2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Robustness in Time Series Prediction based on Local Orbit Instability Method

Hironori Sawayanagi†, Jousuke Kuroiwa†, Haruhiko Shirai§,Tomohiro Odaka¶ and Hisakazu Ogura†

† Graduate Course of Human & Artificial Intelligent Systems, University of Fukui
§ Technical Support Division, University of Fukui
¶ Graduate Course of Nuclear Power & Energy Safety Engineering, University of Fukui
3-9-1 Bunkyo, Fukui, 910-8507, Japan

Abstract—In this paper, we investigate the robustness in the time series prediction method based on the local instability of the orbit, which we proposed previously. In our proposed method, we evaluate the Jacobian matrix by considering the structure of the local instability of the orbit. Thus, the Jacobian matrix is calculated from a certain ellipsoidal neighboring region around a reference data point. From computer experiments, comparing with the conventional method where a spherical region is assumed as the neighboring region, our method can predict future behavior of the systems with higher precision independent from the size of the neighboring region. It suggests us that our method can give the robust prediction.

1. Introduction

The time series prediction is one of important problems in chaotic time series analysis[1, 2, 3, 4]. Recently, we have proposed a novel evaluation method of the Jacobian matrix by considering the structure of the local instability of the orbit[5]. In conventional method, the Jacobian matrix is calculated from a certain spherical neighboring region around the reference data point at each time step[2]. In the evaluation process, the accuracy depends on the number of data points. Thus, at the situation that we can observe behaviors of the system during a long time duration, we can ensure the accuracy of the evaluation. In usual, however, the number of data points which we can observe is limited, then it is hard to archive the desired accuracy.

In addition, even though one sets the size of the neighboring region being larger, the accuracy could not be better. We consider the reason as follows. In the case that the data points are dispersed uniformly, we could succeed to evaluate the Jacobian matrix from a certain spherical neighboring region. In usual, chaotic data forms a certain attractor, and reveals trajectory(orbit) instability toward certain directions, that is, the data points are not dispersed uniformly. Therefore, from data points in the neighboring region with a certain larger size, it is difficult to get effective informations, instability or stability of the orbit.

In order to overcome the problem, we have proposed our evaluation method of the Jacobian matrix by considering the structure of the local instability of the orbit[5]. We have succeeded to give better prediction accuracy for several examples though the case of the limited data number. The size of the neighboring region is one of important factors in evaluating Jacobian matrix, finally ensuring the accuracy in predicting future behavior. Therefore, our purpose of the paper is to show that our evaluation method is robust for the size of the neighboring region.

2. Jacobian Matrix Evaluation Based On Local Orbit Instability

Let us present our method briefly[5]. Let us consider a certain nth order nonlinear dynamical system, sometimes the system reconstructed by Takens's embedding theorem[6]. We denote a n-dimensional state vector in system to be $\boldsymbol{x}(t)$.

In the local linear prediction method[4], a future point of $\boldsymbol{x}(t)$ can be given,

$$\hat{\boldsymbol{x}}(t+1) = \boldsymbol{DF}(\boldsymbol{x}(t))(\boldsymbol{x}(t) - \boldsymbol{x}(s)) + \boldsymbol{x}(s), \tag{1}$$

where $\boldsymbol{DF}(\boldsymbol{x}(t))$ is a Jacobian matrix and $\boldsymbol{x}(s)$ is the nearest neighbor point on the attractor. Thus, it is important to ensure the accuracy in evaluating the Jacobian matrix in order to realize a better prediction.

In our method in evaluating Jacobian matrix, we utilize the information of local instability of trajectory adding to the information of $\boldsymbol{x}(t)$. In the conventional evaluation method of the Jacobian matrix, a spherical neighboring region is assumed as a near neighbor set of $\boldsymbol{x}(t)$. In our method, we extend the radius for certain directions of the spherical neighboring region. The direction is determined by considering local instability of the orbit. The key point of our method is how to evaluate local instability of the orbit. In our method, we employ the concept of local Lyapunov spectra [1, 3].

The local Lyapunov spectra are calculated by

$$\lambda_i(t) = \frac{1}{T}\sum_{\tau=1}^{T} \log|\boldsymbol{e}'^{(i)}(t-\tau)|\ (t \geq t_0), \tag{2}$$

where T is the small number of time steps and t_0 is a certain duration time steps for excluding transient behavior. The

n-dimensional vector $\boldsymbol{e}'^{(i)}(t)$ is given by performing Gram-Schmidt to $\boldsymbol{e}^{(i)}(t)$, and $\boldsymbol{e}^{(i)}(t)$ is given by

$$\boldsymbol{e}^{(i)}(t+1) = \boldsymbol{DF}(t)\boldsymbol{u}^{(i)}(t), \quad (i = 1, 2, \cdots, n), \tag{3}$$

where the orthonormal base vector $\boldsymbol{u}^{(i)}(t)$ is given by performing Gram-Schmidt ortho-normalization to $\boldsymbol{e}^{(i)}(t)$. It should be noted that in the calculation of Eq.(3), the evaluation of $\boldsymbol{DF}(t)$ is done with a spherical neighboring region.

