2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Synchronization Phenomena of Coupled Colpitts Oscillators using Printed Spiral Inductors

Masafumi NUKUSHINA†, Takashi KUNIHIRO†, Tatsuya NANKO†,
Masayuki YAMAUCHI† and Mamoru TANAKA‡

†Department of Electronics and Computer Engineering, Hiroshima Institute of Technology
2-1-1, Miyake, Saeki-ku, Hiroshima 731-5193, JAPAN
‡Deptartment of Electrical and Electronics Engineering, Sophia University
7-1, Kioi-cho, Chiyoda-ku, Tokyo 102-8554, JAPAN
Email: nukushina@gmail.com

Abstract—We make a printed spiral inductor (PS-Inductor) on a board. Recently, we developed a calculating method and an actual measurement method of inductance of PS-Inductors. In this paper, we measured the mutual inductance between two printed spiral inductors using these methods. Further, we observed synchronization phenomena from the circuit experiments in a system coupled by the magnetic fields between two PS-Inductors of two Colpitts-oscillators.

1. Introduction

A lot of synchronization phenomena are analyzed in many fields of science.[1]-[3] Some kind of synchronization phenomena attract the attention of many researchers in many scientific fields. If oscillators are coupled as a system, the synchronization phenomena can be often observed in the system. In biology, synchronization phenomenon of a lot of neurons by pulse signals is observed, and a lot of pacemaker cells which are included in a heart are synchronized. In the field of electrical engineering, synchronization phenomena are used for communications systems. Especially, one of the RFID system communicates using the synchronization phenomenon caused by a magnetic field between a card reader and noncontact IC card.[4] This system uses spiral antennas. Recently, we created a spiral inductor on a printed wiring board and named it a Printed Spiral Inductor (PS-Inductor). An actual measurement method and a simulation method of inductance of PS-Inductor are developed[5]. The inductance is actually measured by making a Colpitts-oscillator with the PS-Inductor, arbitrary capacitors and arbitrary registers. The inductance of the inductor can be calculated by using oscillation frequency, arbitrary capacitance and arbitrary registance. The simulator calculates the magnetic field existing in inside the conductors using finite element method. And the inductance is obtained.

In this paper, a new simulator and a new actual measurement method for calculating mutual inductance, which is generated between PS-Inductors, are developed. Next, two Colpitts-oscillators are coupled by mutual inductance between two PS-Inductors in each oscillator.

The new simulator which is updated from our previous simulator for mutual inductance between two PS-Inductors. The mutual inductance and each inductor are measured using the new simulator and previous actual measurement method.

The synchronization phenomena which can be observed in the system are investigated. 1. When a distance between two PS-Inductors is changed, synchronization phenomena are investigated. 2. When an angle of overlap between two PS-Inductors is changed, synchronization phenomena are investigated.

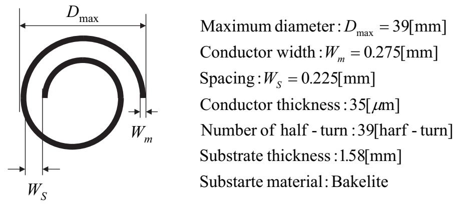


Figure 1: Specification of printed spiral inductor.

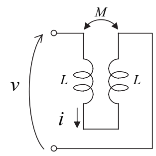


Figure 2: Mutual inductance.

2. Measurement

2.1. Single PS-Inductor

PS-Inductors are created by combining semicircular patterns. In this study, only one pattern of PS-Inductor is created as shown in Fig.1. The PS-Inductor is measured by our previous methods which are actual measurement and simulation[5]. Inductance of the PS-Inductor is around 14.84[μH], which is a result of actual measurement, and around 14.12[μH], which is a result of the simulator.

2.2. Measurement of mutual inductance

We develop an actual measurement method and calculation method for mutual inductance.

