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A queue model for complex networks with adaptive traffic

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Abstract— We here present a queue model to describe TCP network nodes with multiple input and multiple output flows and only one shared and limited buffer. We use this queue model together with performance estimation methods based on the fluid model to estimate TCP network performance by computer simulation and we report simulation results that give the utilization of each network link as a function of the buffer size for a simple network topology.

1. Introduction

Fluid models are today's commonly accepted and widely used technique for performance estimation of complex packet network. This approach neglects the packet-by-packet analysis and adopts a deterministic strategy based on the evaluation of the average network dynamics. In this framework, performance of congestion control strategies can be evaluated and become important in the case of TCP dynamic window flow control often employed in the Internet network. Several methods [1, 2, 3] have been presented to evaluate the rates of flow across complex networks by describing the network node behavior in terms of a queueing system. We here consider a queue model to describe TCP network nodes with multiple input and multiple output flows and only one shared and limited buffer. We consider a multidimensional truncated Markov queue to allow multiple output links (section 3) all subjected to a unique and size-limited buffer (section 4) instead of the classical approach based on M/M/1/K queues. We use this queue model together with performance estimation methods [1, 4, 5] based on the fluid model to estimate TCP network performance by computer simulation and we report simulation results (section 5) that give the utilization of each network link as a function of the buffer size for a simple network topology.

2. Definitions and system model

As introduced in [1], a computer network is classically modeled by a graph $G(V, E)$ with a set V of nodes (vertices) and a set E of directed links (edges). Each node is characterized by a number of input and output links depending on the network topology and the traffic on each input link must be directed to a specific output link depending on the route a packet is assigned to. When an output link of a node is busy, the packet directed to that link is stored in a buffer and if the buffer is full the packet is dropped.

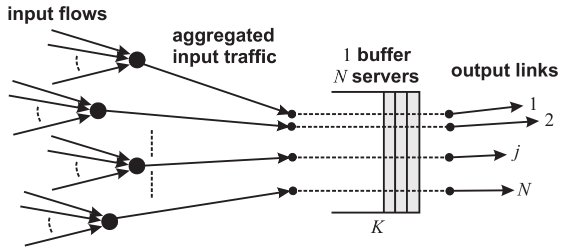

Figure 1: Proposed network node structure.

The node structure described above cannot be modeled either by an M/M/1/K queue or by an M/M/N/K queue because the former only allows to have independent output links and the latter doesn't keep the input aggregated traffic directed to different output links separate. Figure 1 shows the proposed node structure. All the input flows are aggregated to form the traffic directed to each one of N output links (this means that packets belonging to different input flows and directed to the same output link are undistinguishable at node level). The N aggregated inputs are then passed to the corresponding output servers but are stored into a shared buffer of size K. With this system we can model the behavior of a router more accurately. In fact, a router has a shared memory and N output circuits.

To describe such a system by a queueing model, we rely to the truncated multidimensional Markov queue which is made by N M/M/1 Markov queues, one for each output link (it is an N-server system), with a finite number of possible states so that the sum of the queue lengths of these queues is limited by K. The solution to this system allows to express the probability function and the loss probability from which the transfer probability can be calculated and applied [1] to evaluate the average network dynamics.

For a Markov queue, the probability function for the queue length Q to be equal to k is $f_Q(k) = \Pr\{Q = k\}$ and the probability generating function is defined as

$$\phi_Q(z) = \sum_{k=0}^{\infty} f_Q(k) z^k \tag{1}$$

Given (1), the average queue length can be expressed as $\mathbf{E}[Q] = \sum_{k=0}^{\infty} k f_Q(k) = d\phi_Q(z)/dz\big|_{z=1}$.

For M/M/1 queues (infinite buffer size) is

$$f_Q(k) = (1-\rho)\rho^k \tag{2}$$

$$\phi_Q(z) = (1-\rho)/(1-\rho z)$$
$$\mathbf{E}[Q] = \rho/(1-\rho) \tag{3}$$

$\rho = \lambda/\mu \in [0,1[$ is the ratio between arrival and service rates. For M/M/1/K queues (buffer size limited to K) is

$$f_Q(k) = \frac{1-\rho}{1-\rho^{K+1}}\rho^k \tag{4}$$

$$\phi_Q(z) = \frac{1-\rho}{1-\rho^{K+1}}\frac{1-(\rho z)^{K+1}}{1-\rho z}$$

$$\mathbf{E}[Q] = \frac{\rho}{1-\rho} - (K+1)\frac{\rho^{K+1}}{1-\rho^{K+1}}$$

The blocking probability for the M/M/1/K queue is equal to the probability to be in the K-th state, i.e. the probability that a new arrival finds the buffer full, hence

$$p_B = \frac{1-\rho}{1-\rho^{K+1}}\rho^K \tag{5}$$

and the transfer probability is $p_T = 1 - p_B$.

