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Leader-laggard relationship of lag synchronization of chaos in mutually-coupled vertical-cavity surface-emitting lasers

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Abstract – We experimentally observe synchronization of chaos in two mutually-coupled vertical-cavity surface-emitting lasers (VCSELs). We observe in-phase and anti-phase synchronization of polarization-resolved chaotic temporal waveforms under the condition of injection locking. We investigate leader-laggard relationship between two chaotic waveforms of mutually coupled VCSELs and find that the laser with longer wavelength becomes the leader. The leader-laggard relationship is related to the characteristics of injection locking in optically coupled VCSELs.

1. Introduction

Vertical-cavity surface-emitting lasers (VCSELs) have been considered as novel devices for optical communications and wireless local area networks. The laser light emits to the perpendicular direction to the surface in VCSELs. Because of their short cavity length (a few micron) VCSELs emit in a single-longitudinal mode. VCSELs are very sensitive to optical injection or optical feedback due to their short cavity length, in spite of high reflectivity of their facets.

The chaotic dynamics of VCSELs have been intensively investigated for many years [1-6]. The dynamics of polarization mode hopping and polarization switching have also been reported recently [4-6]. VCSELs has attracted increasing interests for the applications of chaotic secure communications in optical local area networks. Synchronization of chaos in unidirectionally coupled VCSELs has been studied numerically [7]. Experimental observation of chaos synchronization in VCSELs has been reported in mutual [8] and unidirectional [9] coupling configurations. A signal transmission embedded on chaotic waveforms in VCSELs has been demonstrated experimentally [10]. However, the detail characteristics of chaos synchronization in coupled VCSELs have not been well understood yet. For example, symmetry breaking and leader-laggard relationship has been observed in mutually-coupled edge-emitting semiconductor lasers [11-13], but not in VCSELs. The polarization dynamics may play a crucial role for the synchronization characteristics in coupled VCSELs.

In this study, We experimentally observe synchronization of chaos in two mutually-coupled VCSELs. We observe in-phase and anti-phase synchronization of polarization-resolved chaotic temporal waveforms under the condition of injection locking. We also investigate leader-laggard relationship between two chaotic waveforms of coupled VCSELs and find the condition which laser becomes the leader or the laggard.

2. Vertical-cavity surface-emitting lasers (VCSEL)

The structure of VCSELs is shown in Fig. 1. The laser light emits to the perpendicular direction to the surface. VCSELs have some advantages compared with edge-emitting conventional semiconductor lasers: low threshold current, compact, high efficiency, large modulation bandwidth, and wafer-scale integration capability for large array configuration.

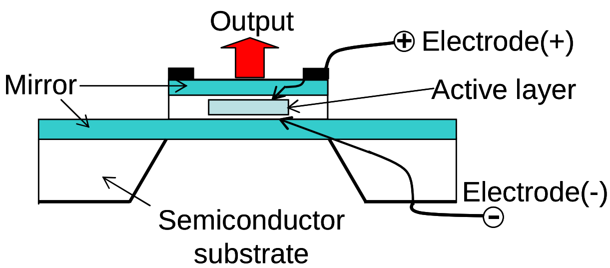

Fig. 1 Schematics of VCSEL structure.

3. Experiment

3.1. L-I characteristics

We measured L-I (Light power – Injection current) characteristics of the two solitary VCSELs (called VCSEL 1 and VCSEL 2). The polarization modes are resolved into two orthogonal components (x-mode and y-mode) by using a polarizer. Here we define x-mode as the first lasing polarization mode after the lasing threshold. We measured the light power for x-mode, y-mode, and total intensities as a function of the injection current, as shown in Fig. 2. The lasing thresholds are $I_{th,1}$ = 0.39 mA and $I_{th,2}$ = 0.37 mA for VCSEL 1 and 2, respectively. One polarization mode (x-mode) starts lasing just above the threshold. For VCSEL 1, the first polarization switching occurs at the injection current of 1.2 mA and the y-mode intensity starts lasing, where the y-mode intensity is larger than the x-mode intensity. The second polarization switching is observed at 6.5 mA and the x-mode intensity becomes larger again. We also observed polarization switching twice in VCSEL 2, although the switching is not as clear as that for VCSEL 1.