2.2.1. *Actual measurement*

Two PS-Inductors is created. One PS-Inductor overlaps the other one. The PS-Inductors is coupled in same turn direction or inverse turn direction. And a Colpitts-oscillator is created using coupled PS-Inductors as one inductor L_{com}. The L_{com} is calculated as follows(see Fig.2):
<when same turn direction>

$$v = L\frac{di}{dt} + M\frac{di}{dt} + L\frac{di}{dt} + M\frac{di}{dt} = 2(L + M)\frac{di}{dt} \qquad (1)$$

$$L_{com} = 2(L + M) \qquad (2)$$

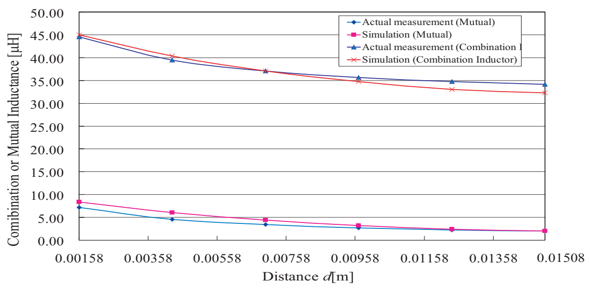


Figure 3: Results of actual measurement and simulation.

<when inverse turn direction>

$$v = L\frac{di}{dt} - M\frac{di}{dt} + L\frac{di}{dt} - M\frac{di}{dt} = 2(L - M)\frac{di}{dt} \quad (3)$$

$$L_{com} = 2(L - M). \quad (4)$$

Therefore, mutual inducatance can be calculated as follows:

$$M = \left| \frac{L_{com}}{2} - L \right|. \quad (5)$$

The combination inductance L_{com} is measured using the previous method. The mutual inductance M is obtained using the L_{com} and Eq.5.

The results of actual measurement of L_{com} are shown as Fig.3.

2.2.2. *Simulation of mutual inductance*

The calculation method of the new simulator is changed to three dimensional calculation from previous simulator which is composed of two dimensional calculation. The new simulator calculates vertical components of magnetic field, which secondary PS-Inductor generates, inside space of conductors of primary PS-Inductor. And the new simulator calculates the sum of the magnetic field from firstly and secondary PS-Inductors and obtains the sum of a base inductance and a mutual inductance($L + M$). The mutual inductance is calculated by subtracting a calculated result of single inductor by previous method from the calculated result($L + M$).

The results of actual measurement of L_{com} are shown as Fig.3.

2.2.3. *Comparison between actual measurement and simulation*

A maximum error between actual measurement and simulation results of combination inductances is around 9.01%. A minimum error is around 0.00781%. An average error is around 1.60%. A maximum error between actual measurement and simulation results of mutual inductances is around 24.4%. A minimum error is around 5.91%. An average error is around 13.6%. Basically, the errors of mutual inductances is small along with distance between primary and secondary PS-Inductors. Because the errors of mutual inductances are influenced by the errors of the measurement results of the single inductance and the combination inductance, the errors of mutual inductances become large value.

3. Synchronization Phenomena

Two Colpitts-oscillators are coupled by mutual inductance between two PS-Inductors in each oscillator(see Fig.4). Each element of two Colpitts-oscillators uses almost the same value. The errors are under the 3%. Phase states between output voltages (v_1 and v_2) are investigated.

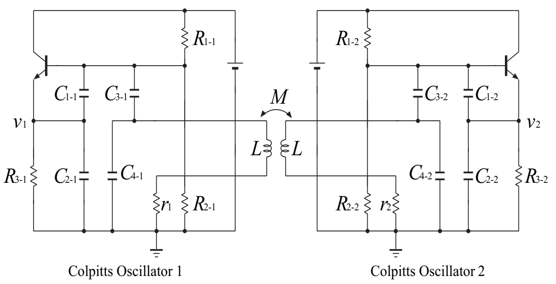


Figure 4: Circuit Model ($R_{1-1} \simeq R_{1-2} \simeq 4.35[\text{k}\Omega]$, $R_{2-1} \simeq R_{2-2} \simeq 9.10[\text{k}\Omega]$, $R_{3-1} \simeq R_{3-2} \simeq 270[\Omega], C_{1-1} \simeq C_{1-2} \simeq 127[\text{pF}], C_{2-1} \simeq C_{2-2} \simeq 438[\text{pF}], C_{3-1} \simeq C_{3-2} \simeq 113[\text{pF}]$ *and* $C_{4-1} \simeq C_{4-2} \simeq 21.5[\text{pF}]$). Each r_1 and r_2 expresses a registor included in each PS-Inductor.

