

# Dynamics of Single Atoms and Molecules Manipulated at Material Surface

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**Abstract**—Nano-technology is a bottom-up technology and shows the possibility of spatial arrangement of atoms and molecules by using sensing probes, for example, SEM, AFM, and so on. The demonstrative experiments have been reported since 1990s. In this paper, we will discuss a model of single particles manipulation from material surface. The model is based on a van der Waals molecule vibrational predissociation of T-shaped model. The probability of the manipulation is also considered with relation to chaotic dynamics.

## 1. Introduction

This paper describes the dynamics of single atoms and molecules, bonding to material surface by van der Waals force, at one of potential wells under external excitation.

Recently, manipulation of single atoms and molecules has been experimentally achieved on surfaces with use of scanning tunneling microscope (STM) and atomic force microscopy (AFM) in lateral processes as well as vertical processes [1, 2, 3, 4, 5]. The lateral processes can transfer atoms and molecules parallel to the surface and the vertical between tip and adatom via vibrational excitation of the target-substrate bond. The processes have been also studied theoretically [6, 7, 8] and numerically [9].

This paper applies van der Waals molecular vibrational predissociation based on T-shaped model [10, 11, 12, 13, 14]. The model was introduced to describe a quantum mechanics which governs the rates of vibrational predissociation of A–B–A triatomic molecules, which are coupled Morse oscillators. Atoms and molecules attached on the material surface are bonded by van der Waals potential, which is characterized by Morse type atom-atom interaction potential. The dynamics of manipulated particles is described by the fragmentation of the atom B from the coupling of A–A.

In the following section, we introduce a Hamiltonian of the triatomic molecules. The system is coupled nonlinear oscillators. Assuming that the rotation and bending modes are neglected, at first, it is shown that the eigen frequency decides the resonance and energy exchange. At high energy, the system possibly shows chaotic vibration. It gives the probability to classical dynamics of atoms and molecules. The fragmentation of atoms from surface is discussed based on the global phase structure.

## 2. Dynamics of Single Atom at Material Surface

In this section, the dynamics of single atoms is discussed based on Hamiltonian mechanics. At first we introduce a model of atoms and molecules alignment for estimating fragmentation of atom from surface bond.

### 2.1. Model of Single Atoms and Molecules

Here we focus on the dynamics of single atoms and molecules attached on material surface. Assume that single atoms or molecules B at a surface is bonding to quadri-atoms A (see Fig.1). The model of molecular vibrational fragmentation has been discussed for T-shaped structure with van der Waals potential. At the surface, we assume the pyramid structure of atoms at steady state. In the pyramid-shape bonding, the rotation and bending modes disappear. Then the DOF (degree of freedom) of B is restricted in the vertical direction to the rectangle plane of A. Hereafter, the system can be modeled by T-shaped structure with diagonal atoms A and B without rotating dynamics.

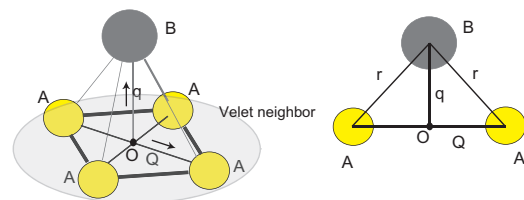


Figure 1: Atoms and molecules alignment at material surface.

The Morse interaction potential limits the distance of the interaction in a short range. In the region, the atoms are called Velet neighbors [15]. The distance between the atoms A is depicted as  $\sqrt{2}Q$  and distance between B and center O of A-plane as  $q$ , which is also on the axis. Then the distance between A and O becomes  $Q$ , which is perpendicular to the  $q$ -axis. Each momentum is given by  $P$  and  $p$ . The Hamiltonian can be written in the form:

$$H = \frac{P^2}{2m} + \frac{p^2}{2\mu} + V_0(Q, q) + V_1(Q, q), \quad (1)$$

where the angle between the axis of A-plane and the vector from the center of mass to B is restricted at rectangle. Then

we neglect the kinetics of angular motion. Here  $m_A$  and  $m_B$  are the masses of A and B, respectively.  $m = m_A/2$  is the reduced mass of A and  $\mu = 2m_A m_B / (m_A + 2m_B)$  the reduced mass of the whole system. The potentials are given as follows [14]:

$$V_0(Q, q) = W_0(Q) + W_0(q),$$

where

$$W_0(Q) = D_{0A}(e^{-2\beta_{0A}(Q-Q_0)} - 2e^{-\beta_{0A}(Q-Q_0)}),$$

$$W_0(q) = D_{0B}(e^{-2\beta_{0B}(q-q_0)} - 2e^{-\beta_{0B}(q-q_0)}),$$

and

$$V_1(Q, q) = W_1(r_+) + W_1(r_-),$$

where

$$W_1(r_{\pm}) = D_1(e^{-2\beta_1(r_{\pm}-r_{\pm 0})} - 2e^{-\beta_1(r_{\pm}-r_{\pm 0})}).$$

$V_1$  is the expansion of van der Waals potential in the Taylor series around equilibrium point.  $r_{\pm}$  are the distance between B and one of the atoms A.  $r$  is a function of  $Q$  and  $q$ . The equilibrium point is given by  $r_0$ . The shape of Morse potential is shown in Fig.2

