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Performance of the Recovered Pseudo-Noise Binary Sequences by Independent Component Analysis for CDMA

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Abstract—The performance of the spreading sequences obtained by recovering Pseudo-Noise binary sequences by ICA for CDMA are investigated. We compared the property of these recovered sequences with the one of the original sequences and the Chebyshev chaotic sequences. For this purpose, we investigate correlation properties and calculate Bit Error Rates (BERs) analytically. Using ICA, we can obtain the spreading sequence which realizes much lower BERs and keeps the correlation property of the original sequences. These are new method, in that we use ICA in information and communication fields not as separator for received signal but as generator of spreading sequences.

1. Introduction

In Code Division Multiple Access (CDMA), each of the signals existing in the same frequency band at the same time is assigned as the spreading sequences which are to be orthogonalized as much as possible. One of the most essential point to gain the performance of CDMA is making up the many spreading sequences which realize the orthogonality as much as possible.

On the other hand, Independent Component Analysis (ICA) attracts much attention as one of the technique of separating of mixed signals [1]. This technique makes it possible to recover the original signals $s_1(t), s_2(t), \cdots, s_n(t)$ from the observed signals $x_1(t), x_2(t), \cdots, x_m(t)$ without knowing how to mix original signals. It is expressed by

$$\begin{aligned} \mathbf{X}(t) &= \mathbf{A}\mathbf{S}(t); \\ \mathbf{X}(t) &= (x_1(t), x_2(t), \cdots, x_m(t)), \\ \mathbf{S}(t) &= (s_1(t), s_2(t), \cdots, s_n(t)), \end{aligned} \tag{1}$$

where $\mathbf{A}$ is an unknown $(m \times n)$ mixing matrix which indicates how to mix original signals. The observer has only the signal data $\mathbf{X}(t)$. Under the assumptions that each original signal $s_i(t)$ is independent variable and their probability distributions are nongaussian, the mixing matrix $\mathbf{A}$ is guessed.

In this paper, we compare the performance for CDMA of the following several sequences: Chebyshev chaotic sequences (Cheby.–original), Chebyshev chaotic sequences recovered by ICA (Cheby.–ICA), Pseudo-Noise (PN) binary sequences constructed by ± 1 like Gold sequences or Kasami sequences (Bin.–original) and these binary sequences recovered by ICA (Bin.–ICA). Here, we generate new sequences from ICA. These sequences recovered by ICA are expected to make a good performance as the spreading sequences for CDMA because the Signal-to-Interference Ratios (SIRs) of the sequences recovered by ICA are amplified as was shown in [2]. For investigating the performance, we focus on correlation properties of these sequences and Bit Error Rates (BERs) for these sequences in synchronous CDMA with the additive white Gaussian noise. Here, we use FastICA algorithm [3] of several methods of ICA, which is contained in IT++ [4]. We use the chaotic signals as for constructing spreading sequences. Each signal is defined as

$$S_{i,j+1} = T_q(S_{ij}), \quad q \geq 2. \tag{2}$$

Here, $T_q(x)$ is the q-th order Chebyshev polynomial defined by $T_q(\cos\theta) = \cos(q\theta)$. It is known that this Chebyshev map is ergodic and it has the ergodic invariant measure

$$\rho(x)dx = dx/(\pi\sqrt{1-x^2}) \tag{3}$$

and it satisfies the orthogonal relation

$$\int_{-1}^{1} T_i(x)T_j(x)\rho(x)dx = \delta_{i,j}\frac{1+\delta_{i,0}}{2}, \tag{4}$$

where $\delta_{i,j}$ is the Kronecker delta function. From the above fact, these chaotic signals generated from $T_q(x)$ of different orders can be used as the naturally orthogonal spreading sequences in CDMA from the ergodic principle [5–7]. Furthermore it is known that these chaotic naturally orthogonal signals can be recovered from mixed state by ICA [8].