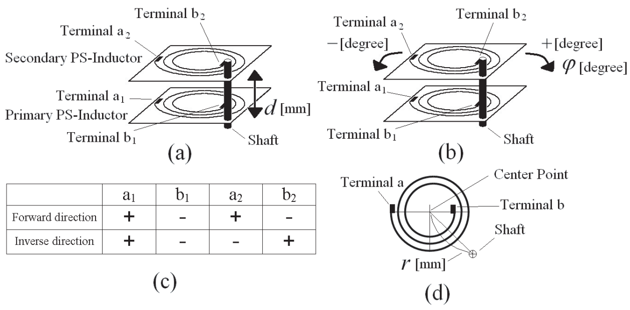


Figure 5: Coupling type of PS-Inductors.

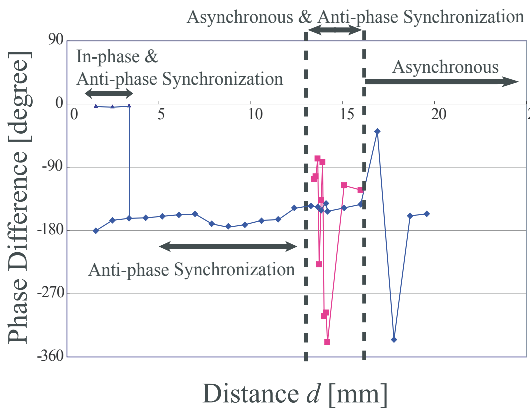


Figure 6: Trajectory of phase difference by changing distance with forward direction.

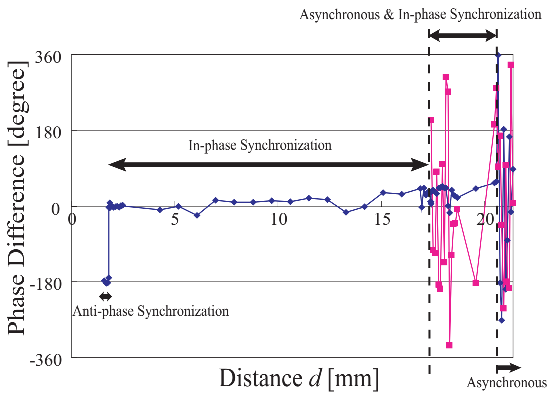


Figure 7: Trajectory of phase difference by changing distance with inverse direction.

3.1. Synchronization phenomena depending on distance d

One PS-Inductor is overlapped the other one in the same turn direction as shown in Fig.5(a). A state coupled in inverse direction between PS-Inductors is made by replacing positive terminal and negative terminal(see Fig.5(c)).

When a distance d between two PS-Inductors is changed, synchronization phenomena are investigated. Intervals of changing the distances are fixed as 0.01[mm]. Minimum distance is 1.58[mm] which is the thickness of substrate.

3.1.1. Forward direction

When terminal a_1 and terminal b_1 are same pole(see Fig.5), anti-phase synchronization and asynchronous state can be observed. Relations between distance d and phase difference are shown in Fig.6.

1. around 1.58[mm]~ 3.6[mm]
   In-phase synchronization and anti-phase synchronization states can be observed.
2. around 3.61[mm]~ 13.45[mm]
   Anti-phase synchronization states can be observed.
3. around 13.46[mm]~ 15.98[mm]
   Anti-phase synchronization and asynchronous states can be observed.
4. around 15.99[mm]~
   Synchronization phenomena are broken and asynchronous states are observed.