Based on the concept of local Lyapunov spectra, if the value of $\lambda_i(t)$ is positive, $\boldsymbol{u}^{(i)}(t)$ represents the unstable direction of the trajectory at time t. On the other hand, if the value of $\lambda_i(t)$ is negative, $\boldsymbol{u}^{(i)}(t)$ represents the stable direction. Therefore, we determine the direction to extend the radius depending on whether $\lambda_i(t)$ is positive or negative.

Now, let us summarize our method.

1. Evaluate the Jacobian matrix from a certain spherical neighboring region based on the conventional method[2].

2. Based on Eq.(2), calculate local Lyapunov spectra.

3. Change the spherical neighboring region into the ellipsoidal neighboring region. The radius of the spherical neighboring region is extended to the direction of $\boldsymbol{u}^{(i)}(t)$ depending on the sign of $\lambda_i(t)$.

4. Evaluate another Jacobian matrix from the ellipsoidal neighboring region.

3. Computer Experiments

3.1. Purposes and Procedure

The purpose of the computer experiment is to investigate the dependence of the prediction accuracy on the size of the neighboring region. In addition, we investigate which direction is better to extend the radius of the spherical neighboring region, stable direction or unstable direction. Therefore, we perform prediction with the change of the radius of the neighboring region for two cases; evaluating the Jacobian matrix with expanding the radius toward (i)the stable direction and (ii) the unstable direction.

3.2. Conditions

In computer experiments, we employ two types of chaotic attractors, Rössler attractor and Ikeda attractor. The Rössler attractor is written by[7],

$$\begin{cases} \frac{dx}{dt} = -y - z \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z(x - c) \end{cases} \tag{4}$$

where $a = 0.15$, $b = 0.2$ and $c = 10$. We generate time series of 3-dimensional state vector, $(x(0), y(0), z(0))^T$, $(x(1), y(1), z(1))^T$, $\cdots$, by calculating Eq.4 based on forth order Runge-Kutta method with the step size of 0.1. The shape of the Rössler attractor is given in Figure 1.

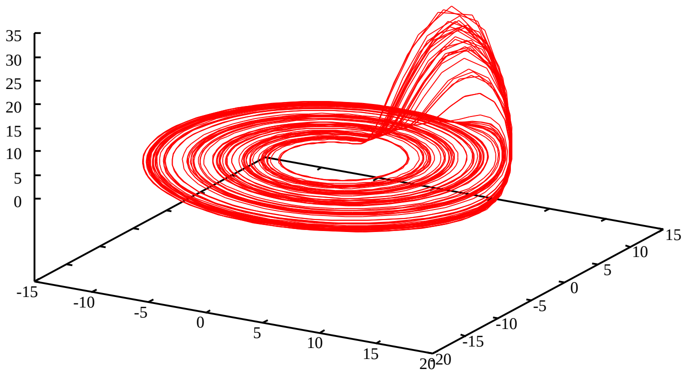

Figure 1: Rössler attractor

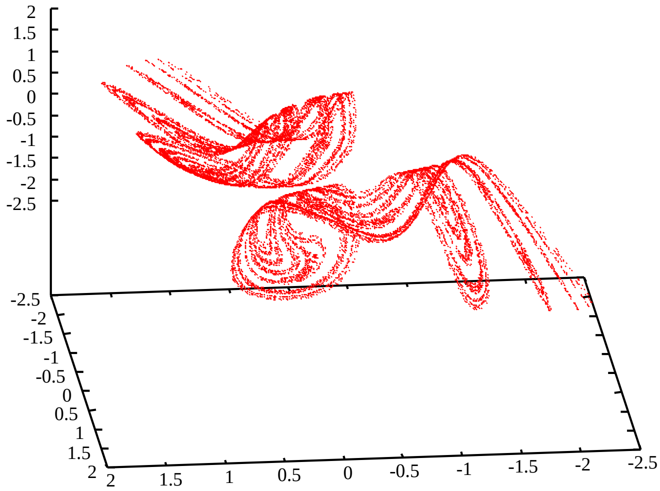

Figure 2: Restructured Ikeda attractor

On the other hand, The Ikeda attractor is presented by[8],

$$\begin{cases} x(t+1) = q + b(x(t)\cos\theta((t)) - y(t)\sin(\theta(t))) \\ y(t+1) = b(x(t)\sin(\theta(t)) + y(t)\cos(\theta(t))), \end{cases} \tag{5}$$

where,

$$\theta(t) = \kappa - \frac{\alpha}{1 + x(t)^2 + y(t)^2}. \tag{6}$$

We set the parameters to be $q = 1$, $b = 0.9$, $\kappa = 0.4$ and $\alpha = 6$. In computer experiments, we reconstruct 3-dimensional attractor from time series of $x(0)$, $x(1)$, $\cdots$ with the time delay of $\tau = 1$ according to the Takens's embedding theorem[6]. The reconstructed Ikeda attractor is given in Figure 2.

