2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Comparison of chaotic sequences in a chaos-based DS-CDMA system

G. Kaddoum[†‡], P. Chargé [‡], D. Roviras[†], D. Fournier-Prunaret[‡]
† IRIT, ENSEEIHT
2, rue Charles Camichel B.P. 7122 31071 Toulouse Cedex 7 - FRANCE
‡ LATTIS, INSA
135 Avenue de Rangueil 31077 Toulouse Cedex 4 - France
Email: Georges.Kaddoum@enseeiht.fr, Daniele.Fournier@insa-toulouse.fr

Abstract– This paper provides a performance analysis of a direct-sequence code division multiple access (DS-CDMA) system when chaotic sequences are used instead of conventional pseudo-noise (PN) spreading codes. Different chaotic maps are compared by observing their statistical properties and calculating bit error rate (BER) in the single user case

1. Introduction

In the past few years, much research effort has been devoted to the study of communications using chaotic functions. It has been shown through many works [1] [2], that the use of chaotic waveforms instead of conventional codes (Gold, Kasami,…) can improve the system performance. Also using chaotic spreading sequences (CSS) has a second advantage over PN sequences, since it allows the increase of the transmission security [3]. All these previous works focus on how chaotic sequences can be used in order to outperform conventional communication systems [4]. Many schemes have been proposed like CSK, DCSK, [1] [2] or chaotic DS-CDMA. Most of these systems require coherent correlators and a threshold detector to decode the signals. When theoretical performance of these systems is derived at the receiver, analytic forms are obtained at the output of the correlators [5]. By often making use of the Central limit Theorem, terms of these forms are approximated by independent and identically distributed (iid) normal random variables. In fact, since chaotic sequences are purely deterministic this hypothesis is not really acceptable. But in many cases, that theoretical performance is in very good agreement with performance provided by long run Monte-Carlo simulations. Note that this approximation is good enough because correlation duration (symbol duration) is always much larger than the chaotic element duration (chip duration), (i.e. the spreading factor is very large).

In this paper we are interested in chaos-based DS-CDMA systems using very short spreading factors (SF). In this particular framework, statistical properties of chaotic maps can no longer be ignored. Correlation properties of chaotic sequences for CDMA have been considered in several papers [6]-[8], where methodologies for the design of optimal-spectrum sequences are provided. In [9] a short analysis shows the correlation merit of the spreading sequence given by the Chebyshev map in terms of SNR improvement.

The proposed work deals with the analysis of statistical properties of several chaotic maps. First, the probability distribution functions of the chaotic times series are provided. It is then shown how these distribution functions affect the performance when these series are used as direct spreading sequences for the DS-CDMA.

The paper is organized as follows. In section 2 we present the different chaotic maps for spreading spectrum. Then, in third section, the transmission system is illustrated. In this section also the performance of CSS, and the lower and upper bound of the BER are presented. Section 4 shows simulation results. Finally, section 5 reports some conclusive remarks.

2. Chaotic generator

For this study, two kinds of one-dimensional map are chosen. The first one is the Chebyshev polynomial function of order 2 (CPF) and the second is a one-dimensional noninvertible piecewise linear map (PWL).

2.1 Chebyshev map

The CPF is given as:

$$x_k = 2x_{k-1}^2 - 1 \tag{1}$$

This map has been chosen because it is a very simple function for which it has been proved that it produces chaotic sequences. Moreover, as it will be emphasized in this paper, the chaotic time series provided by the order 2 Chebyshev map has a specific probability distribution function, which is not uniform.

2.2 Piecewise-linear map

The PWL map is given as [10]:

$$\begin{cases} z_k = K|x_k| + \phi \ [\bmod 1] \\ x_{k+1} = sign(x_k)(2z_k - 1) \end{cases} \tag{2}$$

It depends on parameters K and ϕ. K is a fixed positive integer, ϕ ($0 < \phi < 1$) is a parameter that can be changed to produce different sequences, and the initial condition x_0 will be chosen so that $0 < x_0 < 1/K$.

3. Theoretical performance of CSS

3.1 Emitted signal

A stream of binary data symbols S_i of period T_S is spreaded by a chaotic signal $x_c(t)$ at the emitter side. Chaotic signal $x_c(t)$ is generated as in (1) or (2). A new chaotic sample is thus generated every time interval equal to T_C ($x_k = x_c(kT_C)$). The emitted spreaded signal r(t) is thus equal to :

$$r(t) = \sum_{i=0}^{\infty} \sum_{k=0}^{SF-1} S_i x_{i.SF+k}\, g(t-(iSF+k)T_C) \qquad (3)$$

where $x_{i.SF+k}$ are the chaotic samples corresponding to data symbol S_i, and g is the rectangular pulse of unit amplitude on $[0, T_C]$. Parameter SF is equal to the number of chaotic samples in a symbol duration ($SF = T_S / T_C$) and, by analogy with DS-CDMA, we have called this parameter the Spreading Factor (SF).

