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On Relation between Routing Strategy and Communication Quality in Wireless Multi-hop Networks

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Abstract—In wireless multi-hop networks, a source and a destination usually have some candidate paths between them, and the communication quality depends on the selection of a multi-hop path from the candidates. In this paper, we analyze the quality of a path using a metric called Expected Transmission Count (ETX) in a line network where nodes are located at constant intervals. We theoretically and exactly analyze the route ETX of Optimum Routing (OR), which minimizes the route ETX, in the network as well as those of Shortest Path Routing (SPR) and Longest Path Routing (LPR). We also approximately analyze the route ETX of OR in a two-dimensional network with lattice structure. We compare the ETXs in these networks with those in random networks.

1. Introduction

In wireless multi-hop networks [1, 2], a source node sends data to a destination node through a multi-hop path that consists of some intermediate nodes as relay nodes. The multi-hop path is selected by the routing algorithm used in the network. The shortest path algorithm is often used as a routing algorithm; however, it tends to select long links, which have a low reliability and lead to a poor communication quality [3]. In [4], the authors proposed a path metric called Expected Transmission Count (ETX), which is defined as the expected total number of transmissions required to successfully send a packet from the source to the destination, and showed that the communication quality is improved by selecting a path which has the minimum ETX. Other than ETX, several metrics of quality of a path have been proposed such as Per-hop Round Trip Time (RTT) [5], Medium Time Metric (MTM) [6], and Expected Transmission Time Metric (ETT) [7].

In [8, 9], we theoretically analyzed the mean ETX of a path selected by three routing algorithms, namely Longest Path Routing (LPR), Shortest Path Routing (SPR), and Optimum Routing (OR), which minimizes the ETX, in a one-dimensional wireless multi-hop network where nodes are randomly distributed based on a Poisson process. In the analysis of OR, however, we used an approximate method because the direct analysis of OR is not easy if nodes are randomly distributed. Also, it is more difficult to analyze OR in the two-dimensional network where nodes are randomly distributed.

In this paper, we try to analyze OR in two kinds of networks to overcome the above difficulties. First, we consider a simple one-dimensional network where nodes are located at constant intervals on a line, and exactly analyze the route ETX of OR in this network. At the same time, we analyze the route ETX of LPR and SPR in this network. Second, we apply the results of the first analysis to an approximate analysis of the route ETX of OR in a lattice network as an extension to analysis in two-dimensional networks. We compare the numerical results of ETX in these networks with the numerical results and simulation results in the networks where nodes are randomly distributed to find possibility to apply these results to evaluation of the mean route ETX in random networks.

2. Definitions and Assumptions

Suppose that S and D are source and destination nodes, respectively, and that the distance between S and D equals ℓ. Let d be the communication range of a node. Namely, two nodes have a direct link if the distance between them is smaller than or equal to d, and they do not have a direct link otherwise. Let $u(z)$ be the function of ETX of a link, where z is the length of the link. We assume that $u(z) = \infty$ if $z > d$. We also assume that $u(z)$ is a convex monotonically increasing function, and $u(0) > 0$. Route ETX is defined as the sum of the ETXs of all links in the route.

As mentioned above, we consider two kinds of networks: One is a one-dimensional line network where nodes are located at constant intervals with distance a_1. Denote this network by Network A. Suppose that S is at $x = 0$, and D is at $x = \ell$ on the x axis, where ℓ is multiple of a_1, which is a positive constant, and that relay nodes are located at $x = a_1$, $x = 2a_1$, ..., $x = (\frac{\ell}{a_1} - 1)a_1$. The other is a two dimensional network where nodes are located with lattice structure with lattice constant a_2. Denote this network by Network B. Suppose that S is at $(0, 0)$, and D is at (ℓ_x, ℓ_y), where both ℓ_x and ℓ_y are multiples of a_2, and $\sqrt{\ell_x^2 + \ell_y^2} = \ell$. Suppose that relay nodes are located at (x, y), where $x = 0$, a_2, $2a_2$, ..., $\frac{\ell_x}{a_2}a_2$, and $y = 0$, a_2, $2a_2$, ..., $\frac{\ell_y}{a_2}a_2$.

