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Synchronous Phenomena in Coupled Nd:YAG Lasers

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Abstract—In coupled chaotic oscillators with slight parameter mismatch, there are two transition routes as the coupling strength increases: one is the transition to lag synchronization through phase synchronization (PS) with $\pi/2$ phase shift and the other the direct transition to complete synchronization through PS without phase shift. We reports the experimental observation of the two routes in coupled chaotic Nd:YAG lasers.

1. Introduction

Over the past decade synchronization in coupled chaotic lasers has received much attention because it not only is applicable to such technological areas as secure communications [1, 2, 3] but also deepens our understanding of laser dynamics. In a real system, since parameter mismatch is indispensable, the understanding of synchronization in coupled chaotic lasers is important for the above applications. When two lasers are mutually coupled with each other, due to parameter mismatch, they exhibit various synchronous behaviors such as periodic phase [4], phase (PS) [5], lag (LS) [6], and complete synchronization (CS) [7, 8]. In coupled chaotic oscillators, it is known that there are two routes: one is the transition from nonsynchronous state to LS through PS with $\pi/2$ phase shift [5] and the other direct transition to CS through PS without phase shift [9, 8]. In coupled Nd:YAG lasers, depending on the coupling method, we experimentally find the two routes.

2. Transition to Lag Synchronization

For the transition to LS through PS with $\pi/2$ phase shift, two diode laser pumped Nd:YAG lasers (rod lengths are 4.85 mm and 10.00 mm-long, respectively) are coupled electronically not to make the two systems identical. Each Nd:YAG laser is pumped by a 808 nm laser diode, which is driven by a current source, as shown by the schematic diagram in Fig. 2. Each output coupler whose transmittance is 97 percent at 1064 nm is set about 2.5 cm apart from each YAG rod. The back surfaces of the rods are coated for total transmission at 808 nm and for total reflection at 1064 nm. The front surfaces are coated for total transmission at 1064 nm. About 10 cm apart from each output coupler, a 1064 nm band pass filter is set to block the 808 nm pump beam.

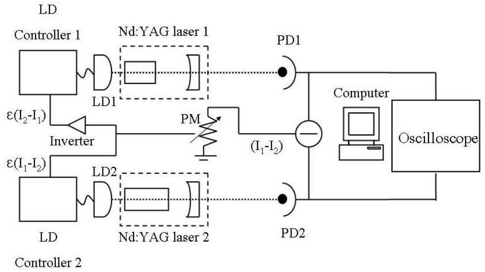

Figure 1: Schematic diagram of the experimental setup in coupled diode laser pumped Nd:YAG lasers. LD, PD, and PM are the diode laser, the Si P-I-N photo diode, and the potentio-meter, respectively.

Each laser signal is detected with a fast Si P-I-N photo diode and monitored with an oscilloscope. From the output signals $\epsilon(\tilde{I}_1 - \tilde{I}_2)$ is obtained by using an electronic circuit after removing the dc components, where $\tilde{I}_{1,2}$ are the unbiased output signals of the two lasers and ϵ is the coupling strength. Then $\epsilon(\tilde{I}_1 - \tilde{I}_2)$ is applied to the LD2 controller and $\epsilon(\tilde{I}_2 - \tilde{I}_1)$ is applied to the LD1 controller. ϵ is adjusted with a potentio-meter with a step size of 0.2 percent. Finally the measured laser signals are transferred to a computer to be analyzed.

In the experiment, the currents of LD1 and LD2 are fixed at 433 mA and 464 mA, respectively. When they are not coupled, the Nd:YAG lasers generate chaotic outputs with different characteristic frequencies of 57 kHz and 62 kHz, respectively. The temporal behaviors of the two laser outputs are shown in Fig. 2(a) for $\epsilon = 0.0$. Two laser exhibit independent time series. For $\epsilon = 0.3$, the time series shows the phase slip by 2π as shown in the box in Fig. 2(b). This slip is called 2π phase jump. When $\epsilon = 0.4$, the phases of the two laser outputs are slightly mismatched occasionally. However, there is no 2π phase slip between the two lasers although the amplitudes are not locked with each other as Fig. 2(c). This is the very PS state. For $\epsilon = 0.75$, the two laser outputs have an almost constant time lag as shown in Fig. 2(d). This is the LS state.

