2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

# Analysis on Chaotic Sequence with Biased Values for Noncoherent Chaos Communication

Shintaro ARAI†, Yoshifumi NISHIO† and Takaya YAMAZATO‡

†Department of Electrical and Electronics Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, JAPAN
Telephone: +81-88-656-7470, Fax: +81-88-656-7471
Email: arai@ee.tokushima-u.ac.jp, nishio@ee.tokushima-u.ac.jp
‡EcoTopia Science Institute, Nagoya University
Furo-cho, Chikusa-ku,Nagoya, 464-8603, JAPAN,
yamazato@nuee.nagoya-u.ac.jp

**Abstract**—In this paper, we analyze a chaotic dynamics generating a chaotic sequence with biased values and apply it to noncoherent chaos communications. We examine a behavior of the chaotic dynamics by increasing the slope of the chaotic map and investigate the invariant measure and the correlation function. A high quality performance of noncoherent chaos communications is obtained by controlling the distribution of the chaotic sequence with biased values. Finally we carry out the computer simulation using its sequence and discuss the obtained results and the future problem for chaos communication.

## 1. Introduction

Recently, a digital communication system using chaos becomes a hot topic [1]- [7]. Especially, it is attracted to develop noncoherent detection systems which does not need to recover a basis signals (unmodulated carries) at the receiver. The differential chaos shift keying (DCSK) [1] and the optimal receiver [2] are well known as a typical noncoherent system. In addition, the correlation delay shift keying (CDSK) [3] similar to the DCSK scheme is also regarded.

Analyzing chaotic sequence as well as its behavior is essential for improving the the performance of chaos communications. A Chaotic sequence is a series of nonperiodic signals generated from nonlinear dynamical systems. These signals are sensitive to initial conditions and difficult to predict the behavior of the future from the past observational signals. Also a chaotic sequence can be generated from a simple model, such as a one-dimensional chaotic map. In our previous research, we investigated a transmitter changing a chaotic sequence depending on an initial value [6]. Moreover we investigated the the performance of chaos communications using the sequence with biased values purposely [8]. As results, it could be observed that its performance was better than that of the conventional transmitter. From these results, we concluded that the chaotic dynamics affect the performance of chaos communications greatly. However, many subjects, such as the behavior of the system and the change of the correlation property, were still not solved. Then we consider that it is important to analyze a behavior of a chaotic sequence for improving the performance of chaos communications.

As analysis methods to characterize chaos, there are many measures of chaos. In this paper, we shall concentrate the invariant measure and the correlation function as measures of chaos. The invariant measure can observe the distribution of the value having the chaotic sequence. The correlation function can examine the irregularity of the chaotic sequence. By calculating the invariant measure and the correlation function, we can detail a chaotic sequence.

In this study, we analyze the chaotic map for the chaotic sequence with biased values using the invariant measure and the correlation function. Moreover, we control the number of biased values of the chaotic sequence by increasing the slope of the chaotic map and observe the behavior of its sequence. Finally we carry out the computer simulation using its sequence and discuss the obtained results and the future problem for chaos communication.

## 2. Chaotic Map With Different Slopes and its Analysis

In this section, we introduce a chaotic map with different slopes and its analysis method. Figure 1 shows the chaotic map with 4 slopes, where this map is made from the Bernoulli shift map well known as a typical 1-dimensional map. Moreover it is based on the reference [7] to make this map. Equation of this map is described by

$$x_{k+1} = \begin{cases} \dfrac{(r_1+1)x_k - q_1 + r_1}{q_1+1} & (-1 \le x_k \le q_1) \\ (r_1-1)x_k/q_1 + 1 & (q_1 < x_k \le 0) \\ (r_2-1)x_k/q_2 - 1 & (0 < x_k \le q_2) \\ \dfrac{(1-r_2)x_k - q_2 + r_2}{1-q_2} & (q_2 < x_k \le 1) \end{cases}, \quad (1)$$