To compare the above two networks with random networks, we consider the following random networks for reference. Network C is a network where nodes are distributed based on a Poisson process with intensity λ_1 on a line. Network D is a network where nodes are distributed based on a Poisson process with intensity λ_2 on a plane. Examples of Network A, B, C, and D are shown in Fig. 1(a), (b), (c), and (d), respectively.

In Network A, we exactly analyze the route ETX of the following three routing algorithms. Longest Path Routing (LPR) selects all nodes between S and D as relay nodes. As a result, LPR maximizes the number of hops. Shortest Path Routing (SPR) selects the node nearest to D and within d of S as a first relay node, and it selects the node nearest to D and within d of the first relay node as a second relay node. In the same manner, the multi-hop path to D is constructed. The path includes the minimum number of relay nodes. Optimum Routing (OR) selects a path which has the minimum route ETX from all candidate paths. We compare the numerical results of the analyses in Network A with those of the mean route ETX of the three algorithms in Network C. The analyses in Network C are shown in [8, 9]. Note that the analysis of OR in Network C is done approximately.

In Network B, we analyze the route ETX of OR approximately based on the analysis in Network A. We also compute the route ETX of Shortest Path with Minimum ETX Routing (SPMR), which selects a path with the minimum ETX from all shortest paths, by computer simulation.

Let $U_{A,L}$, $U_{A,S}$, and $U_{A,O}$ be the route ETX of LPR, SPR, and OR in Network A, respectively. Let $U_{B,S}$ and $U_{B,O}$ be the route ETX of SPMR and OR in Network B, respectively. Let $U_{C,L}$, $U_{C,S}$, and $U_{C,O}$ be the mean route ETX of LPR, SPR, and OR in Network C, respectively. Let $U_{D,S}$ and $U_{D,O}$ be the mean route ETX of SPMR and OR in Network D, respectively.

3. Analysis of Network A

In this section, we theoretically and precisely analyze $U_{A,L}$, $U_{A,S}$, and $U_{A,O}$. First, we consider $U_{A,L}$. LPR selects a route consisting of $\frac{\ell}{a_1}$ links whose lengths are equal to a_1. Therefore, we have

$$U_{A,L} = \frac{\ell}{a_1} u(a_1). \tag{1}$$

Second, we consider $U_{A,S}$. Let ℓ_{max} be the maximum length of a link which can exist in Network A. Because the maximum transmitting range is d, and the length of every pair of two nodes in Network A is a multiple of a_1, $\ell_{max} = \left\lfloor \frac{d}{a_1} \right\rfloor a_1$. Hence, SPR selects a route consisting of $\left\lceil \frac{\ell}{\ell_{max}} \right\rceil - 1$ links whose lengths are equal to ℓ_{max} and one link whose length is equal to $\ell - \left(\left\lceil \frac{\ell}{\ell_{max}} \right\rceil - 1 \right) \ell_{max}$. Therefore,

$$U_{A,S} = \left(\left\lceil \frac{\ell}{\ell_{max}} \right\rceil - 1 \right) u(\ell_{max}) + u\left\{ \ell - \left(\left\lceil \frac{\ell}{\ell_{max}} \right\rceil - 1 \right) \ell_{max} \right\}. \tag{2}$$

Third, we consider $U_{A,O}$. We have the following theorem:

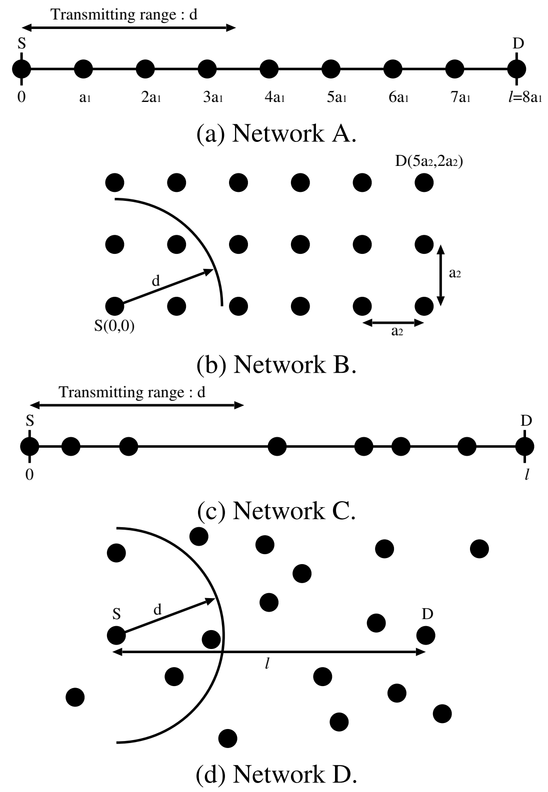


Figure 1: Network models.