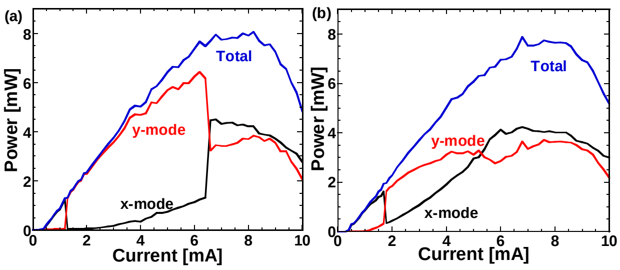

Fig. 2 Light power – Injection current (L-I) characteristics for (a) VCSEL1 and (b) VCSEL2.

3.2. Polarization dynamics in one VCSEL

Figure 3 shows our experimental setup for the observation of the dynamics of two polarization modes. An external mirror is set in front of a VCSEL to obtain self-feedback laser light. Chaos is induced by time-delayed optical feedback. The polarization modes of the VCSEL are resolved into two orthogonal components (x- and y-modes) by using a polarizer. The two polarization-mode dynamics are observed by an oscilloscope through a photodetector. Fig. 4(a) shows the temporal waveforms of the two polarization modes and Fig. 4(b) shows the corresponding radio-frequency (RF) spectra. The two chaotic temporal waveforms are anti-correlated, which is called anti-phase dynamics. The RF spectra show that the fundamental frequency of the chaotic waveforms is 360 MHz, which corresponds to the inverse of the round-trip time (2.8 ns) from the VCSEL to the external mirror.

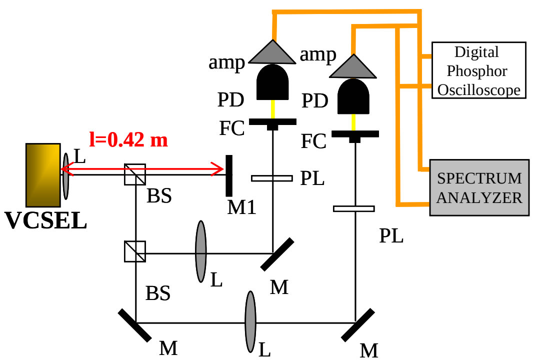

Fig. 3 Experimental setup for the observation of the dynamics of two polarization modes.

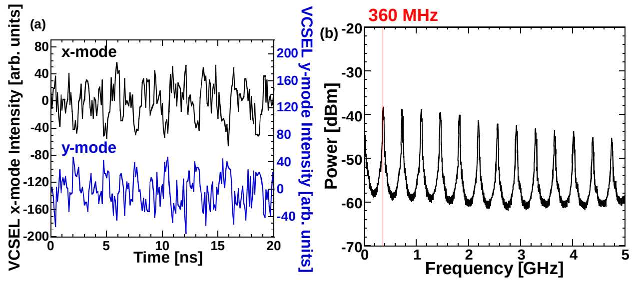

Fig. 4 (a)Anti-phase dynamics of two polarization modes (x-mode and y-mode) and (b) RF spectra of y-mode intensities.

3.3. Chaos synchronization in mutually coupled VCSELs

We used the two VCSELs for our synchronization experiment. Figure 5 shows the experimental setup for chaos synchronization. The two solitary VCSELs (VCSEL 1 and 2) were mutually coupled to each other without an external mirror. The distance between the two VCSELs was set to 0.96 m, corresponding to the one-way coupling delay time of τ = 3.2 ns. A half wave-plate was inserted into the optical path of the two VCSELs to match the polarization direction between the two VCSELs. Each of the two polarization intensities of VCSEL 1 was coupled to that of VCSEL 2. The polarization-resolved temporal dynamics of the two VCSELs were observed by using a digital oscilloscope through two photodetectors with electronic amplifiers. The optical wavelength of the VCSELs was measured by an optical spectrum analyzer. The optical wavelengths of the two VCSELs were controlled by changing the temperature of the VCSELs with a resolution of 0.01 K.