where $q_1$, $r_1$, $q_2$ and $r_2$ are the parameters deciding the slopes { $(-1.0 < q_1 < 0.0)$, $(-1.0 < r_1 < 1.0)$, $(0.0 < q_2 < 1.0)$, $(-1.0 < r_2 < 1.0)$ } . We can change the slopes of the map to change these parameters. In this paper, we consider 2 methods for giving the parameter. One is the parallel shift method ($q_2 = 1 + q_1,\ r_2 = r_1$), i.e. it is made the parallel shift from the left side slope to the right side slope with center on $x_n = 0$. Another is the point symmetry method ($q_2 = (-1) \times q_1,\ r_2 = (-1) \times r_2$), i.e. the left side and the right side slopes are made to be the point symmetry with center on $(x_n,\ x_{n+1}) = (0,\ 0)$. Figures 2(a) and (b) show the chaotic maps with the two methods. As one can see, we can obtain the various maps to change the parameters. By using these maps, we analyze the chaotic dynamics of this one-dimensional chaotic map family. To observe the behavior of the chaotic dynamics using these methods, we investigate the invariant measure and correlation function.

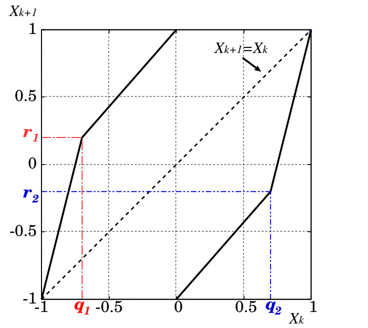



Figure 1: Chaotic map with 4 slopes.

### 2.1. Invariant measure

The invariant measure is the function deciding the iteration density of a map, namely we can observe the distribution of the value of the chaotic sequence. The invariant measure $\rho(x)$ is described by

$$\rho(x) \equiv \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N} \delta(x - f^i(x_0)) \ , \tag{2}$$

where $x_{n+1} = f(x_n)$, $n = 0, 1, 2, \cdots$, $x_0$ is an initial value, $\delta$ is the delta function. If $\rho(x)$ is the system that is not dependent on an initial value, it is called ergodic.

### 2.2. Correlation function

The correlation function is the measure to calculate the correlation between random variables at two different points in space or time. Thus we can observe the irregularity of chaotic sequence to calculate the correlation function. The correlation function $C(m)$ is described by

$$C(m) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} \hat{x}_i \hat{x}_{i+m} \ , \tag{3}$$

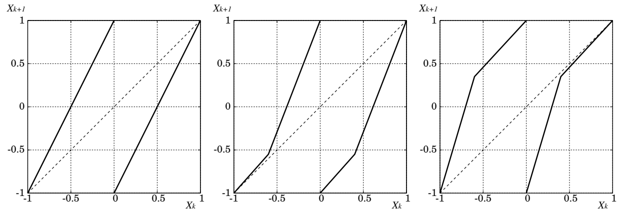
where $\hat{x}_i = f^i(x_0) - \bar{x}$, $\bar{x} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} f^i(x_0)$, $m$ is a difference with a datum point. If the correlation is calculated between random variables at two different points in same sequence, it is called the autocorrelation function. Also if the correlation is calculated between random variables at two different points in different sequence, it is called the cross-correlation function. In this paper, we carry out the autocorrelation analysis since we are not concerned here with a multiplexing system.

### 2.3. Analysis results

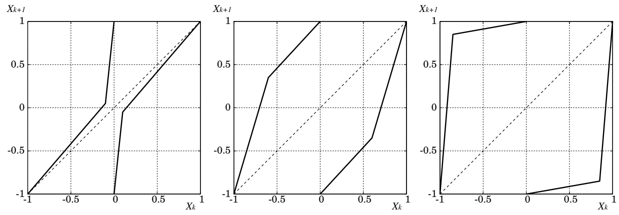
Figures 3 and 4 show the numerical analysis results of the invariant measure and the correlation function. To calculate using the computer, $N$ of each result is assumed to $10^6$.

In the case of the parallel shift of $\rho(x)$ (Fig. 3(a)), we can observe that $\rho(x)$ shifts from $-1$ to 1 according to the parameters ($r_1$). Namely, it can be said that the chaotic sequence with the biased value is distributed to the left or the right with center on $x = 0$ according to the parameters ($r_1$). However, these $C(m)$ (Fig. 4(a)) did not almost change. Moreover, we can also find that the correlation property decreases as compared with the Bernoulli shift map.

