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An Associative Chaotic Neural Network with Gap-Junctions

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Abstract—In the present paper, we incorporate gap–junction connections into associative chaotic neural networks. The constituent neurons in the network are fully connected to other neurons in the network through synaptic weights with a delay. The neighboring neurons in the network are connected through gap–junctions. The gap–junction connections are modeled by taking the difference between internal states of neurons connected through gap–junctions without delay. We investigate retrieval dynamics of the associative chaotic neural network with the gap–junction connections. As a result, the associative chaotic neural network with the gap–junction connections of medium strength is easier to show non-periodic retrieval dynamics than the network without gap–junction connections.

1. Introduction

Most of neural network models are based on the knowledge that the constituent neurons in the brain are mutually coupled by synapses. Typical model for synaptic connections assumes that input signals from other neurons are modeled as a weighted sum of the output of other neurons. Recently, the electrical coupling between neurons so called gap–junction is found in neo-cortex [1] [2] that may have some functional roles in the brain.

A network of model neurons of differential equations with gap–junctions is proposed and it is reported that the network shows chaotic dynamics even when the external inputs to the network is constant in time and uniform in space[3]. The authors of the paper suggest that the chaotic dynamics may play a role in feature binding problems.

In order to investigate possible role of gap–junctions for memory system in the brain, we incorporate gap–junctions into an associative chaotic neural network [4]–[6]. Then we may discuss possible role of gap–junctions in the field of associative memory. As the first step for such a study, in this paper, we construct an associative chaotic neural network whose constituent neurons are mutually coupled not only through synaptic connections but also through gap–junction connections. The weights of the synaptic connections are determined by a conventional auto–associative matrix[7]–[11] with a delayed feedback of the other neurons in the network. The gap–junction connections are modeled by the difference between internal states of the next neighbor neurons without delay.

A network of chaotic elements have been extensively studied [12]. Major differences with the existing studies and the present study are as follows: (1) the network model to be presented in this paper is an extension of conventional artificial neural networks that can be easily adapted to applications; e.g., an associative memory as presented in the rest of the paper. (2) in the present work two-kinds of connections are simultaneously used; that are synaptic connections for outputs of neurons and gap-junction connections for internal states although the existing studies of networks of chaotic maps consider only connections for outputs.

We compare the retrieval dynamics of the associative chaotic neural networks with and without gap–junction connections by numerical experiments. As a result, the network with gap–junction connections is easier to show non-periodic retrieval dynamics than the network without gap–junction connections.

2. Associative Chaotic Neural Network with Gap–junctions

The associative chaotic neural network to be investigated with gap–junction connections in the present paper consists of chaotic neurons [4][5] that exhibits deterministic chaos by itself. In the present paper, in order to distinguish functional role of synaptic connections and gap–junction connections, the connections are assumed to have the following features. The synaptic connections are modeled as weights that are multiplied by the output of the constituent neurons in the network as the same as the most of the neural network models. In this paper, full–connections are assumed for the synaptic connections except for the self–feedback. On the other hand, the gap–junction connections are modeled as the difference between internal states of the next neighbor neurons with a gap–junction weight. From the viewpoint of temporal processing, the synaptic connections and the gap–junction connections are assumed to possess the following difference. The synaptic connections are assumed to have one discrete time step delay for the states updating of the network, on the other hand, the gap–junction connections do not have any delay.

The associative chaotic neural network model with the above assumptions for the connections is represented by the following equations:

$$y_i(t+1) = \eta_i(t+1) + \zeta_i(t+1), \tag{1}$$

$$x_i(t+1) = f\left(y_i(t+1)\right), \tag{2}$$

$$\begin{aligned} \eta_i(t+1) &= k_f \eta_i(t) + \sum_{j=1}^{16} w_{ij} x_j(t-1) \\ &+ g_{i,i-1}\left(y_{i-1}(t) - y_i(t)\right) \\ &+ g_{i,i+1}\left(y_{i+1}(t) - y_i(t)\right), \end{aligned} \tag{3}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \tag{4}$$

where $x_i(t)$ denotes output of the ith neuron at discrete-time t. The variables $\eta_i(t)$ and $\zeta_i(t)$ denote internal states for feedback inputs from the constituent neurons and for the refractoriness, respectively. k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. α and a_i denote refractory scaling parameter and the bias to the ith neuron ($a_i = a$ for every neuron in this paper), respectively.