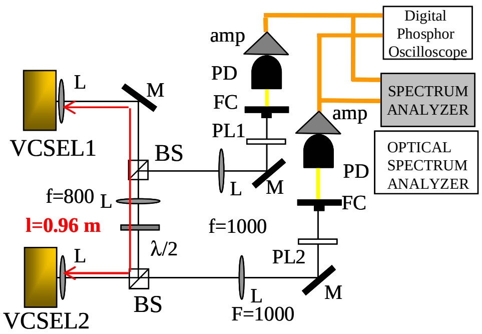

Fig. 5 Experimental setup for chaos synchronization.

We use injection locking to achieve chaos synchronization. Injection locking is a technique to match the optical wavelength between two lasers by injecting a laser beam. Injection locking is achieved when the temperature of the VCSELs are precisely controlled in our experiment. The temperatures are set to 293.00 and 301.67 K for VCSEL 1 and 2, respectively. Using injection locking, the wavelength of the two VCSELs completely matches to 858.036nm as shown in Fig. 6. We observed the polarization-resolved temporal waveforms of the two VCSELs under mutual coupling.

Figure 7(a) shows temporal waveforms of the y-mode intensity for VCSEL 1 and 2. We shifted the temporal waveform of VCSEL 2 with respect to VCSEL 1 by the one-way coupling delay time (τ = 3.2 ns) to obtain the optimal synchronization. Synchronization of chaos is observed between the y-mode of VCSEL1 and y-mode of VCSEL2. The temporal waveforms show in-phase synchronization of chaos. Figure 7(b) shows the correlation plots of the y-mode intensities of the VCSEL 1 and 2. Good linear correlation is observed and high accuracy of chaos synchronization is achieved. Moreover, the RF spectra between the two VCSELs match well to each other (not shown) and the peak frequency appears at 156 MHz, corresponding to the inverse of the round-trip coupling delay time (2τ = 6.4 ns) between the two VCSELs.

We also observed the temporal waveforms between the different modes of the two VCSELs. Figure 7(c) shows the temporal waveforms of the y-mode intensity of VCSEL 1 and the x-mode intensity of VCSEL 2. We shifted the temporal waveform of VCSEL 2 with respect to VCSEL 1 by the one-way coupling delay time (τ = 3.2 ns) to obtain the optimal synchronization, as well as Fig. 7(a). The temporal waveforms are anti-correlated and

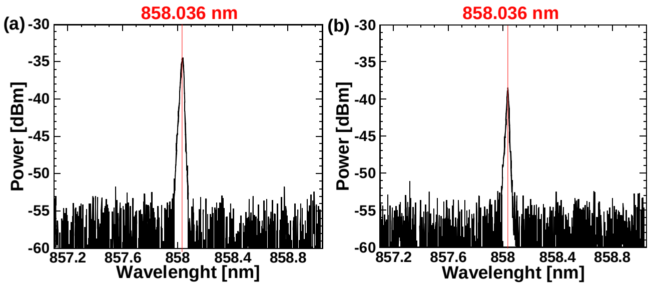

Fig. 6 Optical spectra of (a)VCSEL1 and (b) VCSEL2.

