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Sparse and Passive Reduced-Order Interconnect Modeling by Eigenspace Method

Yuichi Tanji
Kagawa University
tanji@eng.kagawa-u.ac.jp

Abstract— **The passive and sparse reduced-order modeling of RLC networks is presented, where eigenvalues and eigenvectors of the original networks are used, thus, the macromodels obtained from the proposed method are more accurate than the Krylov subspace methods and TBR procedures to a class of problems. Furthermore, the proposed method is applied to post-processing after a reduced order model is obtained from the Krylov subspace methods or TBR procedures without breaking the passivity condition and losing accuracy of the marocmodels. Therefore, the proposed eigenspace method is not only an alternative of other reduced-order macromodeling methods, but also is embedded in these methods enhancing their performances.**

I. Introduction

Increasing clock speed and low resistance metal on high performance integrated circuits make on-chip inductance effects prominent. The complex 3D structural interconnects in integrated circuits are modeled by the RLC circuits equivalent to the electromagnetic fields. Since the RLC circuits are extremely large-scale, the time- and frequency-domain analysis for evaluating the inductance effects wastes huge CPU times. The problem of inductance is not limited to VLSI systems. To gain reliability of overall performance of the electronic systems, the packages and PCBs embedded in the systems are modeled by RLC circuits and the time- and/or frequency-domain analysis must be carried out.

Model Order Reduction (MOR) [1]-[13], [15] for large-scale linear RLC networks are known as a powerful tool for simulating and modeling the interconnects on VLSI's, packages, and board levels. As MOR methods, AWE [1] and CFH [2] were the pioneers. PVL [3] suggested effectiveness of the Krylov subspace method. PACT [4] presented an efficient reduction technique for RC networks preserving passivity of the reduced circuits. PRIMA [5] is a general method for passive reduced-order macromodeling of RLC networks, and many derivations were proposed [6], [7]. Furthermore, the concept is extended to modeling of circuit, microwave, mechanical, and biological systems. In control community, Truncated Balanced Realization (TBR) has been developed as the reduction methods [13]. To obtain the reduced-order model based on TBR, we must solve the Lyapunov equation which needs a large cost. Hence, a method for approximating the gramian which is the solution of the Lyapunov equation, was presented [12]. Although TBR provides more accurate macromodel than the Krylov subspace methods, it does not guarantee passivity of the macromodel. To guarantee the passivity of the reduced circuits on TBR procedures, solving the algebraic Riccati equations was presented in [8], [9]. Recently, a passive reduction method for descriptor system is proposed [10], which is based on the gramian-based method [11] and projection method such as PRIMA. Solving the algebraic Riccati equation to a class often seen for interconnects on VLSI's, packages, and board levels, is very complicated, where the deflated subspace must be considered [8]. On the other hand, the passivity-preserving gramian-based method [10] is easily applicable to this case.

In this paper, the eigenspace method for reduced-order interconnect modeling is presented. The eigenspace method uses a part of eigenvalues and eigenvectors of the original system directly. Therefore, this method is more accurate to a class of problems than the TBR and Krylov subspace methods. However, passivity of the macromodel obtained from the eigenspace method was not guaranteed. Hence, we propose how to preserve passivity of the reduced circuit. Since the eigenspace method is based on the eigen decomposition, it might not be preferable to other MOR methods. However, after reductions using the passivity-preserving Krylov subspace methods [5]-[7] or gramian-based method [10], the eigenspace method can be used for post-processing the reduced-order models obtained from these MOR methods. Namely, the reduced-order models obtained from the Krylov subspace methods and the gramian-based method are composed of dense matrices, thus, the models must be converted into ones with sparse matrices to speed up the simulation using the macromodels. Furthermore, though the models, which are generated by the Krylov subspace methods or the gramian-based method, fit the characteristics of the original circuit at low frequencies, they tend to have erroneous parts at high frequencies. Hence, the low pass filtering is also necessary for the reduced-order models. The eigenspace method offers the sparsification and low pass filtering without breaking the passivity condition and degrading the accuracy of the macromodels.

This paper is organized as follows. Section II provides the eigenspace method. Section III presents how to preserve passivity and sparsify the reduced-order model. Section IV shows the numerical examples. In Section V, conclusions are presented.