The synaptic weights and gap–junction weights from the jth neuron to the ith neuron are denoted by w_{ij} and g_{ij}, respectively. In Eq.(3), $g_{1,0}$ and $g_{16,17}$ are treated as equivalent to $g_{1,16}$ and $g_{16,1}$, respectively, so that the periodic boundary conditions are realized in the network. A delay of a single time step of the updating of the network states is assumed for the synaptic connections in order to make a contrast to the gap–junction connections in the transmission time of signals.

The output function of the neuron is denoted by f; in this paper, we use the logistic function represented by

$$f(y) = \frac{1}{1 + \exp(-y/\varepsilon)} \tag{5}$$

where ε is a parameter for the steepness of the function[4][5]. We examine on the associative chaotic neural network with 16 chaotic neurons. The stored patterns of the network are three 16-dimensional binary patterns whose average firing rate of each pattern is set to be equal to 0.5. Therefore, the synaptic weights are determined by the following equation[8]–[11].

$$w_{ij} = \frac{4}{3} \sum_{p=1}^{3} (x_i^{(p)} - \overline{x})(x_j^{(p)} - \overline{x}) \tag{6}$$

with $w_{ii} = 0$ where $x_i^{(p)}$ is the ith component of the pth stored pattern. $\overline{x}$ denotes spatially averaged value of the stored patterns. In the following numerical experiments, we use the three stored patterns as shown in Fig.1.

The connection weights for the gap–junctions are determined as follows.

$$g_{ij} = \begin{cases} G & (for\ |i-j| = 1), \\ 0 & (otherwise) \end{cases} \tag{7}$$

where G denotes a coupling strength for the gap–junctions. Each constituent neuron in the network is connected only with the nearest neighbor neurons through the gap–junctions with a global parameter G. It has been reported

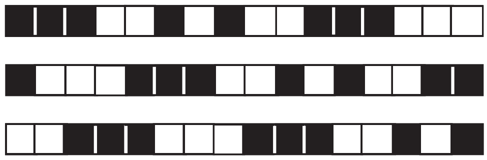

Figure 1: Stored patterns of the associative chaotic neural network with 16 neurons. Each row is a stored pattern vector. The filled and open squares in the rows represent 1 and 0 that correspond to the neuronal outputs, respectively.

that the associative chaotic neural network without neither gap–junctions nor the delay for feedback through synapses exhibits chaotic sequential patterns that include the stored patterns when the parameters of the network are set to certain values[6]. However, setting the parameter values or controlling the network without neither gap–junctions nor the delay for feedback through synapses to exhibit such chaotic sequential pattern retrieval is hard.

3. Retrieval Dynamics of Associative Chaotic Neural Network with Gap-Junctions

We examine the case when the associative chaotic neural network with both gap–junctions and the delay for the feedback through synapse. Namely, the network model of Eqs.(1) – (4) are examined as an associative memory. In the following numerical experiments, the initial states of the network are given by the second stored pattern of Fig.1 with perturbing two bits of the pattern. The initial values for the delay-feedback input to each neuron in the network are set to naught throughout the paper.

In order to show the effect of the gap–junction connections for the network, we firstly show the case when the network without gap–junction connections but with delayed-feedback connections through synapses. Figure 2 shows the time evolution of direction cosine $dc^{(p)}$ between the output pattern of the network with 16 neurons and the stored patterns of Fig.1.