3.2 Received signal

The purpose of the paper being to compare different classes of CSS we will use here a very basic transmission channel like the Additive White Gaussian Noise (AWGN) channel. The additive noise has a power spectral density equal to $N_0/2$. The received signal is first de-spreaded by a replica of the chaotic signal used at the emitter side (perfect synchronisation is assumed) and then integrated on the symbol duration T_S.

After integration, the decision variable associated to symbol i is thus :

$$D_i = S_i T_C \sum_{k=0}^{SF-1} (x_{i.SF+k})^2 + n_i = S_i \xi_{bc}^{(i)} + n_i \qquad (4)$$

where $\xi_{bc}^{(i)}$ is the chaotic signal energy corresponding to the symbol interval number i and n_i is the noise after despreading and integration.

3.3 Theoretical BER

Without loss of generality we will assume in the following that we use a BPSK type modulation.

If the energy of the spreading signal is constant (as it is assumed in DS-CDMA or BPSK cases), the BER is given by the following expression [11]:

$$BER_{BPSK} = Q\left(\sqrt{\frac{2\xi_b}{N_0}}\right) \qquad (5)$$

where ξ_b is the constant transmitted bit energy and $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-u^2}{2}\right)} du$ is the complementary cumulative distribution function of the standardized normal random variable.

When the bit energy is not constant (as it is with chaotic waveforms), the BER is given by the integration of (5) over all possible values of the bit energy:

$$BER_{choas} = \int_0^{+\infty} Q\left(\sqrt{\frac{2\xi_{bc}}{N_0}}\right) p(\xi_{bc}) d\xi_{bc} \qquad (6)$$

where ξ_{bc} is the chaos energy computed on symbol duration T_S.

3.4 Lower and upper bounds of the BER

The lower bound of the BER is given by equation (5) and corresponds to constant bit energy.

The BER is a function of the distribution of the bit energy. The degradation is more important when the distribution of the bit energy is high at low values.

As a benchmark for BER upper bound we will use a Gaussian distributed sequence. Among all sequences used in [12] this Gaussian sequence appeared to be an upper bound of the BER for all types of chaotic maps. The Gaussian sequence is a zero mean sequence with independent samples with variance equal to σ_X^2.

For such Gaussian sequence the BER is given by [12]:

$$BER_{Gauss} = E\left\{Q\left[\left(-\left[\frac{\sigma_X}{\sigma}\right]\sqrt{\chi_{SF}^2}\right)\right]\right\} \qquad (7)$$

where χ_{SF}^2 is a chi-square random distribution with SF degrees of freedom and σ^2 the variance of the Gaussian additive noise.

4. Simulation results

The parameters of PWL are fixed as follow: ($K=3$, $\phi=0.1$) and ($K=2$, $\phi=0.5$).

Following expression (6) it is necessary to get the bit energy distribution before computing the BER.

Figure 1 gives the probability density function (PDF) of the bit energy for four types of spreading sequences and for a spreading factor equal to 10. All sequences have a constant mean power. The histogram of figure 1 has been obtained using one million samples.

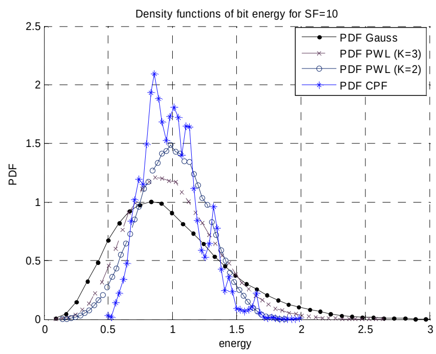

Fig.1 Probability density functions of bit energy for various chaotic sequences (SF=10).

Using the histogram of figure 1 we can compute the BER of (6) by using the following expression:

$$BER_{chaos} \approx \sum_{i=1}^{m} Q\left(\sqrt{\frac{2\xi_{bc}^{(i)}}{N_0}}\right) P\left(\xi_{bc}^{(i)}\right) \qquad (8)$$

where m is the number of histogram classes and $P\left(\xi_{bc}^{(i)}\right)$ is the probability of having the energy in intervals centred in $\xi_{bc}^{(i)}$.

Looking at figure 1 it is clear that the Chebyshev chaotic map will give better results in terms of BER than the other sequences. In contrast, the energy PDF of the Gaussian sequence has the higher distribution values for low energies and it will result in the worse BER.

Figure 2 gives the BER of the three sequences (CPL, PWL (K=2), PWL (K=3)) of figure 1 plus the lower bound BER corresponding to the BPSK case.

Computed BER on figure 2 are computed using the histograms of figure 1 together with equation (8). Simulation results are also plotted on figure 2 showing a perfect agreement.