3.1.2. Inverse direction

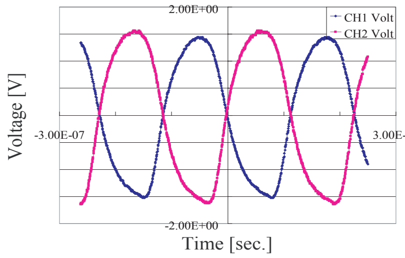
When terminal a_1 and terminal b_1 are different pole(see Fig.5), in-phase synchronization, anti-phase synchronization and asynchronous state can be observed. Relations between distance d and phase difference are shown in Fig.7.

1. around 1.58[mm]~ 1.80[mm]
   Anti-phase synchronization states can be observed(see Fig.8(a)). The phase difference becomes small as distance d becomes large.
2. around 1.81[mm]~ 17.41[mm]
   In-phase synchronization states can be observed(see Fig.8(b)).
3. around 17.42[mm]~ 20.48[mm]
   In-phase synchronization and asynchronous states can be observed. The phase difference becomes large as distance d becomes large.
4. around 20.49[mm]~
   Asynchronous states are observed(see Fig.8(c)).

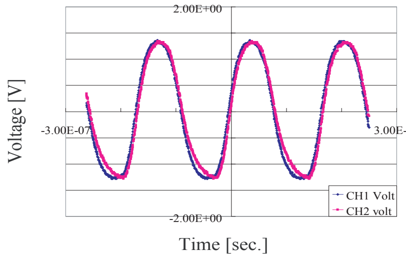
3.2. Synchronization phenomena depending on angle φ

One PS-Inductor is overlapped the other one in the same turn direction as shown in Fig.5(b). A state coupled in inverse direction between PS-Inductors is made by replacing positive terminal and negative terminal(see Fig.5(c)). The PS-Inductors are fixed by shaft. The distance r between center of the PS-Inductor and center of the shaft is fixed at 24.5[mm]. Secondary PS-Inductor can be rotated around the shaft.

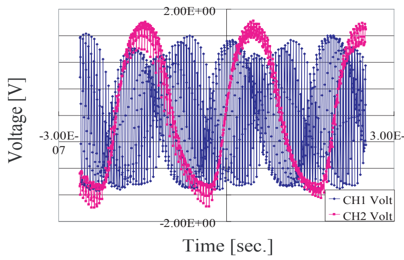
When an angle φ between two PS-Inductors is changed, synchronization phenomena are investigated. Intervals of changing angle φ are fixed as 1[degree]. The distance between primary PS-Inductor and secondary PS-Inductor is fixed at 1.58[mm] which is thickness of substrate.



(a) Changing distance – Inverse direction: anti-phase synchronization waveform (d =1.67[mm]).



(b) Changing distance – Inverse direction: in-phase synchronization waveform (d =8.78[mm]).



(c) Changing distance – Inverse direction: asynchronous state (d =18.68[mm]).

Figure 8: Changing distance in the inverse direction coupling.

3.2.1. Inverse direction

When terminal a_1 and terminal b_1 are different pole(see Fig.5), in-phase synchronization, anti-phase synchronization and asynchronous states can be observed. Relations between the angle φ and a phase difference are shown in Fig.9.

1. around 0[degree]~ 2[degree]
   around 0[degree]~ −3[degree]
   Anti-phase synchronization states can be observed.
2. around 3[degree]~ 19[degree]
   around −4[degree]~ −22[degree]
   In-phase and anti-phase synchronization states can be observed. In-phase or anti-phase synchronization state is decided by initial values.
3. around 20[degree]~ 47[degree]
   around −23[degree]~ −40[degree]
   In-phase synchronization states can be observed. The phase difference becomes large as angle φ becomes large.
4. around −41[degree]~ −43[degree]
   In-phase synchronization and asynchronous states are observed.