The direction cosine is computed by

$$dc^{(p)}(t) = \frac{1}{16} \sum_{i=1}^{16} x_i(t) x_i^{(p)}, \tag{8}$$

for the pth pattern, where the pth patterns for $p = 1, 2, 3$ represent the patterns of Fig.1 from the top to the bottom row, respectively. When the network retrieves the pth exact stored pattern and its reverse one, $dc^{(p)}$ becomes 1 and -1, respectively.

Figure 2 shows that the network without gap–junctions retrieves only one closest stored pattern to the initial pattern and oscillating around it with the above mentioned parameter values. This is the basal state for the following numerical experiments.

Next, examples of retrieval dynamics of the stored patterns of Fig.1 are shown in Figs.3–5 for different gap–junction coupling strength G with the same values for other parameters as in the case of Fig.2. Figures 3–5 show the time evolution of direction cosine $dc^{(p)}$ between the stored patterns and the output pattern of the network with gap–junctions and the delay for the feedback through synapse, when G is set to $0.004, 0.04$, and 0.2, respectively.

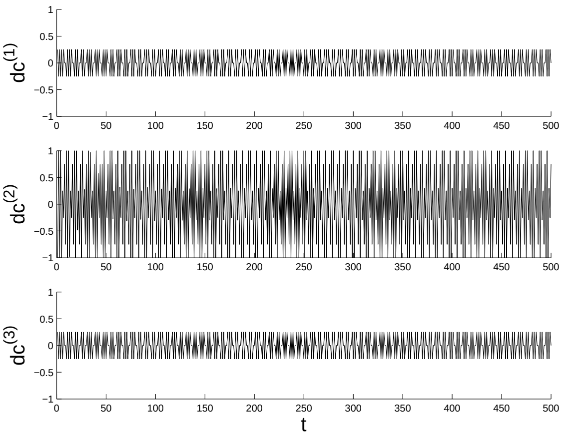

Figure 2: Time evolution of the direction cosine between the three stored patterns and the output pattern of the network without gap–junctions.

From Figs.3–5 we find the following tendency by changing the coupling strength G of the gap–junctions. (a) when G is small, the network converges to a periodic oscillation after transient wandering states for several hundreds of iterations as shown in Fig.3 for $G = 0.004$. The wandering transient duration sometimes includes some of the stored patterns. This result can be seen that the effect of the gap–junctions is too weak and the dynamics is similar to the network without gap–junction connections. (b) when G is large, the network quickly converges to a periodic oscillation which includes one of the stored patterns as shown in Fig.5 for $G = 0.2$. This result can be seen that the effect of the gap–junction connections is dominant for the network dynamics so that groups of neurons in the network are synchronized. (c) when G is a medium value, the network retrieves all the stored patterns in a non-periodic sequence as shown in Fig.4 for $G = 0.04$. Moreover, Fig.6 shows that the retrieval dynamics of the network is not a periodic oscillation at least for 20,000 iterations.

The tendency that the network with a medium coupling strength G of the gap–junctions shows a non-periodic oscillation with retrieval of the stored patterns can be found in other parameter values, namely, in most of the cases we examined, it is easier to find the non-periodic retrieval dynamics of the network with medium strength

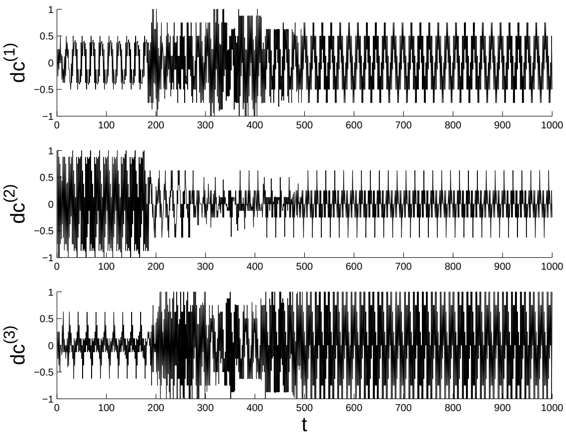

Figure 3: Time evolution of the direction cosine between the three stored patterns and the output pattern of the network with gap–junctions ($G = 0.004$).