Theorem 1. *Assume that $u(z)$ is a convex monotonically increasing function. Define the following constants:*

$$\ell_1 = \left\lfloor \frac{\ell}{k a_1} \right\rfloor a_1, \quad \ell_2 = \left\lfloor \frac{\ell}{k a_1} \right\rfloor a_1 + a_1.$$

Let r_1 be the k-hop route that consists of at most two kinds of links whose lengths are ℓ_1 and ℓ_2. r_1 is uniquely determined, and it has the minimum route ETX in all k-hop routes.

Proof. First, we prove the uniqueness of r_1. Let n_1 and n_2 be the numbers of links whose lengths are ℓ_1 and ℓ_2, respectively. Because r_1 is a k-hop route, $n_1 + n_2 = k$. Also, because the sum of lengths of all links in r_1 is ℓ, $n_1\ell_1 + n_2\ell_2 = \ell$. From these equations, n_1 and n_2 are uniquely determined and

$$n_1 = k - \left(\frac{\ell}{a_1} - k \left\lfloor \frac{\ell}{k a_1} \right\rfloor \right), \quad n_2 = \frac{\ell}{a_1} - k \left\lfloor \frac{\ell}{k a_1} \right\rfloor.$$

Second, we prove that r_1 has the minimum route ETX in all k-hop routes. Assume that the k-hop route r_2 other than r_1 has the minimum ETX in all k-hop routes. Suppose that r_2 consists of k links whose lengths are $\ell'_1, \ell'_2, ..., \ell'_k$, where $\ell'_1 \le \ell'_2 \le ... \le \ell'_k$. From the assumption, $\ell'_1 < \ell_1$ or $\ell'_k > \ell_2$. If $\ell'_1 < \ell_1$, then $\ell'_k > \ell_1$ because $\ell'_1 + (k-1)\ell_1 < \ell$. If $\ell'_k > \ell_2$, then $\ell'_1 < \ell_2$ because $\ell'_k + (k-1)\ell_2 > \ell$. Hence, $\ell'_k - \ell'_1 \ge 2a_1$. Then $u(\ell'_1 + a_1) + u(\ell'_k - a_1) < u(\ell'_1) + u(\ell'_k)$ because $u(z)$ is a convex monotonically increasing function. Let r_3 be the k-hop route consisting of k links whose lengths are $\ell'_1 + a_1$, ℓ'_2, ℓ'_3, ..., ℓ'_{k-1}, $\ell'_k - a_1$. r_3 is a k-hop route and has the route ETX smaller than that of r_2, which is a contradiction. Therefore, r_1 has the minimum route ETX in all k-hop routes. □

From Theorem 1, we have

$$U_{A,O} = \min_{\lceil \frac{\ell}{\ell_{max}} \rceil \le k \le \frac{\ell}{a_1}} \{n_1 u(\ell_1) + n_2 u(\ell_2)\}. \quad (3)$$

4. Analysis of Network B

In this section, we approximately analyze $U_{B,O}$ because it is difficult to precisely analyze $U_{B,O}$. From Theorem 1, we expect that the k-hop route that consists of at most four kinds of links whose vectors are $\vec{\ell_3}$, $\vec{\ell_4}$, $\vec{\ell_5}$, and $\vec{\ell_6}$ approximately minimizes the route ETX in all k-hop routes, where

$$\vec{\ell_3} = \left(\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor a_2, \left\lfloor \frac{\ell_y}{ka_2} \right\rfloor a_2\right), \quad (4)$$

$$\vec{\ell_4} = \left(\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor a_2, \left\lfloor \frac{\ell_y}{ka_2} \right\rfloor a_2 + a_2\right), \quad (5)$$

$$\vec{\ell_5} = \left(\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor a_2 + a_2, \left\lfloor \frac{\ell_y}{ka_2} \right\rfloor a_2\right), \quad (6)$$

$$\vec{\ell_6} = \left(\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor a_2 + a_2, \left\lfloor \frac{\ell_y}{ka_2} \right\rfloor a_2 + a_2\right). \quad (7)$$