In the evaluation process of the Jacobian matrix, we set the duration time of t_0 to be 1000, and T to be 1. We perform prediction of $N = 100$ data points for the Rössler attractor and the reconstructed Ikeda attractor. We predict 1 step future point for each data and evaluate a prediction

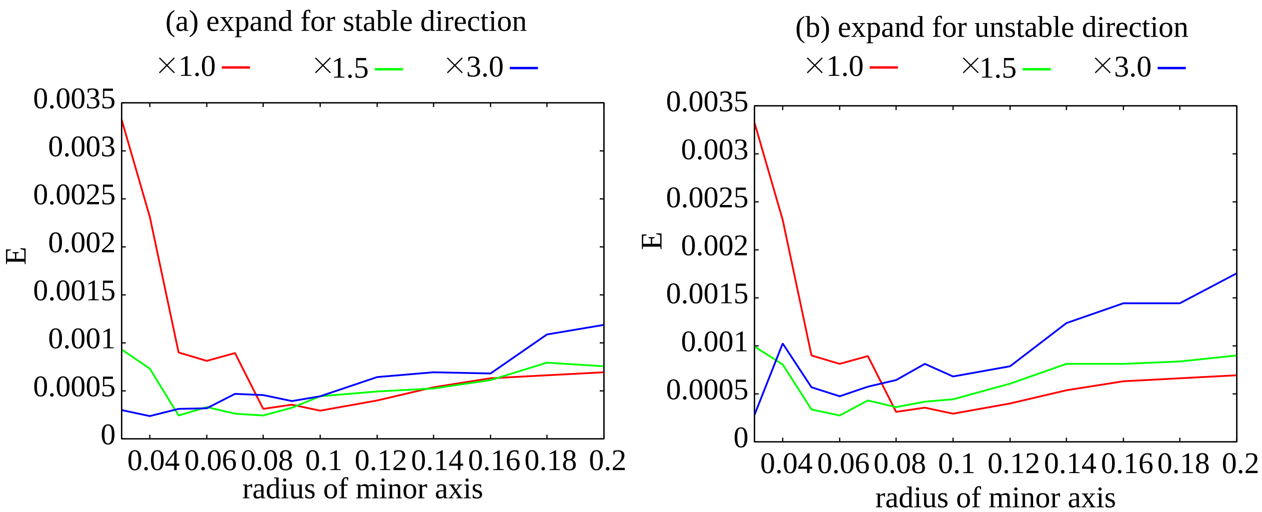

Figure 3: Prediction accuracy for the 1 step future point in the Rössler attractor. (a) The result for extending to stable direction. (b) The result for extending to unstable direction. The horizontal axis represents radius of minor axis and the vertical axis prediction accuracy E. For an instance, the radius 0.1 means that the radius covers 10% of the whole attractor region. The red line represents that the major axis is 1 times as long as the minor axis, that is, we employ the conventional spherical neighboring region. The green line represents that the major axis is 1.5 times as long as the minor axis and the blue line that the major axis is 3.0 times as minor axis.