II. Eigenspace Method

A function of RLC networks in the Laplace-domain such as admittance, impedance, and so on, has real poles and complex conjugate pairs. The reduced-order model of RLC networks can be easily produced by these poles (we call it the eigenspace method). Although the Krylov subspace [5]-[7] and gramian-based [8]-[13] MOR methods provide an efficient model replacing the original system into a small one, these methods are not necessarily effective for all the cases. Fig. 1 shows the approximation results obtained from the eigenspace method, PRIMA [5], and TBR [13] for an RLC ladder network (detail of this example will be presented in Sect. V), where 'exact' is the exact frequency response, 'eigen' is the result obtained from the eigenspace method, 'PRIMA' and 'TBR' are the results of PRIMA and TBR, respectively. The result given by PRIMA is not better than the eigenspace method and the waveform has an incorrect large peak part around $30[Hz]$. The waveform obtained from TBR is greatly different from the exact frequency response. Therefore, the reduced-order modeling by the eigenspace method is effective to a class of problems. The rest of this section reviews the reduced-order modeling of RLC networks by the eigenspace method for the following discussion.

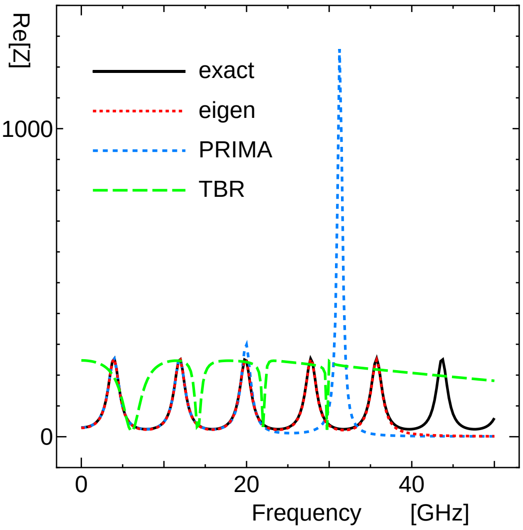

Fig. 1. Approximation results obtained from the eigenspace method, PRIMA, and TBR.

The multi-port RLC networks are formulated by using the modified nodal analysis as follows:

$$\mathcal{G}\boldsymbol{x}(t) + \mathcal{C}\frac{d}{dt}\boldsymbol{x}(t) = \mathcal{B}\boldsymbol{u}(t), \quad (1)$$

$$\boldsymbol{y}(t) = \mathcal{B}^T\boldsymbol{x}(t). \quad (2)$$

The state vector $\boldsymbol{x}(t)$ consists of $[\boldsymbol{v}(t), \boldsymbol{i}(t)]^T$, where $\boldsymbol{v}(t)$ is the node voltages and $\boldsymbol{i}(t)$ is inductor currents. $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ are input and output vectors, respectively.

The coefficient matrices of (1) are expressed as

$$\mathcal{G} = \begin{bmatrix} \boldsymbol{G} & \boldsymbol{A}^T \\ -\boldsymbol{A} & \boldsymbol{0} \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L} \end{bmatrix},$$

where $\boldsymbol{G}$, $\boldsymbol{C}$, and $\boldsymbol{L}$ are the conductance, capacitance, and inductance matrices, and $\boldsymbol{A}$ is the incident matrix associated with the inductor branches. Although the formulation (1) is useful for the Krylov subspace MOR methods [5]-[7], it is not suitable for the gramian-based methods. Then, the following matrix $\mathcal{G}$, which includes an resistance matrix $\boldsymbol{R}$, is used [14].

$$\mathcal{G} = \begin{bmatrix} \boldsymbol{G} & \boldsymbol{A}^T \\ -\boldsymbol{A} & \boldsymbol{R} \end{bmatrix}.$$

Transforming (1) and (2) into the Laplace-domain, we can write the impedance matrix as

$$\boldsymbol{H}(s) = \mathcal{B}^T(\mathcal{G} + s\mathcal{C})^{-1}\mathcal{B}. \quad (3)$$