3. N M/M/1 queues with shared buffer

We are now interested in finding the probability function $f_Q(k,N) = \Pr\{Q = k\}$ of the sum of the queue lengths of N M/M/1 queues, each one with $f_{Q_j}(k_j) = \left(1-\rho_j\right)\rho_j^{k_j}$, where $k = \sum_{j=1}^N k_j$. It can easily be verified that the new probability function is the N-dimensional discrete convolution of the probability functions $f_{Q_j}(k_j)$. Hence, the probability generating function $\phi_Q(z)$ is simply the product of the probability generating function $\phi_{Q_j}(z)$

$$\phi_Q(z) = \prod_{j=1}^N \phi_{Q_j}(z) = \prod_{j=1}^N \frac{1-\rho_j}{1-\rho_j z} \tag{6}$$

We can then develop $\prod_{j=1}^N \left(1-\rho_j z\right)$ by the the Lagrange partial fraction decomposition and write

$$\frac{1}{\prod_{j=1}^N \left(1-\rho_j z\right)} = \sum_{j=1}^N \frac{\rho_j^{N-1}}{\prod_{l=1, l\neq j}^N \left(\rho_j - \rho_l\right)}\frac{1}{1-\rho_j z}$$

From geometric series, we can also write $1/(1-\rho_j z) = \sum_{k=0}^\infty (\rho_j z)^k$ with $|\rho_j z| < 1 \Rightarrow |z| < 1/\rho_j$, to get

$$\phi_Q(z) = \sum_{k=0}^\infty \left\{\sum_{j=1}^N \left(1-\rho_j\right)\rho_j^k \left[\rho_j^{N-1}\prod_{\substack{l=1\\ l\neq j}}^N \frac{1-\rho_l}{\rho_j-\rho_l}\right]\right\} z^k$$

It follows immediately from (1) that

$$f_Q(k,N) = \sum_{j=1}^N f_{Q_j}(k)\mathcal{X}_j(\boldsymbol{\rho},N) \tag{7}$$

where $f_{Q_j}(k) = \left(1-\rho_j\right)\rho_j^k$ and

$$\mathcal{X}_j(\boldsymbol{\rho},N) = \mathcal{X}_j(\rho_1,\ldots,\rho_N,N) = \rho_j^{N-1}\prod_{\substack{l=1\\ l\neq j}}^N \frac{1-\rho_l}{\rho_j-\rho_l}$$

The probability function $f_Q(k)$ is the linear combination of the probability functions $f_{Q_j}(k)$ calculated at the same buffer length k with the coefficients $\mathcal{X}_j(\boldsymbol{\rho},N)$. Note that it must be $\rho_j \neq \rho_l$; $\forall j, l$.

Since $\phi_{Q_j}(0) = \sum_{k=0}^\infty f_{Q_j}(k) = 1$, then from (6) also $\phi_Q(0) = 1$, i.e. the new probability function is already normalized. From this we can get an interesting property of the coefficients $\mathcal{X}_j(\boldsymbol{\rho},N)$

$$\sum_{j=1}^N \mathcal{X}_j(\boldsymbol{\rho},N) = 1 \tag{8}$$

This result will be useful in the next section.

In the special case in which $\rho_j = \rho_l$; $\forall j, l$, from (6) we have

$$\phi_Q(z) = \prod_{j=1}^N \frac{1-\rho}{1-\rho z} = \left(\frac{1-\rho}{1-\rho z}\right)^N$$

and by applying the negative binomial series we get

$$\frac{1}{(1-\rho z)^N} = \sum_{k=0}^\infty \binom{N+k-1}{k}\rho^k z^k$$

and then

$$\phi_Q(z) = \sum_{k=0}^\infty \left\{\binom{N+k-1}{k}\rho^k(1-\rho)^N\right\} z^k$$

It follows immediately that

$$f_Q(k,N) = \binom{N+k-1}{k}(1-\rho)^N\rho^k \tag{9}$$

One can verify that (7) and (9) are equal to (2) when $N = 1$, that is the new queue reduces to the M/M/1 queue.