In order to observe PS and phase jumps, we obtain the following phase increment by using the local maximums of

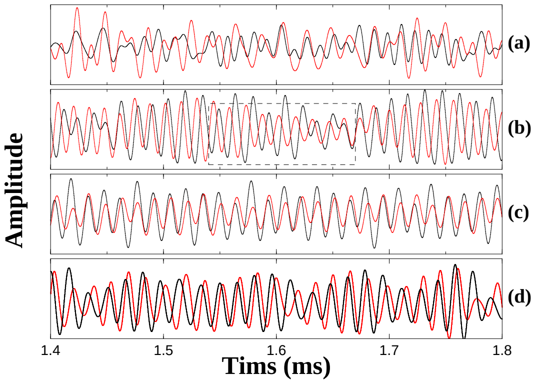

Figure 2: Temporal behaviors of the two laser outputs. Coupling strengths: (a) $\epsilon = 0.0$; (b) $\epsilon = 0.3$; (c) $\epsilon = 0.4$ (PS); and (d) $\epsilon = 0.75$ (LS).

each laser output :

$$\theta^i(t) = 2\pi \frac{t(I^i) - t(I^i_n)}{t(I^i_{n+1}) - t(I^i_n)} + 2\pi n, \tag{1}$$

for $t(I^i_n) \leq t \leq t(I^i_{n+1})$, where I^i_n is the n-th local maximum of the signal of the laser i. From the definition of phase, we calculate the phase difference $\phi(t) = \theta^1(t) - \theta^2(t)$. Fig. 2 shows $\phi(t)$ according to the coupling strength. As the coupling strength increases, we can find that the number of intermittent phase jumps decreases gradually. This jumping is the typically observed in coupled Rössler oscillators when their characteristic frequencies are slightly mismatched. In Fig. 2(c) for $\epsilon = 0.4$, there are no more intermittent 2π phase jumps, and phases are locked within 2π. Also the figure clearly shows a $\pi/2$ phase shift, when the phases of two laser outputs are locked.

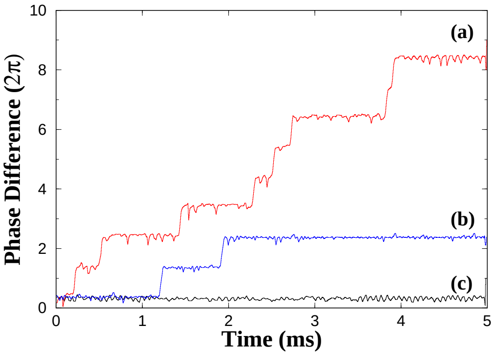

Figure 3: Phase difference of the two laser outputs when (a) $\epsilon = 0.2$, (b) $\epsilon = 0.3$, and (c) $\epsilon = 0.4$.

It is known that when PS state has $\pi/2$ phase shift, the system develops to LS as the coupling strength increases. Because of the phase shift, when LS occurs the time series of two oscillators almost coincide with a lag time τ_L such that $x_1(t) \approx x_2(t + \tau_L)$. The lag time τ_L is a distinguishable behavior from that of CS, of which time series of corresponding dynamical variables of the subsystems completely coincide such that $x_1(t) \approx x_2(t)$.

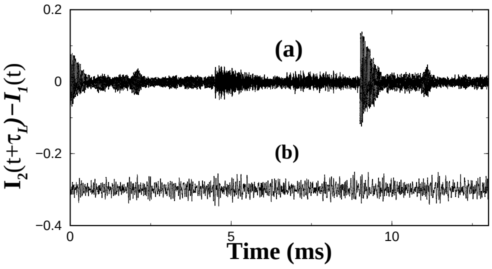

Figure 4: Temporal behavior of the difference of the two signals, $I_2(t + \tau_L) - I_1(t)$ at (a) $\epsilon = 0.65$ and (b) $\epsilon = 0.7$, where $\tau_L = 12.04$ μs is the lag time obtained from the similarity function.

For a stronger coupling strength region $\epsilon > 0.69$, we find the LS state. For $\epsilon = 0.75$, as the time series shown in Fig. 2(d), we can see a lag time, which is related to the $\pi/2$ phase shift. To confirm the LS state, the difference of the two laser outputs is obtained as shown in Fig. 4. Before the transition at $\epsilon = 0.65$, $I_2(t + \tau_L) - I_1(t)$ exhibits small amplitude chaotic fluctuations between the large chaotic bursts as shown in Fig. 4(a), where τ_L is 12.04 μs. This is a typical feature of intermittent LS state. After the transition to LS, no large chaotic bursts can be seen for $\epsilon = 0.75$ as shown in Fig. 4(b). This means that the two laser outputs are almost in coincidence with a lag time τ_L, such that $I_2(t + \tau_L) \approx I_1(t)$. This is the evidence of the transition to LS from PS. So we can understand that the electronically coupled Nd:YAG lasers exhibit the transition to LS through PS with $\pi/2$ phase shift as we increase the coupling strength.