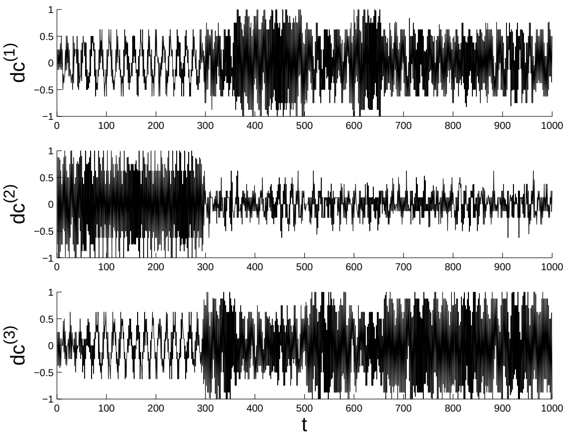

Figure 4: Time evolution of the direction cosine between the three stored patterns and the output pattern of the network with gap–junctions ($G = 0.04$).

gap–junction connections and delayed-feedback connections through synapses than the the network without gap–junction connections but with delayed-feedback connections.

4. Conclusions

In the present paper, we incorporate gap–junction connections into an associative chaotic neural network. The constituent neurons in the network are fully connected to other neurons in the network through synaptic weights with a delay. The neighboring neurons in the network are connected through gap–junctions. The gap–junction connections are modeled by taking the difference between internal states of the neurons connected through the gap–junctions without delay. We investigate retrieval dynam-

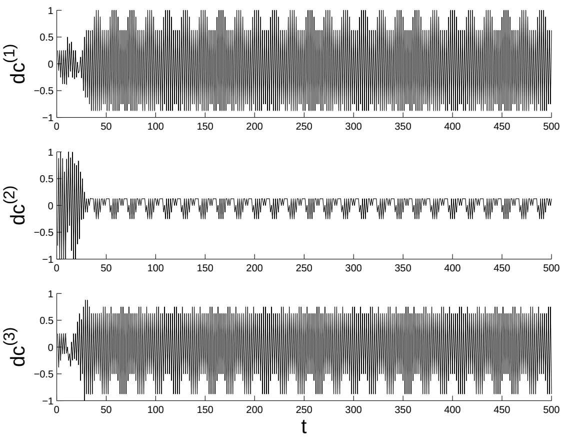

Figure 5: Time evolution of the direction cosine between the three stored patterns and the output pattern of the network with gap–junctions (G = 0.2).

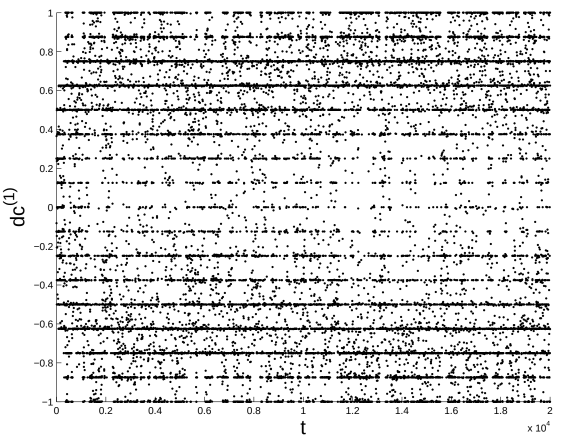

Figure 6: Time evolution of the direction cosine between one of the stored patterns and the output pattern of the network with gap–junctions (G = 0.04) for 20 000 iterations.

ics of the associative chaotic neural network with the gap–junction connections. As a result, we find a tendency that the network with both gap–junction connections of a medium strength and delayed-feedback connections through synapses is easier to show non-periodic dynamics or long-transient duration which looks non-periodic than the the network without gap–junction connections.

An analysis of the non-periodic retrieval dynamics with computing indices like the Lyapunov exponents is a future problem.

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