4. N M/M/1 queues with shared and limited buffer

To set a limit on the buffer size of the multidimensional queue, we must multiply by a normalization constant so that the sum of the probabilities to be in the first $K+1$ states is equal to one, i.e the cumulative probability $\Pr\{Q \leq K\} = 1$. This choice inplies that a packet leaves the buffer only after the service completion. The case in which a packet leaves the buffer when the service starts is much more complicated and allows the buffer to have only few additional free locations (from zero up to N locations), i.e. N server registers are required but they are not shared. With this, from (7) the cumulative probability is

$$\Pr\{Q \leq K\} = \sum_{k=0}^K f_Q(k,N) = 1 - \sum_{j=1}^N \rho_j^{K+1}\mathcal{X}_j(\boldsymbol{\rho},N) \tag{10}$$

where the identity (8) has been applied. Now (7) must be normalized by (10) and the final expression is

$$f_Q(k,K,N) = \sum_{j=1}^N f_{Q_j}(k)\mathcal{Z}_j(\boldsymbol{\rho},K,N) \tag{11}$$

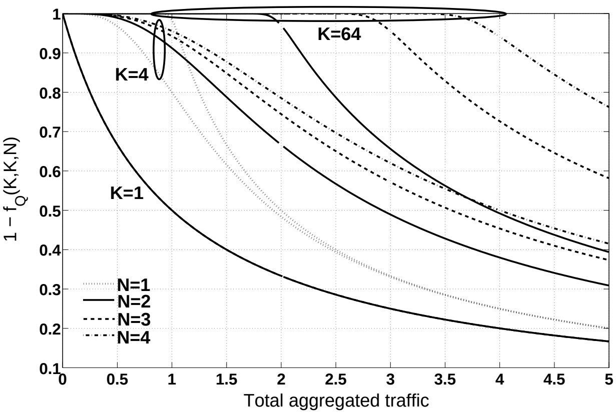

Figure 2: Transfer probability for $N = 1, 2, 3, 4$ and $K = 1, 4, 64$ in case of equal utilizations. Ellipses in the plot group the same values of K together while different values of N are reported with different line styles.

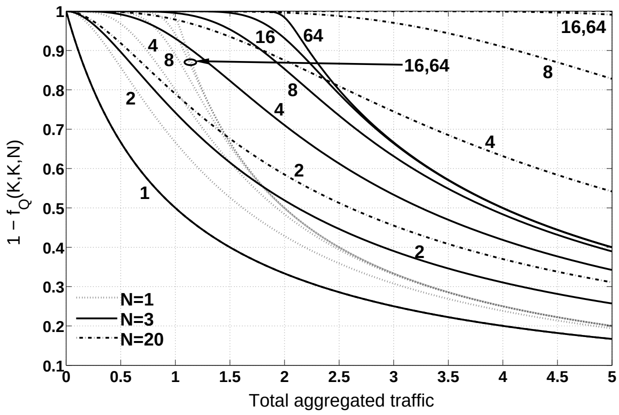

Figure 4: Transfer probability for $N = 1, 3, 20$ and $K = 1, 2, 4, 8, 16, 64$ in case of unequal utilizations. Numbers in the plot indicate the values of K while different values of N are reported with different line styles.

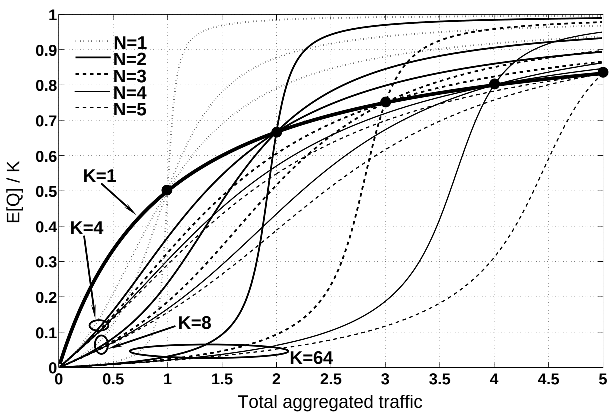

Figure 3: Normalized average queue length for $N = 1, 2, 3, 4, 5$ and $K = 1, 4, 8, 64$ in case of equal utilizations. Ellipses in the plot group the same values of K together while different values of N are reported with different line styles.

