2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Prediction of high-dimensional multivariate information as an amplitude-event dynamical system

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abstract

To predict high dimensional multivariate information, such as event sizes and timings, conventional frameworks have usually treated event sizes or event timings separately. However, in real worlds, we often able to measure the event sizes and the event timings simultaneously. Therefore, it is very natural to consider that if we can measure them, it might be better to handle them simultaneously in order to realize higher predictability. Following the idea, a new prediction framework using both event sizes and timings has already been proposed. However, the framework was applied only to a low-dimensional chaos. Therefor, in the present paper, we applied the framework to high-dimensional chaos with a nonlinear prediction method. As a result, it is shown that the framework exhibits high performance than the conventional framework even if it is applied to high-dimensional chaos.

1 Introduction

In the real world, complex phenomena produced by deterministic nonlinear dynamical systems are ubiquitous. To analyze such complex phenomena, one of the important step is to construct a good model. To model these phenomena, we usually use amplitude information of an observed smooth time series. We can also use non-smooth time series, or event timings, such as interspike interval time series produced from neurons.

However, in the real world, we are often able to measure the amplitude information and the event timings simultaneously. For example, they are seismic events, and financial indices. If we can measure them, it is very natural to expect that if we can handle them simultaneously, we can construct a better model, then realize higher prediction accuracy.

Following the idea, a new prediction framework using such amplitude information and event timings has already been proposed[1]. In Ref.[1], a low-dimensional chaos was analyzed by the zeroth order prediction scheme on the framework. Then, the high predictability is realized by the framework. However, if we apply the framework to real phenomena, such as seismic events or financial indices, it is very important to analyze the validity of the framework to high dimensional data. Therefore, in the present paper, we applied the framework to high-dimensional chaos with a nonlinear prediction method.

2 Prediction Framework

In the conventional frameworks, we usually use observed amplitude values that are equally sampled, or observed timings, such as a spike series. In these cases, we only use one-dimensional observed values produced from an unknown dynamical system for prediction: we use only event sizes to predict event sizes, and only event timings to predict event timings. Then, we can recover the dynamics of an unknown system from the information of observed time series by an embedding theorem[2]. In the following, we described an event size as $V(n)$ and an event timing as $T(n)$ (Fig.1).

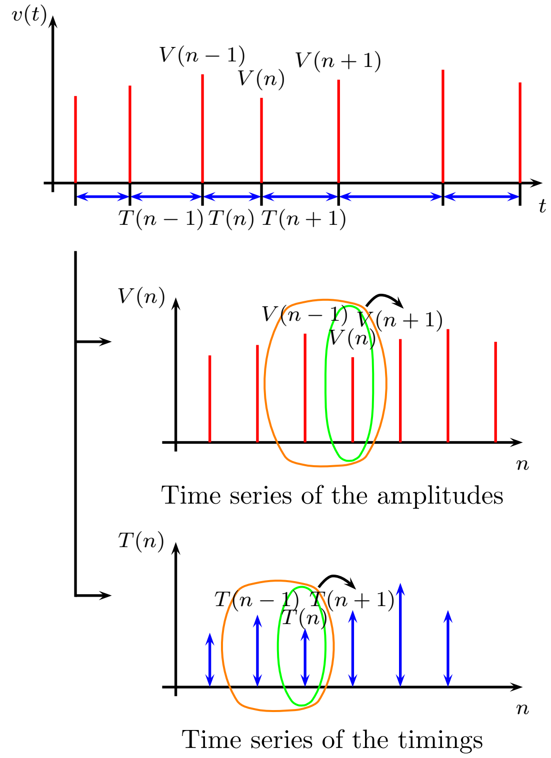


Figure 1 Two types of observed information.

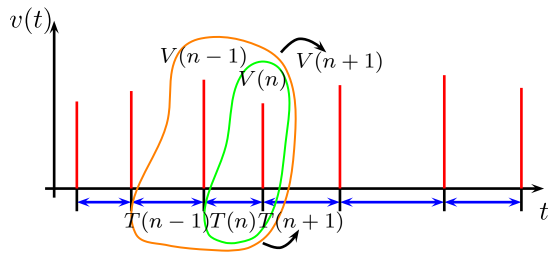


Figure 2 A prediction framework that combines both of amplitude size and event timing.