In the case of the point symmetry of $\rho(x)$ (Fig. 3(b)), its distribution is divided into right and left with center on $x = 0$ according to the parameters $(q_1, r_1)$. In addition, we can observe that $C(m)$ (Fig. 4(b)) increase according to the



(1) (2) (3)
(a) Parallel shift $(q_2, r_2) = (q_1 + 1, r_1)$



(4) (5) (6)
(b) Point symmetry $(q_2, r_2) = ((-1) \times q_1, (-1) \times r_1)$

Figure 2: Chaotic map with each parameter. (1) Bernoulli shift map, (2) $(q_1, r_1) = (-0.6, -0.55)$, (3) $(q_1, r_1) = (-0.6, 0.35)$, (4) $(q_1, r_1) = (-0.1, 0.05)$, (5) $(q_1, r_1) = (-0.6, 0.35)$, (6) $(q_1, r_1) = (-0.85, 0.85)$

parameters $(q_1, r_1)$. However, since $C(m)$ decreases changing alternately according to $m$, like a $(q_1, r_1) = (-0.6, 0.35)$ and $(q_1, r_1) = (-0.85, 0.85)$, we expect that the chaotic sequence having the similar value alternately was generated.

From these results, we can confirm that $\rho(x)$ and $C(m)$ of the chaotic sequence are different each by changing the parameter. Concurrently, we can also find that it is possible to choose any $\rho(x)$ and $C(m)$ by changing the parameters. However, when the chaotic map has four slopes, the deviation of the chaotic sequence is limited to one or two.

In this paper, we perform generating the chaotic sequence with any $\rho(x)$ using the chaotic map with many slopes. Furthermore, we also perform to generate the chaotic sequence which was excellent in the correlation property by increasing slopes.

## 3. Chaotic Map with Many Different Slopes

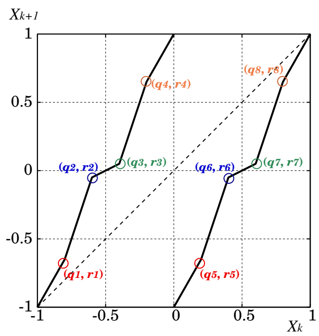
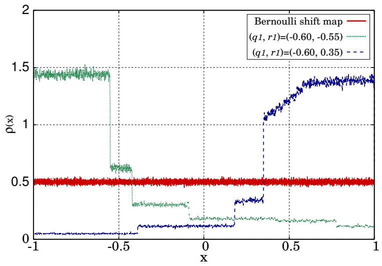
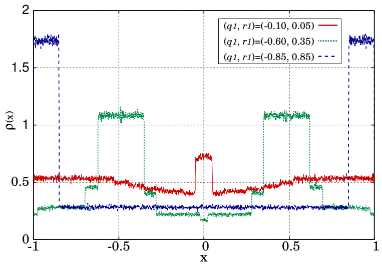



Figure 5: Chaotic map with 10 slopes.

In this section, we introduce a chaotic map with many different slopes. In Sec. 2, we have observed to be different $\rho(x)$ and $C(m)$ of the chaotic sequence depending on the slope of the chaotic map. However, when the chaotic map has four slopes, the deviation of the chaotic sequence is limited to one or two. Thus, we perform generating

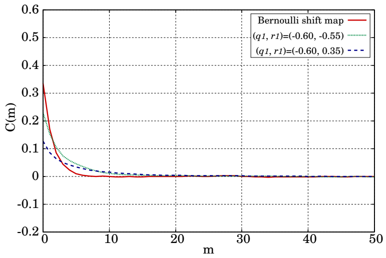



(a) Parallel shift.

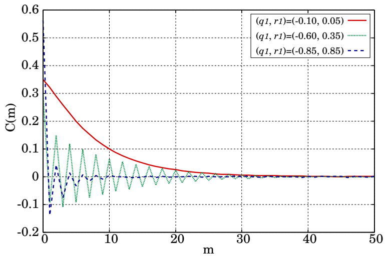



(b) Point symmetry.

