqn.(2) by

$$p_i(t) = \sum_{j=1}^{N} w_{ij} y_j(t). \quad (3)$$

If

$$p_i(t) y_i(t) \leq 0, \text{ for some} si \quad (4)$$



(a) in-phase synchronization (b) opposite-phase synchronization

Figure 2: The wave form from two hysteresis neuron system. The wave form of (a) is observed in the case where the system has an excitatory connection, and (b) is observed in the case where the system has an inhibitory connection.

is satisfied, a stable output vector does not exist in the system and the output must oscillate[1].

We suppose that the connection coefficient has the following relation:

$$\begin{cases} |w_{ii}| > \displaystyle\sum_{j=1, j\neq i}^{N} |w_{ij}|, \\ w_{ii} + \sum_{j=1, j\neq i}^{N} |w_{ij}| < -1, \forall i. \end{cases} \tag{5}$$

Namely, the connection coefficient matrix has diagonally dominant. If the system adopts such assumption (5), the condition (4) is satisfied. Therefore, this system must exhibit oscillatory state.

The fundamental frequency is influenced by the feedback parameter. The coupling parameters play a role as frequency modulation. Figure 2 shows typical wave forms from the two neurons system. In this figure, the upper two wave forms correspond to the state variable, x_1 and x_2, respectively. The lower two wave forms correspond to the output, y_1 and y_2, respectively.

Both results are observed into the following initial configurations. The initial value is adjusted as

$$\begin{array}{c} (x_1(0), y_1(0)) = (+1, +1) \\ (x_2(0), y_2(0)) = (0, +1) \end{array} \tag{6}$$

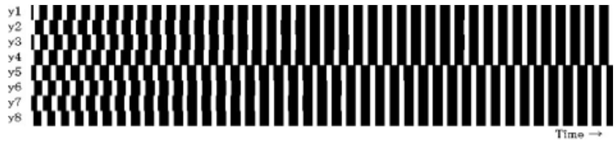
In the case of Figure 2(a), the parameters configuration is

$$\begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 1 & -4 \end{pmatrix}. \tag{7}$$

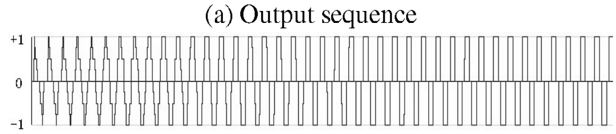
In this case, the cross-connection coefficients have a positive value. Namely, these coefficients can be regarded as excitatory connections. In this case, the system exhibits in-phase synchronization. On the other hand, the parameters configuration of Figure 2(b) is

$$\begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ -1 & -4 \end{pmatrix}. \tag{8}$$

In this case, the cross-connection coefficients have a negative connection coefficient, that can be regarded as an inhibitory connection. In this case, the system exhibits opposite-phase synchronization.



(a) Output sequence



(b) Direction cosine

Figure 3: The behavior of the oscillatory associative memory which stores only one desired pattern, such that $(+1, +1, +1, +1, -1, -1, -1, -1)$.

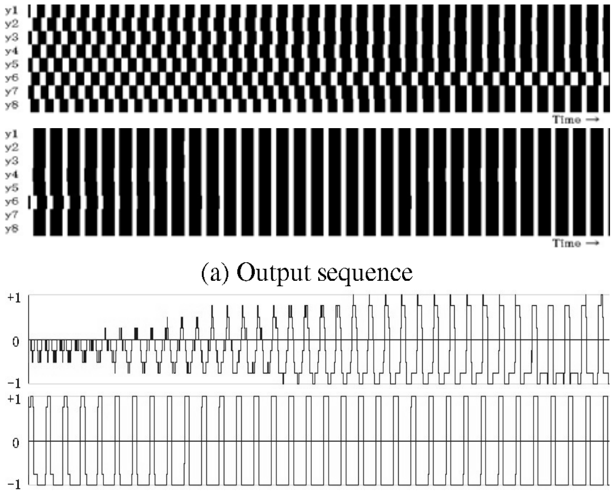
3. Dynamical Associative Memory

In this section, we propose a dynamical associative memory whose output exhibits oscillatory state. By using a Hebbian rule, the connection coefficients are determined as

$$W_{ij} = \frac{1}{m} \sum_{k=1}^{m} S_i^k S_j^k, \tag{9}$$

where $S_i^k \in \{+1, -1\}$ is i-th element of k-th desired pattern, and m denotes the number of desired patterns.