However, there exists a system from which we can observe not only the amplitude values but also the event timings. In such a case, we use a prediction framework using $V(n)$ and $T(n)$ simultaneously which are observed at the same time n (Fig.2)[1].

3 Prediction Method

As for the prediction algorithm, we use the Jacobian-matrix estimate prediction[3].

Let us first consider a nonlinear dynamical system:

$$\boldsymbol{x}(t+1) = \boldsymbol{f}(\boldsymbol{x}(t)), \tag{1}$$

where $\boldsymbol{f}$ is a k-dimensional nonlinear map, $\boldsymbol{x}(t)$ is a k-

dimensional state at time t. To estimate the Jacobian matrix of $\boldsymbol{f}$, we linearize Eq.(1) as follows:

$$\delta\boldsymbol{x}(t+1) = \boldsymbol{Df}(\boldsymbol{x}(t))\delta\boldsymbol{x}(t), \quad (2)$$

where $\boldsymbol{Df}(\boldsymbol{x}(t))$ is the Jacobian matrix at $\boldsymbol{x}(t)$, and $\delta\boldsymbol{x}(t)$ is an infinitesimal deviation at $\boldsymbol{x}(t)$. To evaluate $\boldsymbol{Df}(\boldsymbol{x}(t))$ only with local information at $\boldsymbol{x}(t)$, we first extract a near-neighbor set of $\boldsymbol{x}(t)$. If we denote the i-th near neighbor of $\boldsymbol{x}(t)$ by $\boldsymbol{x}(t_{k_i})$, $(i = 1, 2, \ldots, M)$, two displacement vectors, $\boldsymbol{y}_i = \boldsymbol{x}(t_{k_i}) - \boldsymbol{x}(t)$ and $\boldsymbol{z}_i = \boldsymbol{x}(t_{k_i}+1) - \boldsymbol{x}(t+1)$, can be considered to correspond to $\delta\boldsymbol{x}(t)$ and $\delta\boldsymbol{x}(t+1)$ respectively in Eq.(2). If the norms of $\boldsymbol{y}_i$ and $\boldsymbol{z}_i$ are small enough and the corresponding temporal evolution is small enough, we can approximate the relation between $\boldsymbol{y}_i$ and $\boldsymbol{z}_i$ by the linear equation: $\boldsymbol{z}_i = \boldsymbol{G}(t)\boldsymbol{y}_i$, where the matrix $\boldsymbol{G}(t)$ is an estimation of the Jacobian matrix $\boldsymbol{Df}(\boldsymbol{x}(t))$ in Eq.(2). In order to estimate $\boldsymbol{G}(t)$, we use the least-square-error fitting which minimizes the average square error $S = \frac{1}{M}\sum_{i=1}^{M}|\boldsymbol{z}_i - \boldsymbol{G}(t)\boldsymbol{y}_i|$. In other words, we can estimate $\boldsymbol{G}(t)$ by the following equations: $\boldsymbol{G}(t)\boldsymbol{W} = \boldsymbol{C}$, where $\boldsymbol{W}$ is the variance matrix of $\boldsymbol{y}_i$, and $\boldsymbol{C}$ is the covariance matrix between $\boldsymbol{y}_i$ and $\boldsymbol{z}_i$. If $\boldsymbol{W}$ has its inverse matrix, we can obtain $\boldsymbol{G}(t)$ from $\boldsymbol{G}(t) = \boldsymbol{C}\boldsymbol{W}^{-1}$[3, 4].