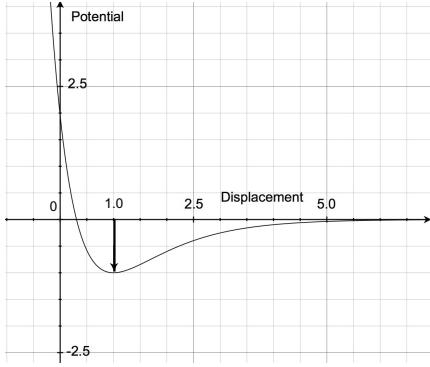


Figure 2: Schematic Morse potential with equilibrium point at 1.0.

Assuming the 2-dimensional symmetry of quadri-atoms A, the potential function around the single atom B possesses the axial symmetric property at steady state. Then, the system seem to be equivalent to T-shaped model. It implies that the fragmentation is limited in the vertical direction to the plane A.

Equation 1 represents the model system Hamiltonian in coordinates  $(Q, q, P, p)$ . In the classical dynamics, we have the relation:

$$\begin{cases} \dot{Q} = \frac{\partial H}{\partial P}, \\ \dot{q} = \frac{\partial H}{\partial p}, \\ \dot{P} = -\frac{\partial H}{\partial Q}, \\ \dot{p} = -\frac{\partial H}{\partial q}. \end{cases} \quad (2)$$

The linearization of the system (2) is based on Taylor expansion around the equilibrium point for  $Q$  and  $q$ .

## 2.2. Analysis based on Action-Angle Formulation [13]

Under the dissipative or excited state, the perturbed Hamiltonian is given as

$$H = H_0 + \varepsilon H_1, \quad (3)$$

where  $\varepsilon$  is a small parameter. The zero-order component of Hamiltonian is given by

$$H_0 = \frac{P^2}{2m} + \frac{p^2}{2\mu} + W_0(Q) + W_0(q). \quad (4)$$

We can rewrite  $H_0$  by an action( $I$ )-angle( $\theta$ ) form. Based on the reference [13], the following relationship is obtained.

$$H_0 = E_A(I_A) + E_B(I_B) \quad (5)$$

$I_A$  and  $I_B$  are action variables, which are obtained by action integral. The potential energy of Morse oscillators,  $E_A$  and  $E_B$ , are defined as follows:

$$\begin{cases} E_A(I_A) = (I_A + \frac{1}{2})\omega_A - (I_A + \frac{1}{2})^2\omega_A^2/4D_{0A} - D_{0A}, \\ E_B(I_B) = (I_B + \frac{1}{2})\omega_B - (I_B + \frac{1}{2})^2\omega_B^2/4D_{0B} - D_{0B}, \end{cases}$$

where

$$\omega_A = (2D_{0A}\beta_{0A}^2/m)^{1/2},$$

$$\omega_B = (2D_{0B}\beta_{0B}^2/\mu)^{1/2}.$$

$D_{0A}$  and  $D_{0B}$  are defined by related Morse potentials. The simple derivatives of  $E_A$  and  $E_B$  give the zero order frequencies in each motion along  $Q$  and  $q$ . That is,

$$\begin{cases} \dot{\theta}_A = -\omega_A + (2I_A + 1)\omega_A^2/4D_{0A} = \Omega_A, \\ \dot{\theta}_B = -\omega_B + (2I_B + 1)\omega_B^2/4D_{0B} = \Omega_B. \end{cases} \quad (6)$$

The energy at which the atom B is separated at infinity depends on  $D_{0B}$ . The external energy input for fragmentation is due to the boundary of trapped motion of  $H$ . The phase structure is schematically described in Fig.3.

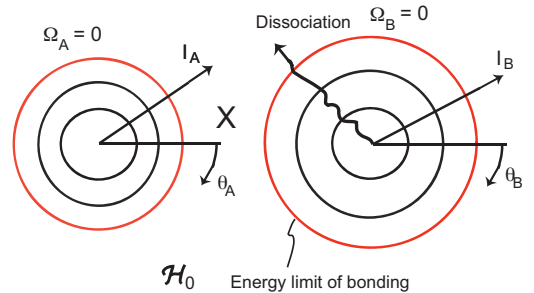


Figure 3: Phase portrait of Hamiltonian.

The maximum values of  $I_A$  and  $I_B$  are obtained by Eq.(4) [17].

$$\begin{cases} I_{A\max} &= -\frac{1}{2} + \frac{2D_{0A}}{\omega_A}, \\ I_{B\max} &= -\frac{1}{2} + \frac{2D_{0B}}{\omega_B}. \end{cases} \quad (7)$$

These are also limits of actions for T-shaped structure in Hamiltonian. That is, the external energy input to the system can dissociate the bonding between atoms A and B.

### 3. Manipulation of Single Atoms by Probe

One of the important topics of nanotechnology is the manipulation of single atoms at the material surface. We have already known the dynamics of single atoms which dissociate from material surface. Here, we will discuss a mathematical formulation of vibratory fragmentation of single atoms by probe based on perturbed Hamiltonian system.