There can be many k-hop routes consisting of $\vec{\ell_3}$, $\vec{\ell_4}$, $\vec{\ell_5}$, and $\vec{\ell_6}$. Here, we choose a k-hop route with a minimum number of $\vec{\ell_6}$ from the candidates because $\vec{\ell_6}$ is the longest in the four vectors and cause an increase of ETX. Denote this route by r_4. Let n_3 be the number of $\vec{\ell_3}$ included in r_4. In the same manner, let n_4, n_5, and n_6 be those of $\vec{\ell_4}$, $\vec{\ell_5}$, and $\vec{\ell_6}$, respectively. We can determine r_4 uniquely and we have

$$n_3 = k - n_4 - n_5 - n_6, \quad (8)$$

$$n_4 = \frac{\ell_y}{a_2} - k\left\lfloor \frac{\ell_y}{ka_2} \right\rfloor - n_6, \quad (9)$$

$$n_5 = \frac{\ell_x}{a_2} - k\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor - n_6, \quad (10)$$

$$n_6 = \max\left\{0, \frac{\ell_x + \ell_y}{a_2} - k\left\lfloor \frac{\ell_x}{ka_2} \right\rfloor - k\left\lfloor \frac{\ell_y}{ka_2} \right\rfloor - k\right\}. \quad (11)$$

We compute the route ETX of r_4 for $\left\lceil \frac{\ell}{d} \right\rceil \le k \le \frac{\ell_x + \ell_y}{a_2}$ and use the smallest one, denoted by $U'_{B,O}$, as an approximation to $U_{B,O}$. Then

$$U'_{B,O} = \min_{\lceil \frac{\ell}{d} \rceil \le k \le \frac{\ell_x+\ell_y}{a_2}} \sum_{i=3}^{6} n_i u\left(\left|\vec{\ell_i}\right|\right). \quad (12)$$

Note that if $a_2 = a_1$, and $\ell_x = 0$ or $\ell_y = 0$, then $U'_{B,O} = U_{A,O}$.

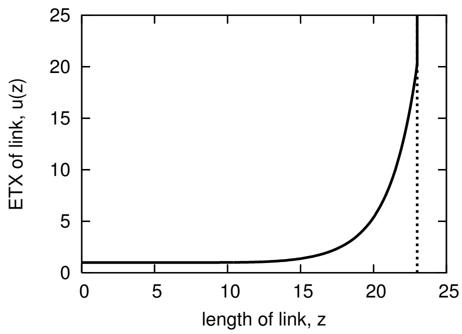


Figure 2: Function of link ETX $u(z)$.

5. Numerical Results

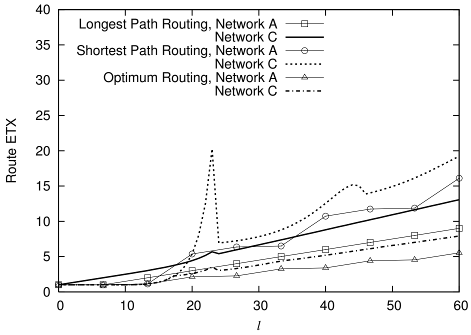
We use $u(z)$ in Fig. 2 as a function of the ETX of a link. We compute this function using the path-loss model in [10]. Also, we set d to 23.

Figure 3 shows the numerical results of $U_{A,L}$, $U_{A,S}$, and $U_{A,O}$ together with the results of $U_{C,L}$, $U_{C,S}$, and $U_{C,O}$ to compare the relay nodes at constant intervals with those at random intervals. For comparison under the same condition, we set $\lambda_1 = \frac{1}{a_1}$ so that the number of nodes per unit length in Networks A and C are the same. In Fig. 3, the horizontal axis is ℓ. Also, $\lambda_1 = 0.15$ in Fig. 3(a), and $\lambda_1 = 0.5$ in Fig. 3(b).