2. Signal-to-Interference Ratio

We can calculate the SIRs for arbitrary sequences recovered by ICA by the formula shown in [2], namely

$$\mathrm{SIR_{ICA}} = \frac{1 + \langle\langle(\overline{Y_0})^2\rangle\rangle}{\sqrt{\langle\langle\sum_{i=1}^{K-1}(\overline{Y_0}\cdot\overline{Y_i})^2\rangle\rangle}}, \tag{5}$$

where $\overline{Y_i} = \sum_{j=1}^{N} Y_{ij}/N$. The quantity Y_{ij} is the i–th sequence data recovered by ICA at the position j and satisfies the relation $\overline{Y_i} = \overline{S_i}/D_{S_i}$, where $\overline{S_i}$ is the finite time N average of the original signal and D_{S_i} is the standard deviation of $\mathbf{S}_i$. Here we define the 0-th sequence as the objective sequence. The quantity K is the sum of the sequences and N is the length of each sequence. The quantities $\mathrm{SIR_{ori}}$ of Cheby.–original and $\mathrm{SIR_{ICA}}$ of Cheby.–ICA are obtained as

$$\mathrm{SIR_{Cheby.-ori}} = \sqrt{\frac{N}{K-1}}, \tag{6}$$

$$\mathrm{SIR_{Cheby.-ICA}} = \frac{N+1}{\sqrt{K-1}}, \tag{7}$$

respectively. The SIRs of the sequences recovered by ICA are much larger than those of the original sequences. Moreover, the data recovered by ICA are the same as those of the original sequences if the number of the sequence data N are large sufficiently. By using ICA, we can obtain the sequences which has the much low interference than conventional ones [2].

3. Correlation Property

It is necessary for using sequences $S_{i,j}$ as spreading sequences for CDMA that sequences have good correlation properties described later. We calculate the auto-correlations and the cross-correlations given by $\sum_{j=1}^{N} S_{i,j}S_{i,j+\Delta}$ and $\sum_{j=1}^{N} S_{i,j}S_{k,j+\Delta}$ $(i \neq k)$, respectively. Here, the sequence code satisfies the relation $S_{i,1} = S_{i,N+1}$ and Δ means the phase difference. For using $S_{i,j}$ as the spreading sequences not only in the synchronous CDMA, it is necessary that the auto-correlation for $\Delta = 0$ is much larger than the one for $\Delta \neq 0$ and the cross-correlations. We investigate the correlation properties of Cheby.–original, Cheby.–ICA, the original Gold sequence as Bin.–original and this Gold sequence recovered by ICA. Figures 1, 2, 3 and 4 show the correlations of these sequences as functions of the phase difference Δ for these code length $N = 127$, respectively. The above sequences recovered by ICA are for $K = 15$. The values of the auto-correlations in these sequences for $\Delta = 0$ are 127. The second largest values of each these sequences in the above data are 36.043, 35.276, 17 and 27.505, respectively. From these figures, we conclude that ICA almost keeps the correlation property of the original sequences.

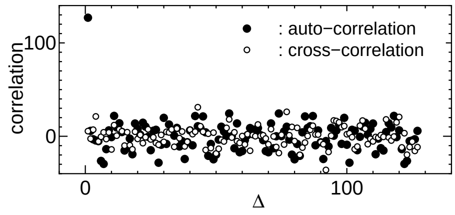

Figure 1: The correlations of Cheby.–original.

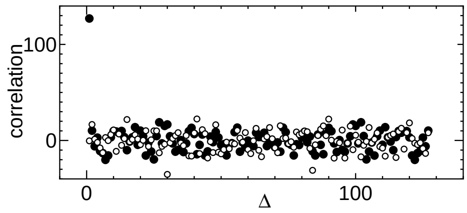

Figure 2: The correlations of Cheby.–ICA.

4. Bit Error Rate and Signal-to-Interference–plus–Noise Ratio

We investigate the BERs analytically in the cases that the above several sequences are used as the spreading sequence in the synchronous CDMA with the additive white Gaussian noise. Here, we used the following sequences as the spreading sequences: Cheby.–original, Cheby.–ICA, Bin.–original and Bin.–ICA. We compared the BERs for these sequences recovered by ICA with the one for Gold sequences as the binary sequences used commonly in the asynchronous CDMA.