The reduced-order impedance matrix is obtained from the eigen decomposition $\mathcal{G}^{-1}\mathcal{C} = \boldsymbol{S}\boldsymbol{\Lambda}\boldsymbol{S}^{-1}$. Considering m real single eigenvalues and n complex conjugate pairs, the q-th ($q = 2n + m$) order models of (3) is described by

$$\boldsymbol{H}_q(s) = \mathcal{B}^T\boldsymbol{S}_{1,q}(\boldsymbol{I} + s\boldsymbol{\Lambda}_q)^{-1}\boldsymbol{S}_{2,q}\mathcal{G}^{-1}\mathcal{B}, \quad (4)$$

where $\boldsymbol{S}_{1,q}$ only includes q columns of $\boldsymbol{S}$ and $\boldsymbol{S}_{2,q}$ has the corresponding q lows of $\boldsymbol{S}^{-1}$. Using the Cholesky decomposition, we can preserve passivity of the reduced-order models of RC or RL circuits. However, passivity of RLC circuits is not guaranteed except for the case that $\boldsymbol{S}_{1,q} = \boldsymbol{S}$, $\boldsymbol{S}_{2,q} = \boldsymbol{S}^{-1}$, and $\boldsymbol{\Lambda}_q = \boldsymbol{\Lambda}$. In the next section, the preservation of passivity will be presented.

III. Passivity and Sparsification

A. Preservation of Passivity

Consider the eigenvalue problem $\mathcal{G}^{-1}\mathcal{C}\boldsymbol{p} = \lambda\boldsymbol{p}$ associated with (1). Using m real single eigenvalues and n complex conjugate pairs, we have the following relation:

$$\mathcal{G}^{-1}\mathcal{C}\boldsymbol{P}_q = \boldsymbol{P}_q\boldsymbol{\Lambda}_q. \quad (5)$$

In (5), we assume the matrices consist of the eigenvalues and eigenvectors, respectively, as

$$\boldsymbol{\Lambda}_q = \mathrm{diag}\left(\lambda_{s,1} \ \dots \ \lambda_{s,m} \ \lambda_{c,1} \ \bar{\lambda}_{s,1} \ \dots \ \lambda_{c,n} \ \bar{\lambda}_{s,n}\right), \quad (6)$$

$$\boldsymbol{P}_q = \left[\boldsymbol{p}_{s,1} \ \cdots \ \boldsymbol{p}_{s,m} \ \boldsymbol{p}_{c,1} \ \bar{\boldsymbol{p}}_{c,1} \ \cdots \ \boldsymbol{p}_{c,n} \ \bar{\boldsymbol{p}}_{c,n}\right], \quad (7)$$

where 's' and 'c' mean single and complex conjugate eigenvalues, respectively, and '‾' means complex conjugate.

Putting $\boldsymbol{x}(t) = \boldsymbol{P}_q\boldsymbol{z}(t)$ and multiplying the conjugate transpose $\boldsymbol{P}_q^H$ of $\boldsymbol{P}_q$, we obtain the reduced-order equations of (1) and (2) as

$$\hat{\mathcal{G}}\boldsymbol{z}(t) + \hat{\mathcal{C}}\frac{d}{dt}\boldsymbol{z}(t) = \boldsymbol{P}_q^H\mathcal{B}\boldsymbol{u}(t), \quad (8)$$

$$\boldsymbol{y}(t) = \mathcal{B}^T\boldsymbol{P}_q\boldsymbol{z}(t), \quad (9)$$

where

$$\hat{\mathcal{G}} = \boldsymbol{P}_q^H\mathcal{G}\boldsymbol{P}_q, \quad \hat{\mathcal{C}} = \boldsymbol{P}_q^H\mathcal{C}\boldsymbol{P}_q.$$

Then, the reduced-order impedance matrix is given by

$$\boldsymbol{H}_q(s) = \mathcal{B}^T\boldsymbol{P}_q\left(\hat{\mathcal{G}} + s\hat{\mathcal{C}}\right)^{-1}\boldsymbol{P}_q^H\mathcal{B}. \quad (10)$$

The impedance matrix of (10) is expressed in a similar form of PRIMA [5] except that conjugate transpose is used rather than transpose. The following theorem is easily verified.

Theorem 1: The multiport network with impedance matrix $\boldsymbol{H}_q(s)$ of (10) is passive.