The network can store and retrieve oscillatory output sequence. We demonstrate the behavior of output when only one desired pattern is stored in the system which contains eight hysteresis neurons. First, we select a desired pattern as $\boldsymbol{S} \equiv (+1, +1, +1, +1, -1, -1, -1, -1)$. In this case, the system exhibits the output sequence as shown in Figure 3. Figure 3(a) shows the output sequence which generates from the system. The horizontal axis denotes time evolution, and the vertical axis denotes the output of each neuron. White pixel corresponds to the negative output, and black pixel corresponds to the negative output. The initial value of the output vector is given as $\boldsymbol{y}(0) \equiv (+1, -1, +1, -1, +1, -1, +1, -1)$. Figure 3(b) shows



(a) Output sequence

(b) Direction cosine

Figure 4: The behavior of the oscillatory associative memory which stores only one desired pattern, such that $(+1, +1, +1, +1, +1, +1, +1, +1)$.

a direction cosine between the desired pattern and the output vector such as

$$direction\ cosine = \frac{\boldsymbol{S} \cdot \boldsymbol{y}}{|\boldsymbol{S}||\boldsymbol{y}|}. \qquad (10)$$

If the direction cosine indicates $+1$ or -1, the system retrieves the desired pattern. In this case, the system retrieves the output sequence which consists with the desired pattern.

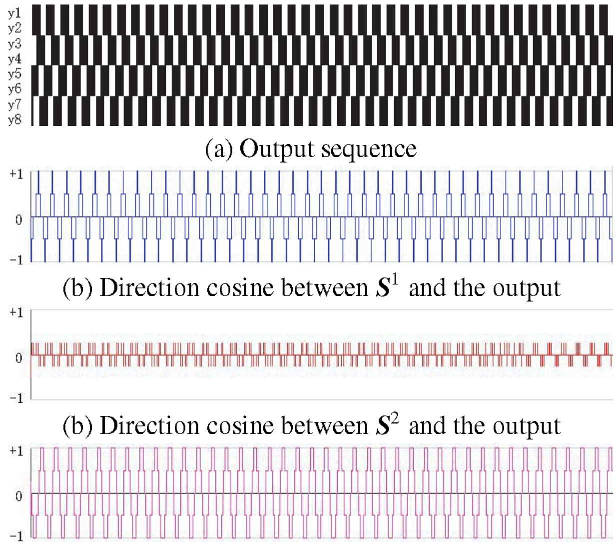
Next, we demonstrate another example, and the behavior shows in Figure 4. The desired pattern is selected as $\boldsymbol{S} \equiv (+1, +1, +1, +1, +1, +1, +1, +1)$. In this case, the transition time is longer than the case of Figure 4.

Finally, we examine two case where the system contains some kinds of desired patterns. First case is that the desired memories have an orthogonal relation. Figure 5 shows the output sequence from eight neurons system with three kinds of desired patterns. The three desired patterns are as follows:

$$\boldsymbol{S}^1 \equiv (+1, +1, +1, +1, -1, -1, -1, -1)$$
$$\boldsymbol{S}^2 \equiv (-1, +1, -1, +1, -1, +1, -1, +1)$$
$$\boldsymbol{S}^3 \equiv (-1, -1, +1, +1, -1, -1, +1, +1)$$

	$\boldsymbol{S}^1$	$\boldsymbol{S}^2$	$\boldsymbol{S}^3$
$\boldsymbol{S}^1$	-	4	4
$\boldsymbol{S}^2$	4	-	4
$\boldsymbol{S}^2$	4	4	-

Table 1: The hamming distance between each desired memory.