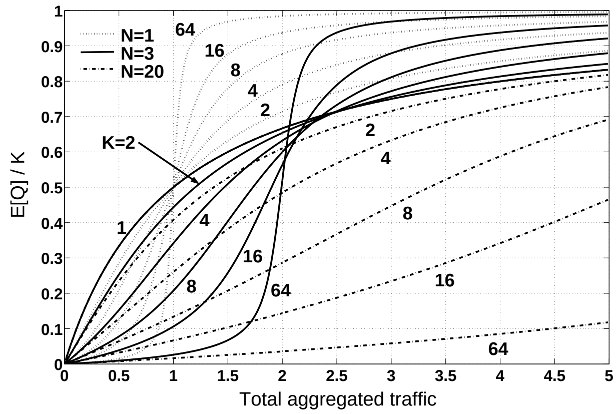

Figure 5: Normalized average queue length for $N = 1, 3, 20$ and $K = 1, 2, 4, 8, 16, 64$ in case of unequal utilizations. Numbers in the plot indicate the values of K while different values of N are reported with different line styles.

where

$$\mathcal{Z}_j(\boldsymbol{\rho}, K, N) = \frac{\mathcal{X}_j(\boldsymbol{\rho}, N)}{1 - \sum_{i=1}^{N} \rho_i^{K+1} \mathcal{X}_i(\boldsymbol{\rho}, N)}$$

The probability function of the new queue is still a linear combination as it was in (7) but with the new coefficients $\mathcal{Z}_j(\boldsymbol{\rho}, K, N)$.

In the special case in which $\rho_j = \rho_l;\ \forall j, l$, from (9) we have

$$f_Q(k, K, N) = \frac{\binom{N+k-1}{k}\rho^k}{\sum_{k=0}^{K}\binom{N+k-1}{k}\rho^k} \qquad (12)$$

One can verify that (11) and (12) are equal to (4) when $N = 1$, i.e. $\mathcal{Z}_j(\boldsymbol{\rho}, K, N)$ reduces to the factor $1/(1 - \rho^{K+1})$ and the new queue reduces to the M/M/1/K queue.

Thanks to the form of (11), the average queue length follows from (3) straightforwardly

$$\mathbf{E}[Q] = \sum_{k=0}^{K} k f_Q(k, K, N) = \sum_{j=1}^{N} \frac{\rho_j}{1 - \rho_j} \mathcal{Z}_j(\boldsymbol{\rho}, K, N)$$

The blocking probability is

$$p_B = \sum_{j=1}^{N} \left(1 - \rho_j\right) \rho_j^K \mathcal{Z}_j(\boldsymbol{\rho}, K, N)$$

and the transfer probability is $p_T = 1 - p_B$.

Finally we notice that setting $K \to \infty$ the function (11) reduces to (7). In fact, since utilizations must be such that $0 < \rho_j < 1; \forall j$ when the buffer is infinite due to obvious stability constraints, then $\lim_{K\to\infty} \rho_j^{K+1} = 0$. When the buffer is finite, the system is stable even for $\rho_j > 1$.

Figures 2 and 3 report the transfer probability $p_T = 1 - p_B = 1 - f_Q(K, K, N)$ and the normalized average queue length $\mathbf{E}[Q]/K$ respectively as a function of the total aggregated traffic ρ_T defined in general as $\rho_T = \sum_{j=1}^{N} \rho_j$ and specialized here to $\rho_T = N\rho$, i.e. all utilizations are equal to ρ. The first fact to be observed is that when $K = 1$ the curves don't depend on N, i.e. only one packet can be served regardless of the number of available servers as expected. Secondly, the transfer probability for a large buffer size is $p_T \approx 1$ for $\rho_T < N$ as expected, i.e. a large queue can transfer N flows with equal utilization without overloading

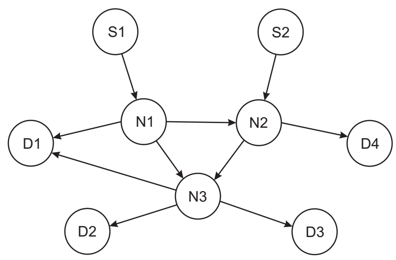

Figure 6: Topology of the simple network for performance evaluation.