Figure 3: Invariant measure with each parameter.

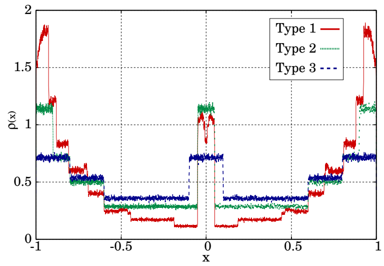



(a) Parallel shift.

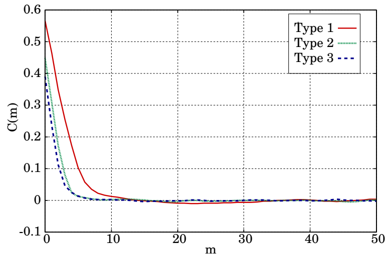



(b) Point symmetry.

Figure 4: Correlation function with each parameter.




(a) Invariant measure.




(b) Correlation function.

Figure 7: Analysis results of Chaotic map with 10 slopes.

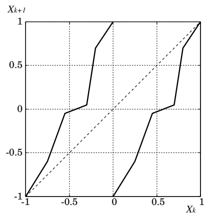
the chaotic sequence with any distribution increasing the slopes of the chaotic map. In this paper, we generate the chaotic sequence with the biased values having 3 distribution including many values of a near $(-1,\ 0,\ 1)$ based on results in Sec. 2.3. Figure 5 shows the chaotic map with 10 slopes to generate the chaotic sequence with 3 distribution. This map can change the frequency of 3 distribution, i.e. $\rho(x)$ of 3 distribution by changing the altitude of the slope. To investigate the distribution by different occurrence rate, 3 types of parameters are used. These parameters are described below. Also, Figures 6(a), (b) and (c) show the chaotic map with each parameter.

- Type 1
  $(q_1,\ r_1) = (-0.75,\ -0.6)$, $(q_2,\ r_2) = (-0.55,\ -0.05)$, $(q_3,\ r_3) = (-0.3,\ 0.05)$, $(q_4,\ r_4) = (-0.2,\ 0.7)$, $(q_5,\ r_5) = (0.25,\ -0.6)$, $(q_6,\ r_6) = (0.45,\ -0.05)$, $(q_7,\ r_7) = (0.7,\ 0.05)$, $(q_8,\ r_8) = (0.8,\ 0.7)$

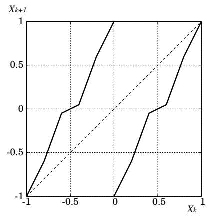
- Type 2
  $(q_1,\ r_1) = (-0.8,\ -0.6)$, $(q_2,\ r_2) = (-0.6,\ -0.05)$, $(q_3,\ r_3) = (-0.4,\ 0.05)$, $(q_4,\ r_4) = (-0.2,\ 0.6)$, $(q_5,\ r_5) = (0.2,\ -0.6)$, $(q_6,\ r_6) = (0.4,\ -0.05)$, $(q_7,\ r_7) = (0.6,\ 0.05)$, $(q_8,\ r_8) = (0.8,\ 0.6)$

- Type 3
  $(q_1,\ r_1) = (-0.8,\ -0.6)$, $(q_2,\ r_2) = (-0.6,\ -0.1)$, $(q_3,\ r_3) = (-0.4,\ 0.1)$, $(q_4,\ r_4) = (-0.2,\ 0.6)$, $(q_5,\ r_5) = (0.2,\ -0.6)$, $(q_6,\ r_6) = (0.4,\ -0.1)$, $(q_7,\ r_7) = (0.6,\ 0.1)$, $(q_8,\ r_8) = (0.8,\ 0.6)$