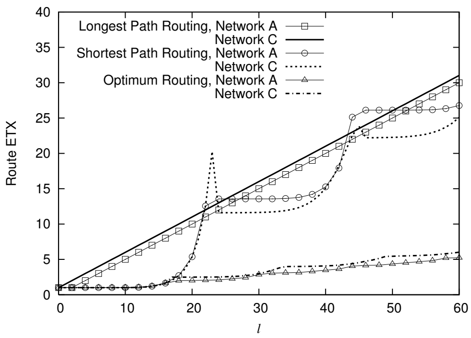
From Fig. 3, we can confirm that OR significantly reduces the route ETX compared with LPR and SPR for large λ_1 while the route ETX of each algorithm is close to each other for small λ_1 in both Networks A and C. We can also confirm that the difference between $U_{A,O}$ and $U_{C,O}$ tends to be smaller as λ_1 increases, and $U_{A,O}$ is close to $U_{C,O}$ for a large λ_1.

Figure 4 shows the numerical results of $U'_{B,O}$ with the simulation results of $U_{B,O}$ to verify the relation between the numerical results of Eq. (12). In this figure, we also show the numerical results of $U_{A,O}$ and the simulation results of $U_{D,O}$ to observe the effects of network structure on the route ETX. We also show $U_{B,S}$ and $U_{D,S}$ to observe how much OR can reduce ETX compared with other routing methods. We set the parameters as follows: $a_1 = a_2$, $\lambda_2 = \frac{1}{a_2^2}$, and $\ell_x = \ell_y = \frac{\ell}{\sqrt{2}}$. In Fig. 4, the horizontal axis is ℓ. Also, $\lambda_2 = 0.0025$ in Fig. 4(a), and $\lambda_2 = 0.04$ in Fig. 4(b).

In Fig. 4, the numerical results of $U'_{B,O}$ is close to the simulation results of $U_{B,O}$. Then we can confirm that the analysis of $U'_{B,O}$ is valid. We can also confirm that OR greatly reduces the route ETX compared with SPMR in Network B and D especially when λ_2 is large. For a small density of nodes, $U_{B,O}$ is close to $U_{D,O}$ while it is quite different from $U_{A,O}$. For a large density of nodes, $U_{B,O}$ is close to both $U_{D,O}$ and $U_{A,O}$. From these results, it is considered that $U'_{B,O}$ can be used as an approximation to not only $U_{B,O}$ but also $U_{D,O}$. Also, for a large density of nodes, even $U_{A,O}$ can be used as an approximation to $U_{B,O}$ and $U_{D,O}$.

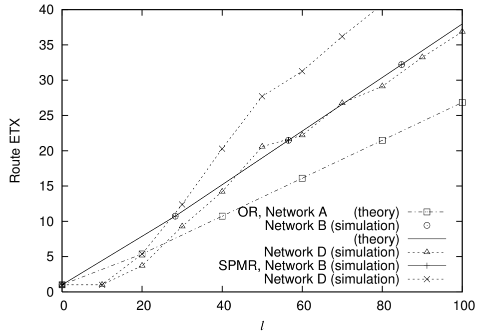


(a) $\lambda_1 = 0.15, a_1 = \frac{20}{3}$.

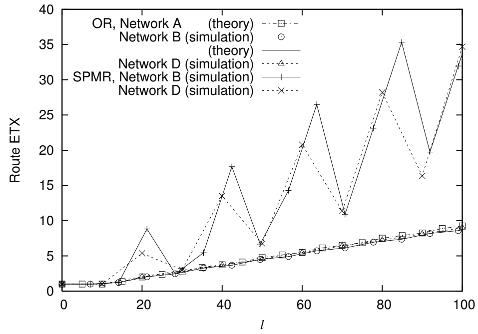


(b) $\lambda_1 = 0.5, a_1 = 2$.

Figure 3: Route ETX in Network A and mean route ETX in Network C.



(a) $\lambda_2 = 0.0025, a_1 = a_2 = 20$.



(b) $\lambda_2 = 0.04, a_1 = a_2 = 5$.

Figure 4: Route ETX in Network A and B and mean route ETX in Network D.

6. Conclusions

We theoretically and exactly analyzed the route ETX of Longest Path Routing, Shortest Path Routing, and Optimum Routing in a line network where nodes are located at constant intervals. We approximately analyzed the route ETX of Optimum Routing in a lattice network. From the numerical results and simulation results, we showed that this approximation was valid. We compared the route ETX in the above networks with that in random networks. From the comparison, we showed that we can approximately compute the route ETX in a two-dimensional random network in the same manner as that in the lattice network. We also showed that we can approximately compute it by the exact method for the line network for a large density of nodes.

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