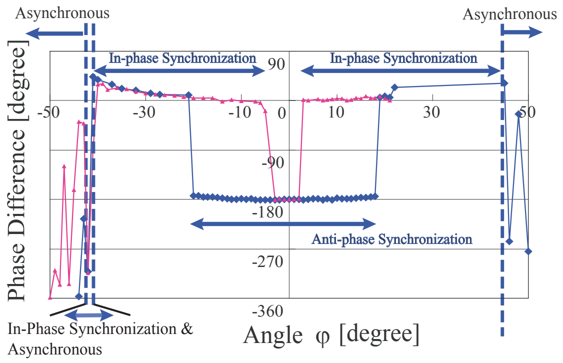


Figure 9: Trajectory of phase difference by changing angle with inverse direction.

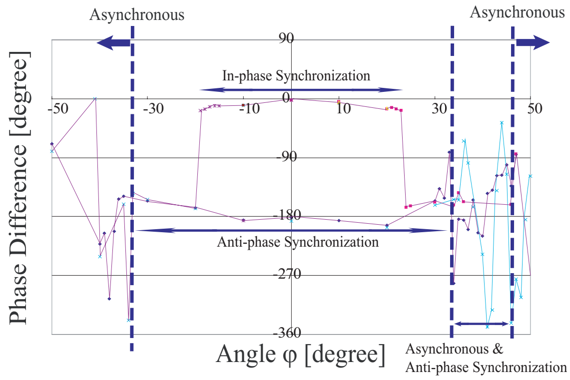


Figure 10: Trajectory of phase difference by changing angle with forward direction.

5. around 48[degree]~ 180[degree]
   around −44[degree]~ −180[degree]
   The synchronous states are broken and asynchronous states are observed.

When the angle φ is changed from 0[degree] to 19[degree], the phase state is anti-phase synchronization. The phase state changes from anti-phase synchronization to in-phase synchronization at 20[degree]. When the angle φ is changed from 20[degree] to 47[degree], the phase state is in-phase synchronization. When the angle φ is changed from 47[degree] to 3[degree], the phase state is in-phase synchronization. The phase state changes from in-phase synchronization to anti-phase synchronization at 2[degree]. A trajectory of the phase difference is like a hysteresis.

3.2.2. Forward direction

When terminal a_1 and terminal b_1 are same pole(see Fig.5), anti-phase synchronization and asynchronous state can be observed.

1. around 0[degree]~ 23[degree]
   around 0[degree]~ −19[degree]
   In-phase and anti-phase synchronization states can be observed. In-phase or anti-phase synchronization state is decided by initial values.
2. around 24[degree]~ 33[degree]
   around −20[degree]~ −33[degree]
   Anti-phase synchronization and asynchronous states are observed. The anti-phase synchronization state or the asynchronous state is decided by initial values.
3. around 46[degree]~ 180[degree]
   around −34[degree]~ −180[degree]
   The synchronous states are broken and asynchronous states are observed.

4. Conclusion

Firstly, the mutual inductances between PS-Inductors were measured by new actual measurement method and the new simulator. Results of the simulator and results of the actual measurement were similar in regards to changing patterns as the distance between PS-Inductors changes. The average of errors between the simulator and the actual measurement of combination inductance was 1.6%. However, the average of errors of mutual inductances was over 10%. The simulator and actual measurement method should be upgraded.

Next, the synchronization phenomena were observed when two Colpitts-oscillators were coupled using the mutual inductance. 1. When a distance between two PS-Inductors was changed, synchronization phenomena were investigated. The synchronization states were changed by distance between the PS-Inductors. In other word, synchronization states were changed by changing mutual inductance. 2. When an angle of overlap between two PS-Inductors were changed, synchronization phenomena were investigated. The phenomenon, which in-phase or anti-phase synchronization was dicided by initial values, was observed. This phenomenon was not able to be observed when the distance was changed.

We think an arbitrary signal can be modeled if many oscillators are coupled as same method and it is short time. These are three reasons as follows: First, some arbitrary signals are applied to coupled-oscillators system by using mutual inductance easily. Second, synchronization states can be changed by initial values. Third, some oscillators can be coupled easily and unnecessary oscillator can be taken off easily.

5. Acknowledgements

This research is supported by the Grants-in-Aid for Young Scientific Research (B) (No. 19760270) from the Japan Society for the Promotion of Science.

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