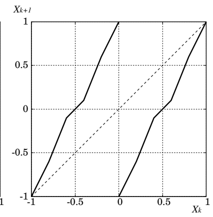
Figure 7 (a) and (b) show the analysis results of the chaotic map with each type parameter. In Fig. 7(a), we can observe that 3 distribution including many values of a near $(-1,\ 0,\ 1)$. Moreover, we can also confirm the different occurrence rate of $(-1,\ 0,\ 1)$ according to the parameters. In Fig. 7(b), it can be observed that $C(m)$ increase according to the parameters as compare with Fig. 4. In addition, it can be said that the chaotic sequence is generated without having the similar value alternately unlike Fig. 4(b). Therefore,




(a) Type 1




(b) Type 2




(c) Type3

Figure 6: Chaotic map with 10 slopes (each parameter).

by giving the many slopes for the chaotic map, we can control precisely the distribution of the chaotic sequence and obtain the high correlation properties.

## 4. Noncoherent Chaos Communication

In this section, we carry out the numerical simulation of a noncoherent chaos communication using the chaotic map with 10 slopes. In this paper, we perform the DCSK simulation as a noncoherent chaos communication. As a reason, it is because that DCSK is one of the noncoherent correlation-based communication systems, and its system is very simple. First we introduce DCSK operation. Next we explain the numerical simulation and discuss the obtained results.

Figure 8 shows the block diagram of a DCSK transmitter (a) and receiver (b). In this scheme, the transmitter out-

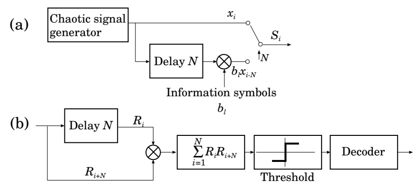



Figure 8: DCSK operation. (a) transmitter. (b) receiver.

puts a chaotic sequence $x_i$ followed by the same sequence multiplied by the information symbol $b_l(\pm 1)$. To transmit 1-bit information, $N$ chaotic signals are generated, where $N$ is the chaotic sequence length. Therefore, the transmitted signal is given by

$$S_i = \begin{cases} x_i & (1 \leq i \leq N) \\ b_l x_{i-N} & (N+1 \leq i \leq 2N) \end{cases} \tag{4}$$

Also, the transmitted signal can be written as $\mathbf{S} = (S_1 \ S_2 \ \cdots \ S_{2N})$ by vector. Since the noise $\mathbf{n} = (n_1 \ n_2 \ \cdots \ n_{2N})$ is added to the transmitted signal by the channel, the received signal can be written as $\mathbf{R} = (R_1 \ R_2 \ \cdots \ R_{2N}) = \mathbf{S} + \mathbf{n}$.

On the receiving side, it is evaluated by the correlation of 2 signals, which are obtained by dividing the received signals into two halves (length $N$). Thus, the output of the correlation can be written as

$$C_1 = \sum_{i=1}^{N} R_i R_{i+N} \quad . \tag{5}$$

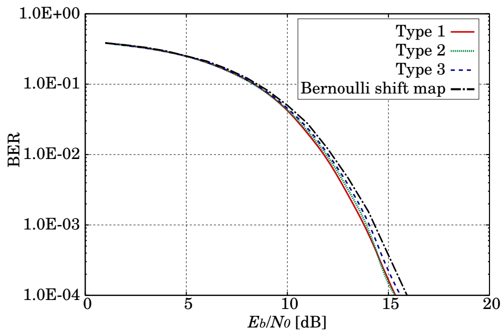
The decoded symbol is decided as "+1" or "−1" depending on $C_1$ being larger or smaller than 0.

To generate the transmitted signal, the DCSK transmitter needs to switch correctly by the chaotic sequence length $N$. Therefore, the sophisticated switch is required, and it is regarded as the important issue to design DCSK.

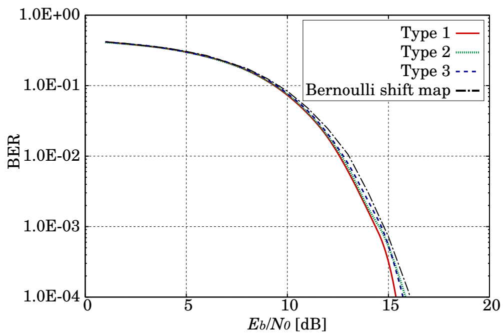
## 5. Simulation results and Discussions

The simulation conditions are as follows. In the transmitting side, the chaotic sequence length $N$ are 16 and 32. Also, as the parameters deciding the slopes of the chaotic sequence, we use 3 type parameters in Sec. 3, i.e. Type 1, Type 2 and Type3. In the channel, noise is assumed to be AWGN. Based on these conditions, the system performance is evaluated by plotting the BER against $E_b/N_0$ when $10^4$ bits of information are transmitted.