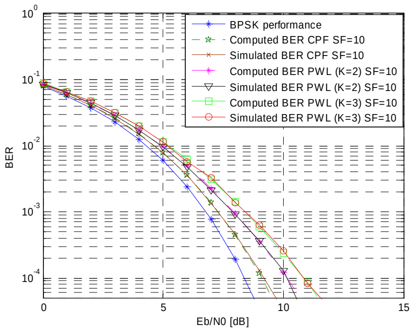

Fig.2. Computed and simulated BER performance of different chaotic sequences (SF=10)

BER on figure 2 are in good agreement with the PDF energy of figure 1.

When the spreading factor decreases (SF=5 on figure 3) performance of different sequences tends to worsen compared to the BPSK lower bound. This is quite evident because when SF is lower the energy distribution tends to be broader.

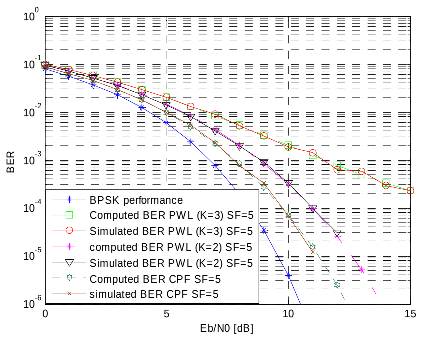

Fig.3. Computed and simulated BER performance of different chaotic sequences (SF=5)

On the other hand, when the spreading factor is very high (SF=50 in figure 4) all BER tend to be equal to the lower BER bound. This means that the energy distribution tends to be very sharp around the mean energy when SF is high. This is illustrated on figure 4.

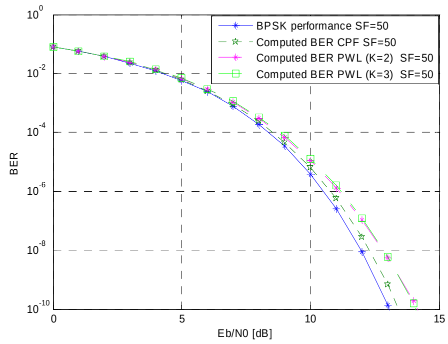

Fig.4. Computed BER performance of different chaotic sequences (SF=50)

Figure 5 shows BER lower and upper bounds, together with the performance of CSS for SF=1, 2, 5, 10. As we can see on figure 5 the performance improvement for the piecewise linear map is increasing regularly with increasing SF. On the other hand, for the Chebyshev map, the improvement is very important between SF equal one and SF equal two. This improvement of BER performance can be explained by the correlation between successive samples in the Chebyshev sequence.

In figure 6 the histogram of the couple of two consecutive square samples $[\,x_k^2, x_{k+1}^2\,]$ is plotted for the Chebyshev case. As we can see on figure 6 two consecutive samples are highly correlated. The probability of having two samples very close to zero is very low. This means that having bit energy very low, for SF=2, has a very low probability. On the contrary, the probability of having two samples of the form (0, +1) or (+1, +1) is high yielding to a high bit energy (almost one value is high). This is the reason for the great improvement of BER from SF equal to one to SF equal to two for the Chebyshev map.

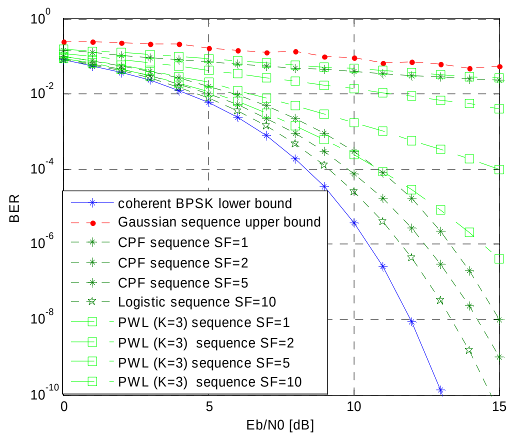

Fig.5 Lower and upper bound simulation

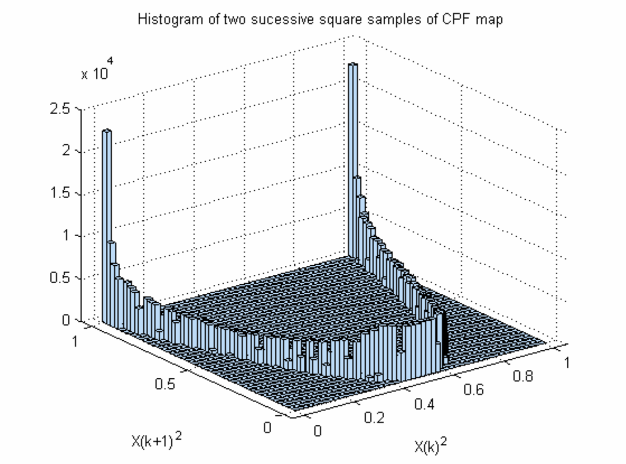

Fig.6 Histogram of two successive square samples of Chebyshev chaotic map

For the piecewise linear map we have less dependence between consecutive samples and, in addition, the probability to find two successive samples close to zero is high, as shown in figure 7. This behaviour explains why the improvement for PWL is lower than the CPF one when SF goes from one to two.