Now, we want to predict a future value of $\boldsymbol{x}(T)$. Then, we first search $\boldsymbol{x}(T_{k_0})$ the nearest neighbor of $\boldsymbol{x}(T)$. Next, we calculate a displacement vector $\boldsymbol{y}' = \boldsymbol{x}(T) - \boldsymbol{x}(T_{k_0})$, and we estimate the Jacobian matrix $\boldsymbol{G}(t_{k_0})$ at $\boldsymbol{x}(t_{k_0})$ by the above procedure with $\boldsymbol{x}(T_{k_i})(i = 1, 2, \ldots, M)$ and their corresponding temporal evolution. If we define $\hat{\boldsymbol{x}}(T+1)$ as the predicted future value of $\boldsymbol{x}(T)$, we can denote the predicted displacement vector $\hat{\boldsymbol{z}}' = \hat{\boldsymbol{x}}(T+1) - \boldsymbol{x}(T_{k_0}+1)$ by $\hat{\boldsymbol{z}}' = \boldsymbol{G}(T_{k_0})\boldsymbol{y}'$. Then, we can predict $\hat{\boldsymbol{x}}(T+1)$ as follows: $\hat{\boldsymbol{x}}(T+1) = \boldsymbol{G}(T_{k_0})(\boldsymbol{x}(T) - \boldsymbol{x}(T_{k_0})) + \boldsymbol{x}(T_{k_0}+1)$. Repeating the scheme for p time iteratively, we can predict the p step future of $\boldsymbol{x}(T)$[5].

4 Making Event Series

In order to confirm the effectiveness and validity of the improved prediction framework, we first produce an event series from a time series that shows chaotic behavior. We use this event series to check the validity of the improved framework. We define the event sizes as the local maxima of $x(t)$, and the event timings as the intervals between two successive the maxima (Fig.3).

We use both of the event sizes and the event timings in a reconstructed state space simultaneously. Because both dynamic ranges are not same and the difference brings leads to results, we normalized the event sizes and timings before the prediction.

5 Simulation for High-Dimensional Data

In Ref.[1], Kanno et al. discussed a prediction problem of a low-dimensional chaos by the zeroth order prediction on the improved framework. In the present paper, we applied the improved framework to the Mackey-Glass equation[6] which shows a higher dimensional chaos then the one used in Ref.[1]. In addition, we applied the Jacobian-matrix estimate prediction[3] which is the first order prediction.

At first, we produced an event series from the Mackey-Glass equation which is described by the following delay differential equation:

$$\frac{dx}{dt} = \frac{ax(t-\Delta)}{1 + x(t-\Delta)^c} - bx,$$

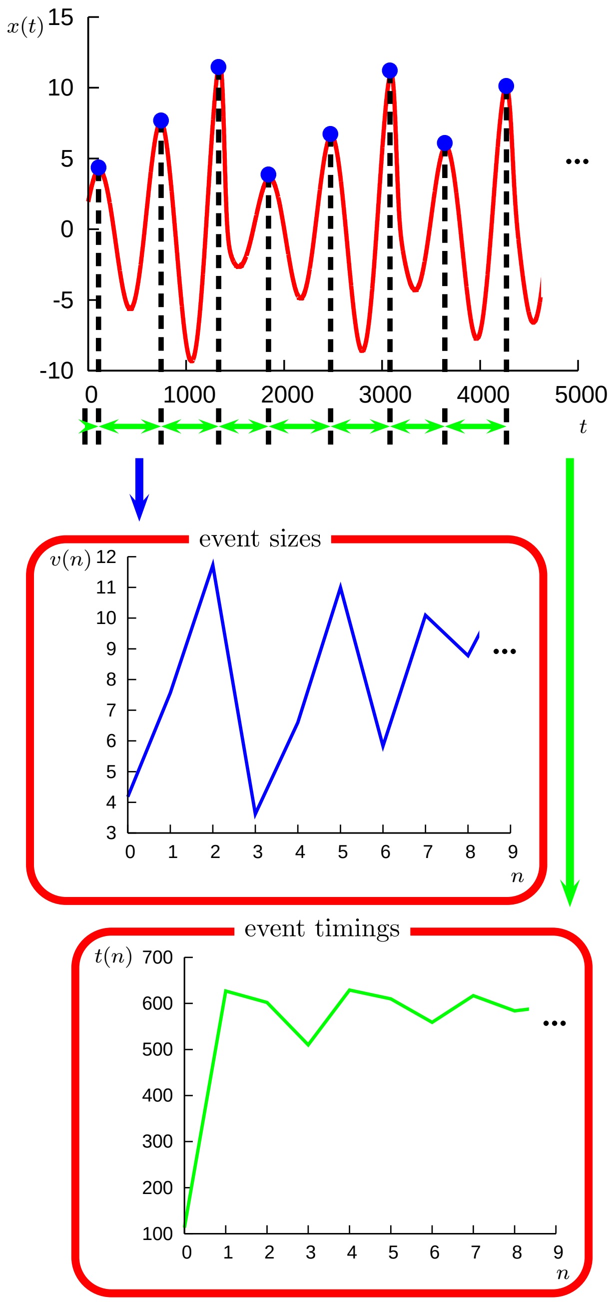


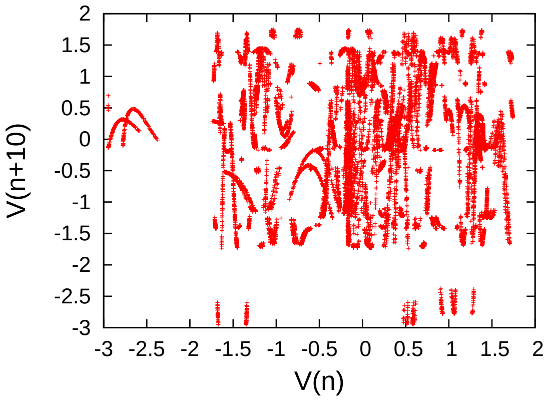
Figure 3 How to make event series to check the validity of the method.

where $a = 0.2, b = 0.1, c = 10$, and $\Delta = 20$. If we use these parameters, the Mackey-Glass equation produces chaos[7].

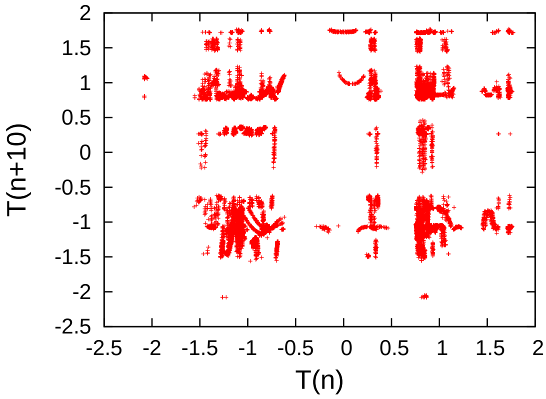
Then, we predicted the event sizes and event timings by the conventional framework or the improved framework, and compared those prediction performance. In the present paper, we use a normalized time series that consists only of event sizes with $d = 2$ and $\tau = 10$, a normalized time series that consists only of event timings with $d = 2$ and $\tau = 10$, and a normalized time series that consists event size and timing (Fig.4). Figure 4 clearly shows that if we use both event sizes and event timings, it is relatively easier to unfold intersections on reconstructed attractors.

6 Evaluation Method

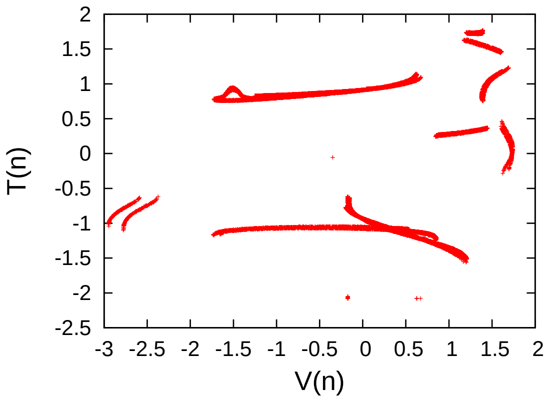
To evaluate the improved prediction framework, we used the normalized root mean square error which is described



(a)



(b)

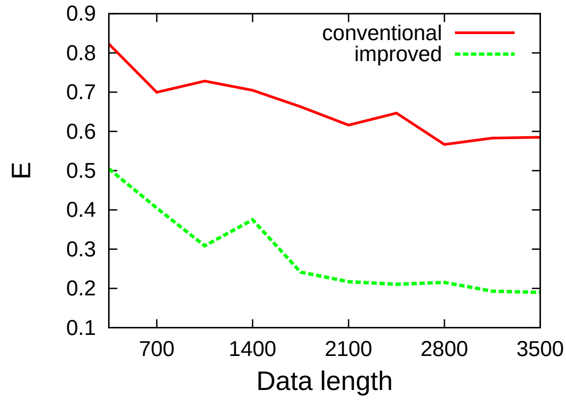


(c)