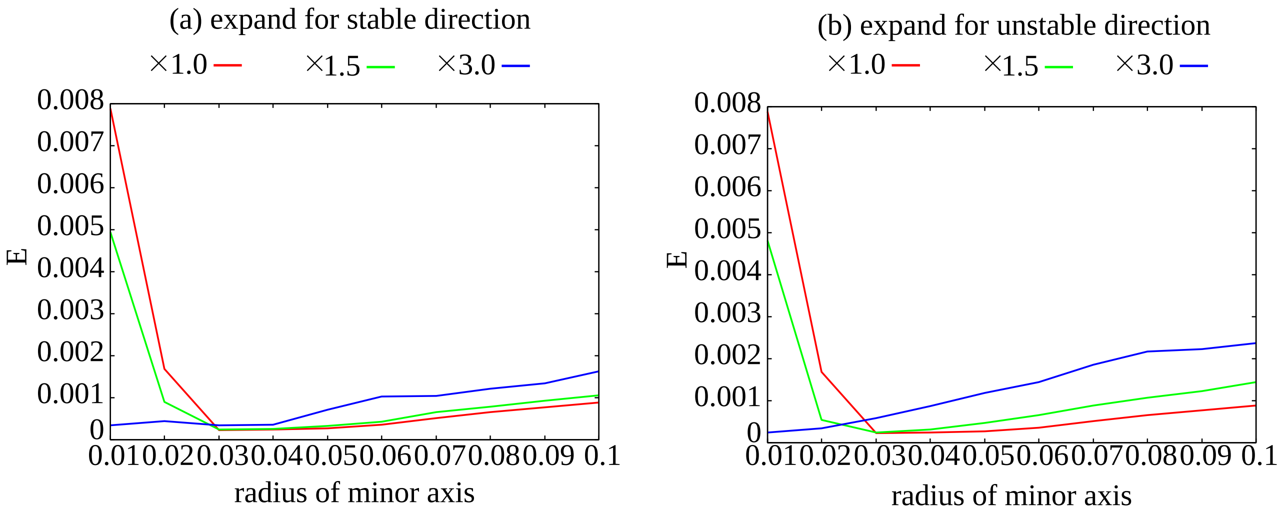

Figure 4: Prediction accuracy for 1 step future point in the reconstructed Ikeda attractor. (a) The result for extending to stable direction. (b) The result for extending to unstable direction. In details, see Figure 3.

accuracy by,

$$E = \frac{1}{N}\sum_{t=t_0}^{t_0+N-1} \frac{\sqrt{\sum_{i=1}^{3}(\hat{x}_i(t+1)-x_i(t+1))^2}}{2R}, \qquad (7)$$

where $\hat{x}_i(t+1)$ is a prediction value, $x_i(t+1)$ is the true value given by the attractor, $N = 100$ and R is the radius of the major axis of the corresponding attractor. Smaller value of E means that the prediction accuracy is higher.

3.3. Results

The result for the Rössler attractor is given in Figure 3. For the small radius, the prediction accuracy extremely gets worse in the conventional method with the spherical neighboring region. On the other hand, the prediction accuracy does not strongly depend on the radius in our method. Furthermore, the smallest value of E is 1.6×10^{-4} in the case that the radius of the minor axis is 0.04, the radius of the major axis is 2.4 times as long as that of the minor axis and the expanding directions are stable ones.

The result for the reconstructed Ikeda attractor is given in Figure 4. Similarly, for the small radius, the prediction accuracy extremely gets worse in the conventional method. On the other hand, the prediction accuracy does not depend on the radius in our method with the major radius of 3 *times* the minor axis.

From the results, our method can give a better prediction accuracy among various radius of the neighboring region comparing with the conventional method. Thus, our method is robust for the radius size in predicting future data points.

4. Conclusions

In the paper, we present our evaluation method of the Jacobian matrix by considering the structure of the local instability of the orbit. We apply our method to time series prediction. We investigate prediction accuracy with the change of the radius size of the neighboring region. Results are as follows:

- Our method reveals good prediction accuracy among various radius of the neighboring region comparing with the conventional method. Thus, our method is robust for the radius size in predicting future data points.
- For the Rössler attractor, our method can give the highest prediction accuracy in the case that the radius of the minor axis is 0.04, the radius of the major axis is 2.4 times as long as that of the minor axis and the expanding directions are stable ones.

Therefore, our method is practical in time series prediction. In the near future, we apply our method to real data.

References

[1] H. Fujisaka, “Statistical dynamics generated by fluctuations of local Lyapunov exponents,”*Prog. Theor. Phys.*, vol.70, pp.1264–1275,1983.

[2] M. Sano,Y. Sawada, “Measurement of the Lyapunov Spectrum from a Chaotic Time Series,”*Physical Review Letters*, vol.55, pp.1082–1086, 1985.

[3] B. Eckhardt,D. Yao, “Local Lyapunov exponents in chaotic systems,”*Physica D: Nonlinear Phenomena*,vol.65, pp.100–108, 1993.

[4] T. Ikeguchi,K. Aihara, “Nonlinear modeling of biological time series,”*Elsevier Science B.V.*, 1994.

[5] H. Sawayanagi, J. Kuroiwa, H. Ogura, T. Odaka I, Takahashi, H. Shirai, “Local trajectory instability and chaotic temporal series prediction,” *Proc. of 2007 RISP International Workshop on Nonlinear Circuits and Signal Processing*(NCSP’07, Shanghai, China), pp.345–348, 2007.

[6] F. Takens, and others, “Detecting strange attractorsin turbulence,” *Lecture Notes in Mathematics*, vol.898, pp366–381, 1981.

[7] O. E. Rössler, “An equation for continuous chaos,” *Physics Letters*, vol.57A, No.5, pp.397–398,1976.

[8] K. Ikeda, “Multiple-valued stationary state and its instability of the transmitted light by a ring cavity system,” *Optics Communications*, vol.30, No.2, pp.257–261, 1979.