(a) Output sequence

(b) Direction cosine between $\boldsymbol{S}^1$ and the output

(b) Direction cosine between $\boldsymbol{S}^2$ and the output

(b) Direction cosine between $\boldsymbol{S}^3$ and the output

Figure 5: The behavior of the oscillatory associative memory which stores three desired patterns. Each stored desired memory is orthogonal vector.

The hamming distance between each desired memory is shown in Table 1. Namely, each desired memory is orthogonal vector. In this case, the system converges to a periodic output sequence which contains two desired memories $\boldsymbol{S}^1$ and $\boldsymbol{S}^3$. Note that this output sequence does not contain the desired memory $\boldsymbol{S}^2$ In order to confirm this phenomenon, we carry out such numerical simulation 100 times. In each simulation, three desired memories are selected randomly, and all desired memories satisfy an orthogonal relation. Table.2 shows each rate. As a result, the output sequence contains all desired memories is 48% in 100 trial.

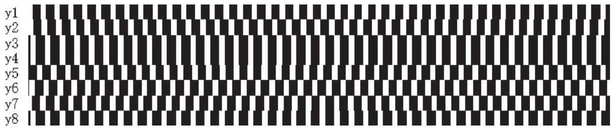
Next, we show another result. The three desired patterns are as follows:

$$\boldsymbol{S}^1 \equiv (-1, +1, +1, +1, +1, -1, -1, +1)$$
$$\boldsymbol{S}^2 \equiv (-1, +1, +1, +1, -1, +1, -1, +1)$$
$$\boldsymbol{S}^3 \equiv (-1, -1, +1, +1, -1, -1, +1, +1)$$

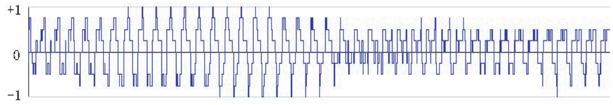
In this case, the hamming distance between each desired memory is shown in Table 3. In this case, the system exhibits that the output wanders in desired patterns $\boldsymbol{S}^1$, $\boldsymbol{S}^2$ and $\boldsymbol{S}^3$ in turn. When the desired memories do not have orthogonal relation, the system exhibits wondering motion.

All desire memories are contained	48%
2 desired memories are contained	41%
Only 1 desired memory is contained	11%
No desired memory is contained	0%

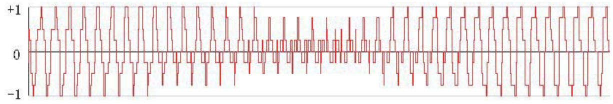
Table 2: The retriving rate of the desired memories in the output sequence.



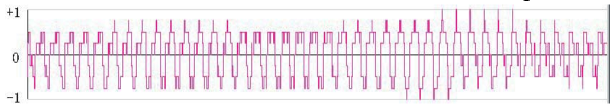
(a) Output sequence



(b) Direction cosine between S^1 and the output



(b) Direction cosine between S^2 and the output



(b) Direction cosine between S^3 and the output

Figure 6: The behavior of the oscillatory associative memory which stores three desired patterns.

In order to confirm this phenomenon, we carry out such numerical simulation 100 times. The desired memories of each simulation are three, and all desired memories do not have an orthogonal relation. Table.4 shows each rate. As a result, almost all output sequences contain all desired memories with wondering motion.

4. Conclusions

In this article, we observed the synchronization phenomena from the oscillatory associative memory. The proposed dynamical associative memory consists of the hysteresis neuron which can be regarded as a relaxation oscillator. First, we clarified that the system exhibits in-phase synchronization oscillation when the cross-connection coefficient denotes excitatory connection. Also, the system exhibits opposite-phase synchronization when the cross-connection coefficient denotes inhibitory connection. Based on this results, we proposed the oscillatory associative memory. The oscillatory associative memory exhibits an interesting wondering motion which is depended on the kind of desired memories. The theoretical analysis is one of our future problems.

	S^1	S^2	S^3
S^1	-	2	3
S^2	2	-	3
S^2	3	3	-

Table 3: The hamming distance between each desired memory.

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All desire memories are contained	93%
2 desired memories are contained	0%
Only 1 desired memory is contained	7%
No desired memory is contained	0%

Table 4: The retriving rate of the desired memories in the output sequence.