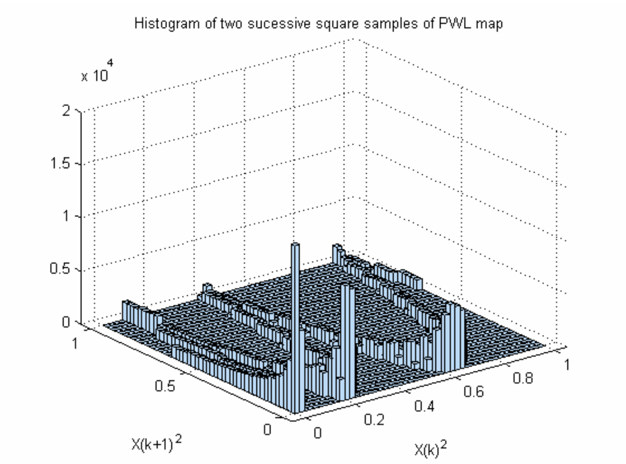

Fig.7 Histogram of two successive square samples of piecewise linear map for $K = 3$, $\phi = 0.1$

5. Conclusion

In this paper we have proposed a new methodology for selecting the optimal chaotic spread spectrum sequence for synchronous CDMA systems. The performance in terms of BER is based on the distribution of the bit energy. The best performance (lower bound) is obtained when bit energy is constant (BPSK case). For CSS the best choice are the chaotic sequences with the minimum distribution of the energy around the mean value, particularly for low energy values. In the paper we have also pointed out the importance of chaotic samples dependence for the performance, particularly for the CPF case. The shape of energy distribution shows that CPF chaotic waveforms outperform PWL ones in terms of BER performance. The difference in performance is highlighted for low spreading factor. When SF is high (SF=50) the difference is much lower because all CSS tend to have the same type of energy distribution.

References

[1] M. P. Kennedy, R. Rovatti, and G. Setti, Eds., chaotic Electronics in Telecommunications. London, U.K.: CRC, 2000. vol 49, pp. 1495-1499, Oct. 2002.

[2] F. C. M. Lau and C. K. Tse, Chaos-Cased Digital Communication Systems. Heidelberg, Germany: Springer-Verlag, 2003.

[3] T. Yang "A survey of chaotic secure communication systems" International Journal of Computational Cognition, Volume 2, Number 2, Pages 81–130, June 2004.

[4] K. Umeno, K. K. Kitayama, "Improvement of SNR with chaotic spreading sequences for CDMA" IEEE Information Theory Workshop, South Africa, June, 1999.

[5]G. Kolumban, "Theoretical Noise Performance of correlation-Based Chaotic Communications Schemes," IEEE Transactions on Circuits and Systems, vol. 47, no. 12, pp. 1692-1701, December2000

[6] C.C. Chen, K. Yao, K. Umeno, and E. Biglieri, Design of spread-spectrum sequences using chaotic dynamical systems and ergodic theory" IEEE Trans. Circuits Syst., I, Fundam. Theory Appl. 48, No.9, pp. 1110-1114, 2001.

[7] T. Kohda and A. Tsuneda. "Pseudonoise Sequences by Chaotic nonlinear maps and their correlations Properties", IEICE Trans. Commun. Vol. E76-B. pp. 855-862 Aug. 1993.

[8] R. Rovatti, G. Setti, G.Mazzini, "Toward Sequence Optimization for Chaos-Based Asynchronous DS-CDMA Systems," in Proc. IEEE GLOBECOM'98, pp. 2174-2179, 1998

[9] K. Umeno and K. Kitayama, "Spreading sequences using periodic orbits of chaos for CDMA" Electronics Letters, Volume 35, Issue 7, p. 545-546, April 1999

[10] P. Chargé, D. Fournier-Prunaret, "An M-ary Orthogonal-CSK communication system" International Symposium on Nonlinear Theory and its Applications NOLTA 2006 Bologna, Italy, Septembre 2006

[11] J.G. Proakis, Digital Communications. Boston, MA: McGraw-Hill, 2001

[12] A. J. Lawrance, G. Ohama, "Exact calculation of bit error rates in communication systems with chaotic modulation" IEEE Transaction on circuits and systems-I: fundamental theory and applications, Vol. 50, NO. 11, November 2003.