#### 3.1. Dynamics of Atoms between Surface and Probe

The manipulation of single atoms has been achieved by STM and AFM [1, 2, 3, 4, 5]. The schematic structure is described in Fig.4.

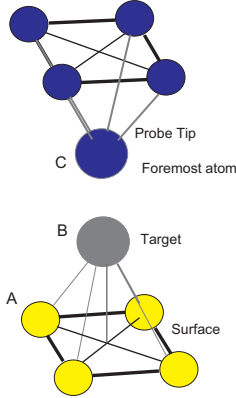


Figure 4: Manipulation of single atom by probe.

The manipulation implies the energy exchange between an atom bonding to surface and probe. The manipulation of atoms is governed by the probability of dissociation of atomic bond. However, the dynamics and probability have not been well understood [18]. If the probe is rigid or consists of heavy atoms with strong bond to bulk, the dynamics is simply modeled by T-shaped configuration of atoms and molecules at surface. Then, the vibration of probe gives us an external energy input to the original Hamiltonian system.

#### 3.2. Probability of Hamiltonian System

The dynamics of single atoms, which are manipulated by probe, is explained with relation to the bond between

T-shaped structure. The nonlinear resonances under external excitation are related to energy transfer between modes and external vibration. Moreover, the limit of resonance depicts the appearance of instability depending on the nonlinearity. On the other hand, we know that there is chaotic region around resonant boundary [12, 13, 14]. Under the external Hamiltonian perturbation  $\Delta H$  might generate vibratory fragmentation and manipulation of atoms as shown in Fig.5. Therefore, the instability of the resonance around the boundary is strongly related to the global phase structure. When the system become dissipative, the complexity is due to the intersection of stable and unstable manifolds. Bonding instabilities are also reported between atoms at material surface and the foremost tip of probe [19]. The instability does not generate the probability but the abrupt jump of dynamics. This might be related to the uncertainty in capture and release of atoms.

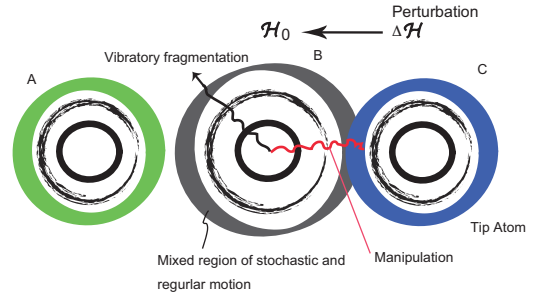


Figure 5: Phase portrait of perturbed Hamiltonian system.

In the classical treatment, the probability of fragmentation is defined by the rate of initial conditions from which trajectories come out of the region surrounded by homoclinic orbit. Consider initial conditions in the space  $(Q_i, q_i, P_i, p_j)$ ;  $i$  and  $j$  show the indexes of meshed initial condition space. Their Hamiltonian trajectories are generated by

$$\begin{cases} \frac{p_i^2}{2m} + W_0(Q_i) &= E_A(I_A), \\ \frac{p_j^2}{2\mu} + W_0(q_j) &= E_B(I_B). \end{cases} \quad (8)$$

These equations possibly show the stochastic region in initial condition space [13]. The dissipation to the coupled Morse oscillators loosens the homoclinic orbit corresponding to the energy limit of bonding. At the same time, the homoclinic intersection and folded manifolds appear in the global phase structure. In the case, the probability is strongly governed by the structure in spite of the uncertainty of Hamiltonian system [14].

#### 3.3. Energy Input for Manipulation

When the single atoms are vibrated by the forcing of probe, the dynamics can be approximated by Eq.(2) with

dissipation and forcing terms. To achieve the vibratory fragmentation, the external energy must be as much as

$$\Delta E_B \geq E_B(I_{B\max}) - E_B(I_{B0}), \quad (9)$$

where  $I_{B0}$  is the initial action value. At the same time the atom B must be captured in the bond of probe tip. As for the foremost atom (C) of tip, the situation coincides with energy exchange between A–B and B–C in the bond A–B–C. After the fragmentation of the bond A–B, the atom B generates the new bonding B–C. In this process, the dissipation works for the stabilization of dynamics. Between release and capture, chaotic dynamics cannot be avoided as mentioned above. Then, the manipulation is governed by the uncertainty of dynamics.

#### 4. Concluding Remarks

This paper discusses the dynamics of single atoms and molecules manipulated by probe at material surface along the theory of classical mechanics of vibrational fragmentation for T-shaped molecules. The Hamiltonian approach can estimate the minimum energy that corresponds to fragmentation of single atoms from other bond. At the material surface, the model is effective in low dimensional approximation through summing Velet neighbors. We showed the possibility that the formulation can be applied to the manipulation by probe, describing the new bond between a target atom and a foremost atom of tip. The dynamics also has chaos based on the nonlinearity of interaction of atoms under Morse potential. The probability of manipulation might be governed by chaos in the dynamics. The quantitative discussion is our next phase. Moreover, the control of dissipation for the stability of manipulation is inevitable. The method for describing dissipation is also an important topic in the atomic dynamics .

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