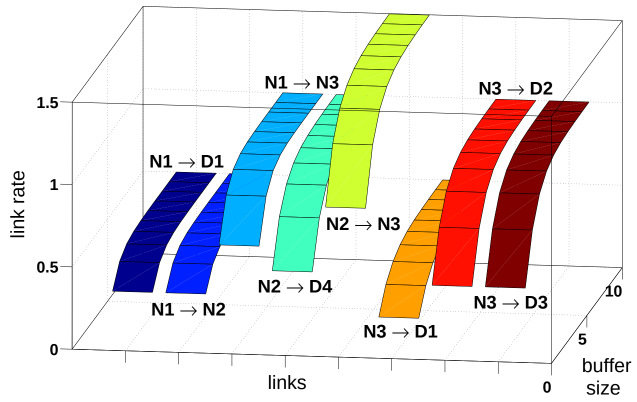

Figure 7: Link rate versus the buffer size for the network in Figure 6 where each source rate is 500 Kbps and each link capacity is 3 Mbps.

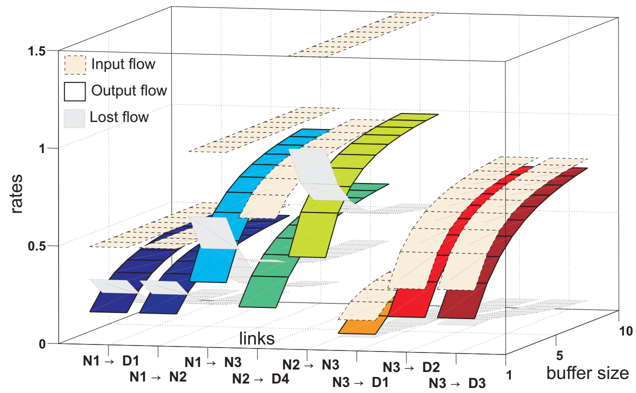

Figure 8: Input, output and lost rates versus the buffer size for the network in Figure 6 where each source rate is 500 Kbps and each link capacity is 1 Mbps.

the buffer if each utilization is less or equal to the capacity of one of the output links.

As a final remark, the normalized average queue length in Figure 3 increases with N when $\rho_T = N$ and its value lies on the curve $\rho/(\rho+1)$. This means that as N increases and the flows share the buffer equally, the buffer average occupation increases even if the capacity is not saturated showing a clear aggregation effect of the adopted queue model. More precisely, if the flows were independent, the expected normalized average queue length would be 0.5 when $\rho_j = 1$, $j = 1, \ldots, N$ which is not the case when the flows share a limited resource.

As a different example, Figures 4 and 5 report a case in which utilizations are strongly unequal because they are defined as $\rho_j = \rho_T(j/\sum_{l=1}^{N} l)$. In this case, the aggregation effect is clearly present in the transfer probability which is less than one before ρ is approximately equal to N, that is since the servers are not used equally then the blocking probability is higher than in the previous case. This situation is confirmed in Figure 5 for the $N = 3$ case by the fact that the queue saturates for $\rho_T < N$, i.e. only a few flows occupy most of the buffer so that when $\rho_T = N$ the buffer has already been saturated by the high-rate flows causing losses to happen also for the low-rate flows.

5. Numerical example

Figure 6 shows the network topology we propose as a simple numerical example. Two sources $S1, S2$ generate 8 flows that pass through 3 intermediate nodes $N1, N2, N3$ and reach 4 destination nodes $D1, D2, D3, D4$. The flows are: S1 → N1 → D1, S1 → N1 → N3 → D2, S1 → N1 → N3 → D3, S1 → N1 → N2 → D4, S2 → N2 → N3 → D1, S2 → N2 → N3 → D2, S2 → N2 → N3 → D3, S2 → N2 → D4. Whith this, the intermediate nodes are differently loaded and have a different number of output links.

Figure 7 shows the output rate of each link of interest as a function of the buffer size where each node has the same buffer size, each output link has the same capacity and each source generates flows with the same rate. The output rate is calculated according to the conservation of flow principle and the transfer probability. With the chosen output link capacity of 3 Mbps and source rate of 500 Kbps, the system is far from the loss boundary. Depending on the number of flows passing trough a certain link, the 3 rate levels of 0.5 Mbps, 1 Mbps and 1.5 Mbps can be recognized for a sufficient buffer size. At lower K, the flows experience losses and link rates are different even if they are composed by the same number of flows.

Figure 8 shows the input, output and lost rates for an insufficient link capacity of 1 Mbps. Additionally, due to losses happened in nodes N1 and N2, the input rate of node N3 is already lower than its maximum value.

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