3. Transition to Complete Synchronization

To observe the direct transition to CS, we couple two different TEM$_{00}$ mode Nd:YAG lasers, which have a 125 mm long YAG rod. As the experimental setup is given in Fig. 3 each laser output is injected into the other laser cavity individually, and the coupling strength is controlled with a Glan-Thompson polarizer and Brewster windows, which are places inside each laser cavity. The reflectivities of the output couplers are 85% and 75%, respectively. The optical signals are detected at the back mirrors with fast p-i-n photo diodes. The measured signals are stored in a memory digital storage oscilloscope to be analyzed. The currents of the two lasers are set at 10.0 Ampere and 10.5 Ampere near the threshold, respectively.

The temporal behaviors of the two lasers are shown in Fig. 3 for four cases of the coupling strength. When the lasers are uncoupled, that is, the rotation angle of the Glan-

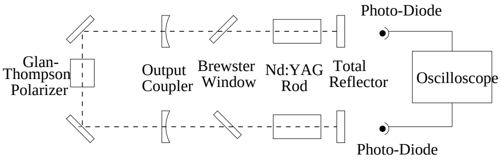

Figure 5: Schematic diagram of the experiment in two coupled Nd:YAG lasers.

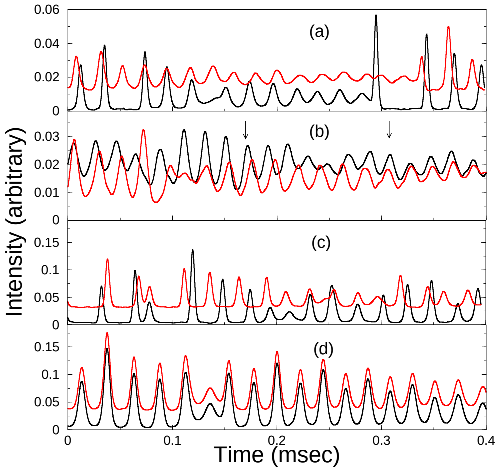

Figure 6: Temporal behaviors of two laser outputs for four cases of the coupling strength: the angles of the G1an-Thompson polarizer are (a) 90 degrees, (b) 45 degrees, (c) 15 degrees, and (d) 5 degrees.

Thompson polarizer is 90 degrees, the temporal behaviors of two lasers are different as shown in Fig. 3(a). When the angle is 45 degrees, the phases begin to be locked as shown in Fig. 3(b). When the angle is 15 degrees, the pulsations are slightly dephased from time to time within $\pm 2\pi$. This means the phase difference is bounded within $\pm 2\pi$. When the angle is 5 degrees, just before the full coupling, their phases come to be locked with each other, as shown in Fig. 3(d). Here we find that there is no time delay between the two laser outputs and that the amplitudes almost coincide. This means that there is no LS since the phases and amplitudes coincide with each other without lag time. From these time series we can understand that the phase difference of the two chaotic laser outputs develops from non-synchronous state to CS through PS without a phase shift.

In order to observe PS and CS, we obtain the phase increment of each laser output from I versus $\dot{I}$ space by using Yalcinkaya and Lai's algorithm [10], where $\dot{I}$ is the time derivative of the laser intensity I:

$$\theta_{1,2} = \tan^{-1}\left(\frac{\Delta \dot{I}_{1,2}}{\Delta I_{1,2}}\right),$$

$$r_{1,2} = \sqrt{(\Delta I_{1,2})^2 + (\Delta \dot{I}_{1,2})^2}. \qquad (2)$$

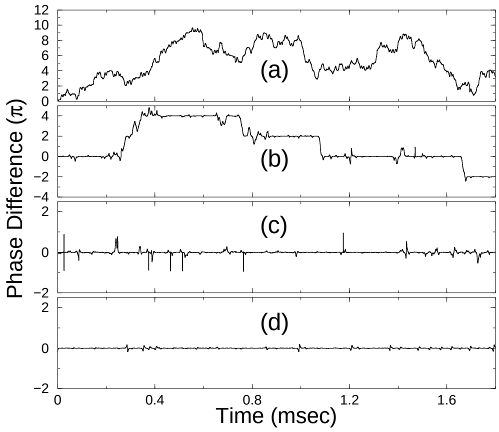

Figure 7: Temporal behaviors of phase difference according to the angle of Glan-Thompson polarizer: (a) non-synchronous state when the angle is 90 degrees, (b) phase jump state when 20 degrees, (c) phase synchronization when 15 degrees, and (d) complete synchronization when 0 degree.