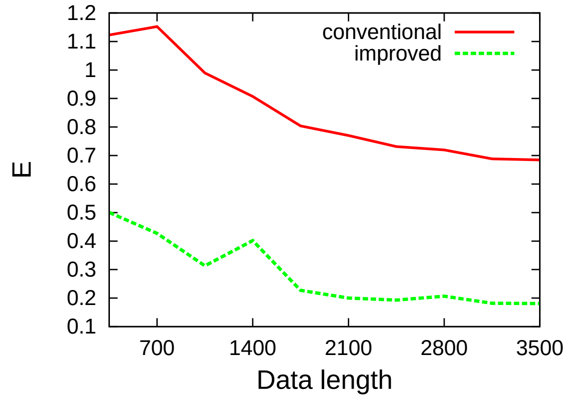
Figure 4 (a) A reconstructed attractor of a normalized time series that consists only of event size with $d = 2$ and $\tau = 10$. (b) A reconstructed attractor of a normalized time series that consists only of event timings with $d = 2$ and $\tau = 10$. (c) A reconstructed attractor of a normalized time series that consists event size and timing.

by the following equation:

$$E = \frac{\sum_{n=1}^{N} (z(n) - \hat{z}(n))^2}{\sum_{n=1}^{N} (z(n) - \bar{z}(n))^2},$$



(a)



(b)

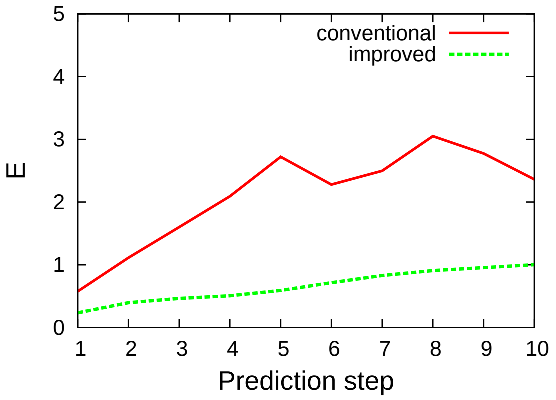
Figure 5 (a) Results of event size prediction, and (b) Results of event timing prediction. The horizontal axis shows the number of data (maxima), and the vertical axis shows the prediction accuracy E.

where $z(n)$ is a true time series, $\hat{z}(n)$ is a predicted time series, $\bar{z}(n)$ is a mean value of $z(n)$, and N is the data length of $z(n)$. In this metric, If E is close to zero, the prediction accuracy is better.

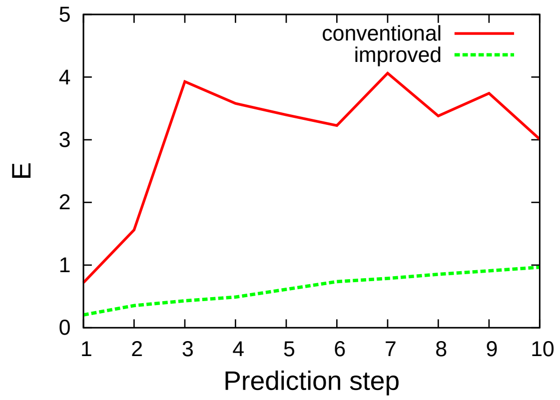
7 Prediction Results

We show the prediction results of the event series produced by the Mackey-Glass equation. At first, we predicted the one-step futures of the event sizes and timings for each data length by the conventional framework or the improved framework, and compared those prediction accuracy. In Fig.5(a), we show the results of the root mean square errors of the predicted event size. The prediction accuracy of the improved framework is better than the conventional frameworks. Moreover, in Fig.5(b), we show the results of the root mean square errors of the event timing prediction. The prediction accuracy of the improved framework becomes good as well as the event size case.