By calculating Signal-to-Interference–plus–Noise Ratios (SINRs), we can calculate the BERs analytically under the assumption that additive white Gaussian noise exists as was used in [9]. The quantity BER can be evaluated by using the following formula

$$\mathrm{BER} = Q(\mathrm{SINR}); \quad Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} e^{-\frac{u^2}{2}}du. \tag{8}$$

Thus, we calculate each of the SINRs for these se-

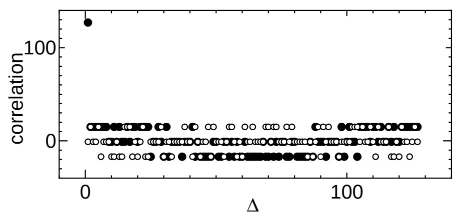

Figure 3: The correlations of the original Gold sequences.

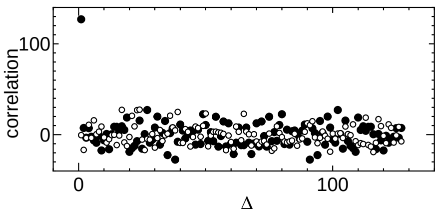

Figure 4: The correlations of Gold sequences recovered by ICA.

quences. The quantity SINR is defined as

$$\mathrm{SINR} = Z_{\mathrm{S}}/\sqrt{\mathrm{Var}\{Z_{\mathrm{IN}}\} + \mathrm{Var}\{Z_{\mathrm{N}}\}}, \tag{9}$$

where

$$Z_{\mathrm{S}} = \langle\langle \sum_{j=1}^{N} (A_{0j})^2 \rangle\rangle, \tag{10}$$

$$\mathrm{Var}\{Z_{\mathrm{IN}}\} = \langle\langle \sum_{k=1}^{K} (\sum_{j=1}^{N} A_{0j}A_{kj})^2 \rangle\rangle, \tag{11}$$

$$\mathrm{Var}\{Z_{\mathrm{N}}\} = \langle\langle \left(\int_0^N n(t)a_0(t) \right)^2 \rangle\rangle. \tag{12}$$

Here, A_{kj} is the k-th spreading sequence at time j, the quantity $a_k(t)$ is the continuous representation of A_{kj} and $n(t)$ is the value of the additive white Gaussian noise. The quantities Z_{S}, Z_{IN} and Z_{N} are the objective signal's output, the interference output and the noisy output of the correlation receiver, respectively. The 0-th signal is set as the objective. We calculate the SINR for the normalized sequence $\{X_{kj}\}$ which satisfies the following relation

$$\langle\langle \sum_{j=1}^{N} X_{kj} \rangle\rangle = \langle\langle \sum_{j=1}^{N} (A_{kj}/\gamma_k)^2 \rangle\rangle = N, \tag{13}$$

where γ_k is the normalization constant. Thus, the SINR' for the normalized sequence in the chip–synchronous system is obtained as

$$\mathrm{SINR}' = \frac{N}{\sqrt{\mathrm{Var}\{Z_{\mathrm{IN}}\} \cdot (N/Z_{\mathrm{S}})^2 + \sigma^2 N}}. \tag{14}$$

Here, we use the following relation

$$\mathrm{Var}\{Z_{\mathrm{N}}\} \cdot (N/Z_{\mathrm{S}})^2 = \sigma^2 N. \tag{15}$$

This formula can be also used in the synchronous CDMA, where the phase differences of these sequences are hold in the chip–synchronous CDMA as the cross-correlations take minimum values. Substituting the properties of the spreading sequences as were obtained in Section 2 for (14), the SINRs using normalized spreading sequences as Cheby.–original in the chip–synchronous CDMA and Cheby.–ICA in the synchronous CDMA are obtained as

$$\sqrt{\frac{N}{K - 1 + \sigma^2}}, \tag{16}$$

$$\frac{N + 1}{\sqrt{K - 1 + \sigma^2(N + 1)^2/N}}, \tag{17}$$

respectively. We also calculate the SINRs for the optimal binary sequences in the chip–synchronous system. In [10], the SIR, namely (14) for $\sigma^2 = 0$, for the original these sequences is calculated as