Figure 3 shows the phase difference of the two laser outputs of $\phi = \theta_1 - \theta_2$ for the four cases of the rotation angle of the polarizer. When the two lasers are uncoupled at 90 degrees of the rotation angle, the phase difference increases or decreases irregularly as is shown in Fig. 3(a). Here we can not find any intermittent phase locking length, because the two laser outputs are not correlated with each other. But when the angle is decreased down to 20 degrees, the phase difference is locked at $\pm 2n\pi$ quite a long time and jumps by $\pm 2\pi$ intermittently, as shown in Fig. 3(b). This jumping behavior is the typical one appearing in coupled hyper-chaotic Rössler oscillators or coupled Lorenz oscillators [9]. At 10 degrees, the phase difference is locked within $\pm 2\pi$ without jumps and without phase shift as shown in Fig. 3. This is the very PS state in this laser system. Here we can observe intermittent chaotic bursts in the time series. When the two lasers are fully coupled at 0 degree of the polarizer angle, the phase difference is almost zero without intermittent chaotic bursts, as shown in Fig. 3(d). This is the very CS, where we can observe no lag time. Thus we can understand that the phase difference of the coupled Nd:YAG lasers exhibits intermittent chaotic bursts before CS. These are the evidence of the direct transition from PS to CS.

4. Discussion

The PS state with a $\pi/2$ phase shift and the intermittent PS state with 2π phase jumps are observed in the coupled Rössler oscillators [5], while the PS state without a phase shift and the intermittent PS state with $\pm 2\pi$ phase jumps are observed in the coupled Hyperchaotic Rössler oscillators [9]. It is known that these differences are caused by the different transition mechanism to PS. While PS with a $\pi/2$ phase shift is governed by type-I intermittency in the presence of noise [11], PS without a phase shift is governed by type-II intermittency in the presence of noise [9]. Accordingly, as for coupled ones, while the electronically coupled Nd:YAG lasers have 2π phase jumps in the intermittent PS state and $\pi/2$ phase shift in the PS state between two laser outputs [12], the optically coupled ones have $\pm 2\pi$ phase jumps in the intermittent PS state and no phase shift in the PS state [13]. Because of the different mechanisms, we can thus classify the transition route into two types: the transition to LS from PS with $\pi/2$ phase shift in the elcectronically coupled Nd:YAG lasers [14] and the direct transition to CS from PS without phase shift in the optically coupled Nd:YAG lasers [15].

In coupled lasers, comparatively, LS has a much shorter history of studies than CS, because many kinds of coupled chaotic lasers directly transit from PS to CS without LS even though there is a parameter mismatch [8, 15]. For example, in optically coupled Nd;YAG lasers, a PS state directly develops to CS state without LS state [15]. So it is not easy to observe LS in laser systems experimentally. One case of observation of LS in a laser system is the coupled diode lasers with time-delay feedback [16, 17]. The experimental observation of LS in electronically coupled Nd:YAG lasers pumped by diode lasers is another case of LS in coupled lasers.

5. Conclusion

We have investigated synchronous phenomena in coupled Nd:YAG lasers experimentally and found two transition routes depending on the coupling method. When two chaotic Nd:YAG lasers are electronically coupled with each other, the lasers exhibit the transition from nonsynchronous state to LS through intermittent PS with gradually increasing 2π phase jumps, and then PS with $\pi/2$ phase shift, as the coupling strength increases. When two Nd:YAG lasers are optically coupled, they exhibit the transition from nonsynchronous state to CS through intermittent PS with irregular $\pm 2\pi$ phase jumps, and then PS without phase shift, as the coupling strength increases. The finding of these two transition routes will be helpful for deep understanding of laser dynamics and for applications of synchronization of chaotic lasers.

Acknowledgments

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