Next, we predicted the event sizes and timings in each prediction step by the conventional framework or the improved framework, and compared those prediction accuracy. Here, the data length was set as 2100. In Fig.6(a), we show the results of the root mean square errors of the predicted event size. The prediction accuracy of the improved framework is better than the conventional frameworks. Moreover, in Fig.6(b), we show the results of the root mean square errors of the event timing prediction. The

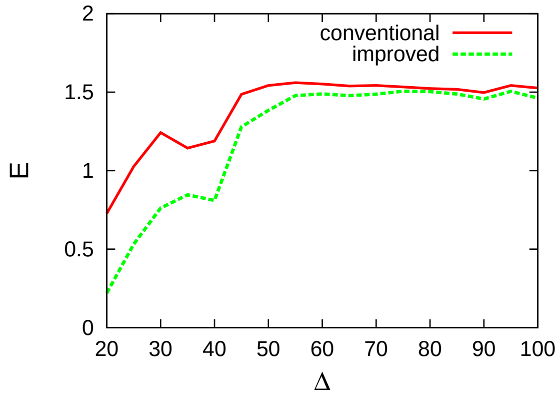


(a)

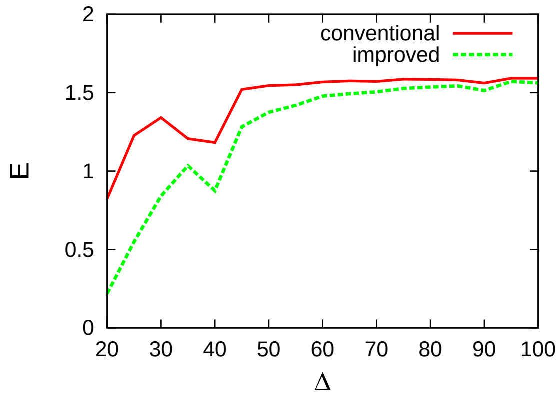


(b)

Figure 6 (a) Results of event size prediction, and (b) Results of event timing prediction. The horizontal axis shows prediction step, and the vertical axis shows the prediction accuracy E.



(a)



(b)

Figure 7 (a) Results of event size prediction, and (b) Results of event timing prediction. The horizontal axis shows parameter Δ of the Mackey-Glass equation, and the vertical axis shows the prediction accuracy E.

prediction accuracy of the improved framework becomes good as well as the event size case.

Finally, we examined the predictability in case of changing Δ. We compared the improved framework with the conventional one for several values of Δ of the Mackey-Glass equation. In Fig.7(a), we show the results of the root mean square errors of the predicted event size. Moreover, in Fig.7(b), we show the results of the root mean square errors of the event timing prediction. When we predicted the event sizes or timings, because the prediction accuracy of the improvement framework is higher for comparatively larger Δ, we can confirm that the improvement framework is also valid for a higher dimension dynamical system.

8 Conclusions

In the present paper, we applied the improved prediction framework[1] to a high-dimensional chaos with the first order prediction, and compared the its performance to the conventional one. As the results, we confirmed that the improved framework is more appropriate than conventional frameworks even for a high-dimensional case. As a future work, we have to treat another time series, especially real time-series data. The research of TI was partially supported by Grant-in-Aids for Scientific Research (C) (No.17500136) from JSPS.

References

[1] Y. Kanno and T. Ikeguchi, “Nonlinear Prediction on Maximum Timing of Complex Time Series,” Proceeding of 9th International Conference on Neural Information Processing(ICONIP ’02), 2002.

[2] T. Ikeguchi, and Y. Kanno, “Nonlinear analysis on complex time series with amplitude and event information,” Proceedings of ITC-CSCC, **2**, pp.1192-1195, 2001.

[3] M. Sano and Y. Sawada, “Measurement of the Lyapunov Spectrum from a Chaotic Time Series,” Physical Review Letters, **55**(10), pp.1082-1085, 1985.

[4] J. P. Eckmann, S. Oliffson Kamphorst, D. Ruelle, and S. Ciliberto, “Lyapunov Exponents from Time Series,” Physical Review A, **34**(6), pp.4971–4979, 1986.

[5] J. D. Farmer and J. J. Sidorowich, “Predicting Chaotic Time Series,” Physical Review Letters, **59**(8), pp.845–848, 1987.

[6] M. C. Mackey and L. Glass, “Oscillation and Chaos in Physiological Control Systems,” Science, **197**, pp.278-289, 1977.

[7] J. D. Farmer, “Chaotic Attractors of an Infinite-dimensional Dynamical System,” Physica 4D, pp.366-393, 1982.