Figure 9 plots the BERs versus $E_b/N_0$ for each type parameter. To compare the performance of the chaotic sequence with biased values, Fig. 9 shows the performance of the conventional DCSK using the Bernoulli shift map. From these results, we can observe that the both BERs ($N = 16, 32$) improve as compare with the conventional DCSK according to the parameters. Especially, Type 1's performance is the best in Fig. 9. This reason for improving its performance is considered that we can control the distribution of the chaotic sequence with biased values so that the correlation property improved. In Figs. 4(a) and 7, we have showed the correlation function of the Bernoulli shift map and the chaotic map with 10 slopes, respectively. We recognize from these figures that $C(0)$ of the chaotic sequence with biased values is higher than that of the Bernoulli shift map. The higher $C(0)$ represents that the autocorrelation property of the proposed chaotic map is high, namely, the chaotic sequence with biased values become strong to interference of the noise even if it does not give the distribution in the extreme. From these results, it would be expected to be strong to a multipass fading. In addition, it might be able to obtain a stable BER which does not depend on an initial value by giving the biased values for the chaotic sequence. Therefore, it can be said that the chaotic sequence which changed the distribution of biased values is very effective in the chaos communication systems.




(a) $N = 16$.




(b) $N = 32$.

Figure 9: Simulation results.

## 6. Conclusions

In this study, we have analyzed the chaotic map for the chaotic sequence with biased value deeply and carried out the computer simulation using its sequence. As results, we have obtained the higher autocorrelation property and the better BER performance by controlling the distribution of the chaotic sequence with biased values, namely, the performance of the chaos communication is improved by the chaotic sequence with biased values depending on its distribution. Consequently, it would be expected that the performance of chaos communication is improved further by controlling the number or frequency of its distribution for any purpose of communication.

## References

[1] G. Kolumbán, B. Vizvári, W. Schwarz, and A. Abel, "Differential chaos shift keying: A robust coding for chaos communication," *Proc. NDES'96*, pp. 87-92, Jun. 1996.

[2] M. Hasler and T. Schimming, "Chaos communication over noisy channels," *Int . Journal of Bifurcation and Chaos*, vol. 10, no. 4, pp. 719-736, Apr. 2000.

[3] M. Sushchik, L. S. Tsimring and A. R. Volkovskii, "Performance Analysis of Correlation-Based Communication Schemes Utilizing Chaos," *IEEE Trans. Circuits and Systems Part I*, Vol. 47, pp 1684-1691, Dec. 2000.

[4] F. C. M. Lau and C. K. Tse, *Chaos-Based Digital Communication Systems*, Springer, 2003.

[5] W. M. Tam, F. C. M. Lau and C. K. Tse, "Generalized correlation-delay-Shift-Keying Scheme for Noncoherent Chaos-Based Communication Systems," *IEEE Trans. Circuits and Systems Part I*, vol. 53, no. 3, pp. 712-721, Mar. 2006.

[6] S. Arai and Y. Nishio, "Variable Sequence Length Transmitter for Noncoherent Chaos Shift Keying," *RISP Journal of Signal Processing*, vol. 10, no. 4, pp. 255-258, Jul. 2006.

[7] G. Cimatti, R. Rovatti and G. Setti, "Chaos-Based Generation of Optimal Spreading Sequences for DS-UWB Sensor Networks," *Proc NOLTA'06*, pp. 583-586, Sep. 2006.

[8] S. Arai and Y. Nishio, "Research on Differential Chaos Shift Keying Changing Deviation of Chaotic Sequence," *Proc NCSP'07*, pp. 349-352, Mar. 2007.