$$\sqrt{\frac{N^3}{(K - 1)(N^2 + N - 1)}}. \tag{18}$$

Using this and (14), we can obtain the SINR for the optimal binary sequences as

$$\frac{N}{\sqrt{(K - 1)(N + 1 - 1/N) + \sigma^2 N}}. \tag{19}$$

In addition, we calculate the SINRs for the recovered binary sequences in the synchronous CDMA. Here, we assume that the differences of the number of two values ± 1 which construct each of the simultaneous binary sequences are the same. We introduce the parameter q which represents the number of $+1$ minus the one of -1 in the binary sequences. When $q = 1$, these sequences mean the Maximal–length sequences (M-sequences). Under this assumption, the SINR' for the recovered these binary sequences can be calculated. The quantities Z_{S} and $\mathrm{Var}\{Z_{\mathrm{IN}}\}$ can be obtained as

$$Z_{\mathrm{S}}^{\mathrm{bin.}} = \frac{N^3}{N^2 - q^2}, \tag{20}$$

$$\mathrm{Var}\{Z_{\mathrm{IN}}^{\mathrm{bin.}}\} = \frac{N^2 q^4 (K - 1)}{(N^2 - q^2)^2}, \tag{21}$$

respectively. Substituting these values for (14), the SINR' of the binary sequences recovered by ICA can be obtained as

$$\mathrm{SINR}'^{\mathrm{bin.}} = \frac{N}{\sqrt{(K - 1)q^4/N^2 + \sigma^2 N}} \tag{22}$$

Finally, we have obtained analytical forms of the BERs using these sequences by substituting these SINRs for the formula (8).

We investigate the dependencies of the above BERs on the bit energy per noise power and the number of users. Figure 5 shows the BERs using the above spreading sequences as functions of the bit energy per noise power. Each lines is drawn by using the analytical forms of the BERs. Figure 6 shows the BERs in the same system as functions of the number of users. The means of these symbol are the same in Figure

5. From these figures, we have confirmed that these sequences recovered by ICA can be used as the spreading sequences. The BERs for these sequences recovered by ICA are much lower than those of the original sequences.

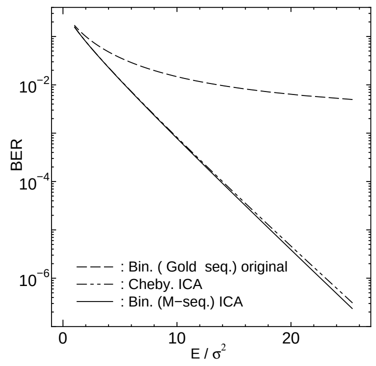

Figure 5: BERs as functions of the bit energy per noise power for $K = 15$ and $E = N = 127$.

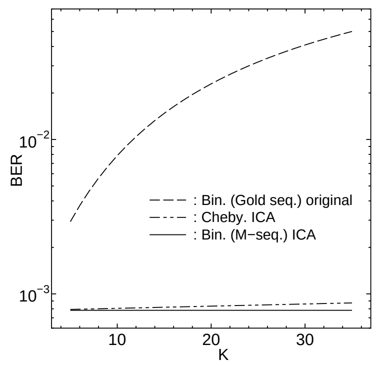

Figure 6: BERs as functions of the number of users K for $E/\sigma^2 = 10$.

5. Conclusions

In the present paper, we have investigated that the performance for CDMA of the spreading sequences which are the recovered PN binary sequences by ICA and compared these sequences with the Chebyshev chaotic sequences. We have found that the spreading sequences which realize the lower BER can be obtained easily by ICA. The PN binary sequences like Gold sequences which are commonly used in the asynchronous CDMA can be transformed as the sequences which have much lower interference between the simultaneous sequences in the synchronous system. Moreover, the sequences recovered by ICA almost keep the correlation property of the original sequences. By the comparison between the Chebyshev chaotic sequences and the above PN binary sequences in the correlation property, it is expected that Cheby.–ICA sequences can be used in the asynchronous CDMA usefully because the correlation property of Cheby.–ICA is similar to the one of Gold sequences. From these facts, we have confirmed that the useful spreading sequences can be obtained by ICA.

Acknowledgment

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