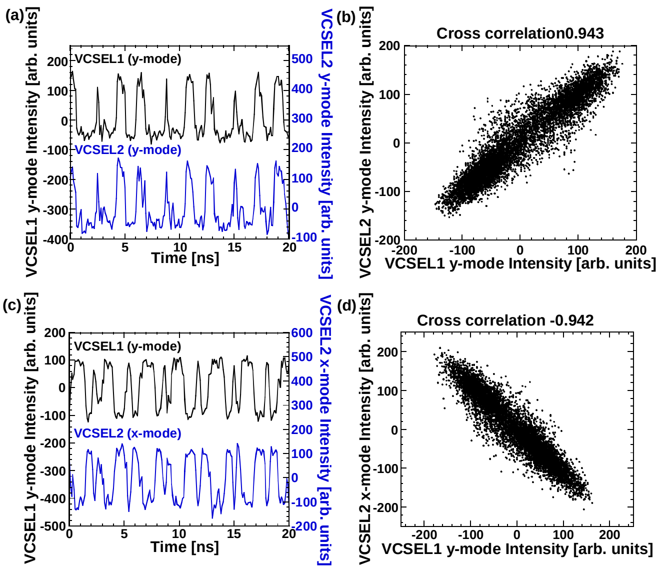

Fig. 7 Temporal waveforms and correlation plots for (a),(b) y-mode of VCSEL 1 and y-mode of VCSEL2, and (c),(d) y-mode of VCSEL 1 and x-mode of VCSEL 2.

anti-phase synchronization of chaos is observed. Figure 7(d) shows the correlation plot between the y-mode of VCSEL 1 and the x-mode of VCSEL 2. The correlation plot has a negative slope and high accuracy of anti-phase synchronization of chaos is observed. The RF spectra look very similar between the two VCSELs. We speculate that strong anti-phase dynamics in a solitary VCSEL is responsible for the anti-phase synchronization of chaos.

3.4. Leader-laggard relationship

We found that one of the VCSELs leads to the other in time by the one-way coupling delay time τ = 3.2 ns in Fig.7. A question arises which VCSEL plays a leading role as a leader to the other VCSEL (laggard). It is important to investigate the condition that determines the leader-laggard relationship in mutually-coupled time-delayed lasers for understanding coupled time-delayed nonlinear systems [11-13].

To investigate which VCSEL becomes the leader, we shifted the temporal waveform of VCSEL 2 ($I_2(t)$) with respect to VCSEL 1 ($I_1(t)$) by the one-way coupling delay time (τ = 3.2 ns) to both the forward and backward directions in time (i.e., $I_2(t+\tau)$ and $I_2\ (t-\tau)$). We calculated the cross correlation value between $I_1(t)$ and $I_2(t+\tau)$ (called C_+), and that between $I_1(t)$ and $I_2(t-\tau)$ (called C_-) as follows,

$$C_+ = \frac{\langle (I_1(t)-\bar{I}_1)(I_2(t+\tau)-\bar{I}_2)\rangle}{\sigma_1\sigma_2} \quad (1)$$

$$C_- = \frac{\langle (I_1(t)-\bar{I}_1)(I_2(t-\tau)-\bar{I}_2)\rangle}{\sigma_1\sigma_2} \quad (2)$$

where $I_{1,2}$ is the total intensity of the two temporal waveforms of VCSEL 1 and 2, $\bar{I}_{1,2}$ is the mean value of the two temporal waveforms, and $\sigma_{1,2}$ is the standard deviation of the two temporal waveforms. The angle brackets denote time averaging. The best in-phase and anti-phase synchronization is obtained at C = 1 and C = -1, respectively.

We compared C_+ with C_- . When C_+ is higher than C_- we can determine that VCSEL 1 is the leader and VCSEL 2 is the laggard, and vice versa. Figure 8 shows the leader-laggard relationship of the y-mode and x-mode temporal waveforms between VCSEL 1 and 2. When the solid black curve (C_+) is larger than the dotted red curve (C_-), VCSEL 1 is the leader and VCSEL 2 is the laggard, and vice versa. We found that VCSEL 1 becomes the leader at negative $\Delta\lambda$ ($\lambda_2 < \lambda_1$) and VCSEL 2 is the leader at positive $\Delta\lambda$ ($\lambda_2 > \lambda_1$), where $\Delta\lambda = \lambda_2 - \lambda_1$ and λ_i is the optical wavelength of VCSEL i. Therefore the VCSEL with longer wavelength becomes the leader.

To reveal the role of injection locking on the behavior of leader–laggard, we put an optical isolator in the optical path between the two VCSELs and look at the optical spectra for the two different directions of optical isolation. We set the condition of the optical wavelength where VCSEL 1 becomes the leader and VCSEL 2 is the laggard, i.e. λ_2 - λ_1 = - 0.012 nm ($\Delta\lambda$ < 0). Figure 9 shows the optical spectra of two VCSELs under the two different directions of the unidirectional coupling. When the light beam is injected from VCSEL 1 to 2, injection locking is achieved as shown in Fig. 9(a). However, when the coupling is set from VCSEL 2 to 1, injection locking is not achieved. This result indicates that injection locking is achieved only at the negative detuning ($\Delta\lambda$ < 0), which can also be observed in edge-emitting semiconductor lasers [14]. Injection locking occurs from the VCSEL with longer wavelength to the VCSEL with shorter wavelength when they are coupled even in the condition of mutual coupling. Therefore the laser with longer wavelength locks the optical wavelength of the other laser and leads to the other laser in time. The leader-laggard relationship is thus determined by the characteristics of injection locking in optically coupled VCSELs.

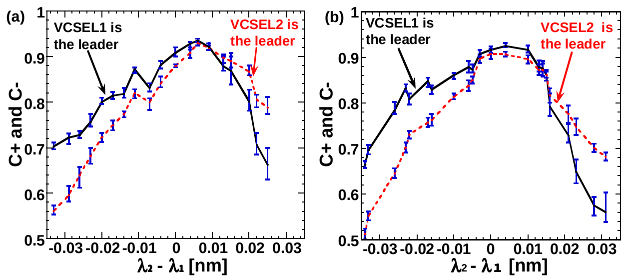

Fig. 8 Learder-laggard relationship between the synchronized chaotic waveforms between the two VCSELs for (a) y-mode and (b) x-mode intensities. When the solid black curve is higher than the dotted red curve VCSEL 1 is the leader and VCSEL 2 is the laggard, and vice versa. The VCSEL with longer wavelength becomes the leader.

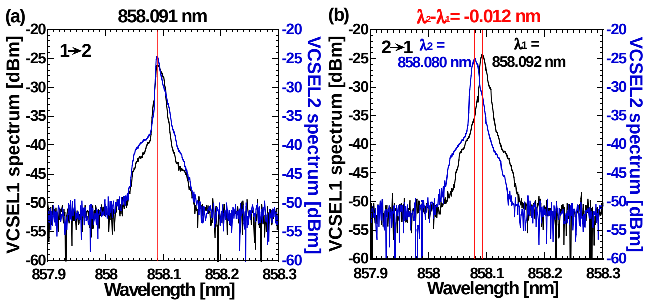

Fig. 9 Optical spectra of the y-mode intensity of VCSEL 1 and 2 when an optical isolator is inserted in the optical path of the two VCSELS to obtain unidirectional coupling (a) from VCSEL 1 to 2 and (b) from VCSEL 2 to 1. The wavelength of VCSEL 1 is longer than that of VCSEL 2. Injection locking is achieved in (a), but not in (b).

4.Conclusion

We have experimentally observed synchronization of chaos in two mutually-coupled VCSELs under injection locking. We have observed in-phase synchronization of chaos between the same polarization modes of the two VCSELs and anti-phase synchronization of chaos between the different polarization modes of the two VCSELs. We have investigated leader-laggard relationship between the two VCSELs and found that the laser with longer wavelength becomes the leader. The leader-laggard relationship is strongly related to the characteristics of injection locking in optically coupled VCSELs.

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