Proof) A network with impedance or admittance matrix $\boldsymbol{Y}(s)$ is passive, if $\boldsymbol{Y}(s) + \boldsymbol{Y}^H(s)$ is nonnegative definite for all complex values of s satisfying $Re(s) > 0$. As [5], we have

$$\boldsymbol{H}_q(s) + \boldsymbol{H}_q^H(s) = \mathcal{B}^T\boldsymbol{P}_q\left(\hat{\mathcal{G}} + s\hat{\mathcal{C}}\right)^{-1} \times\boldsymbol{P}_q^H\left[(\mathcal{G} + s\mathcal{C}) + (\mathcal{G} + s\mathcal{C})^H\right]\boldsymbol{P}_q\left(\hat{\mathcal{G}} + s\hat{\mathcal{C}}\right)^{-H}\boldsymbol{P}_q^H\mathcal{B}.$$

Assume that the original network is passive, then, $(\mathcal{G} + s\mathcal{C}) + (\mathcal{G} + s\mathcal{C})^H$ is nonnegative definite. Therefore, $\boldsymbol{H}_q(s) + \boldsymbol{H}_q^H(s)$ is nonnegative definite.

□

Moreover, the following useful property for accurate approximation holds.

Theorem 2: The eigenvalues of the matrix $\hat{\mathcal{G}}^{-1}\hat{\mathcal{C}}$ is equal to q values of $\mathcal{G}^{-1}\mathcal{C}$.

Proof) From (5), we obtain $\boldsymbol{P}_q^H\mathcal{C}\boldsymbol{P}_q = \boldsymbol{P}_q^H\mathcal{G}\boldsymbol{P}_q\boldsymbol{\Lambda}_q$. This means

$$\hat{\mathcal{G}}^{-1}\hat{\mathcal{C}} = \boldsymbol{\Lambda}_q, \quad (11)$$

where $\boldsymbol{\Lambda}_q$ includes q eigenvalues of the matrix $\mathcal{G}^{-1}\mathcal{C}$.

□

B. Sparsification

The MNA equation of (8) and (9) in the time-domain and the impedance matrix (10) have dense matrices $\hat{\mathcal{G}}$ and $\hat{\mathcal{C}}$. These dense matrices make the succeeding simulation inefficient. Furthermore, $\hat{\mathcal{G}}$ and $\hat{\mathcal{C}}$ are complex matrices, and the MNA equation of (8) and (9) must be converted into a real one.

From (11), (8) is rewritten by

$$\boldsymbol{z}(t) + \boldsymbol{\Lambda}_q \frac{d}{dt}\boldsymbol{z}(t) \quad = \quad \boldsymbol{W}_1 \mathcal{B} \boldsymbol{u}(t). \tag{12}$$

Assume that $\hat{\mathcal{G}}$ and $\boldsymbol{\Lambda}_q$ are invertible, then, $\hat{\mathcal{C}}$ is also invertible from (11). Hence, we can describe as

$$\begin{aligned} \boldsymbol{W}_1 &= \hat{\mathcal{G}}^{-1}\boldsymbol{P}_q^H \\ &= \boldsymbol{\Lambda}_q \hat{\mathcal{C}}^{-1}\boldsymbol{P}_q^H \\ &= \left[\boldsymbol{w}_{s,1} \ \ldots \ \boldsymbol{w}_{s,m} \ \ \boldsymbol{w}_{c,1} \ \ \bar{\boldsymbol{w}}_{c,1} \ \ldots \ \boldsymbol{w}_{c,n} \ \ \bar{\boldsymbol{w}}_{c,n} \ \right]^T. \end{aligned}$$

Since $\boldsymbol{x}(t) = \boldsymbol{P}_q \boldsymbol{z}(t)$ is a real vector, $\boldsymbol{z}(t)$ is expressed as $\boldsymbol{z}(t) = [z_{s,1} \ \ldots \ z_{s,m} \ z_{c,1} \ \bar{z}_{c,1} \ \ldots \ z_{c,n} \ \bar{z}_{c,n} \ ]$. Adding the equations related with $z_{c,1}$ and $\bar{z}_{c,1}$ in (12) and subtracting them [15], we can obtain the real equation equivalent to (12) as

$$\tilde{\boldsymbol{z}}(t) + \tilde{\boldsymbol{\Lambda}}_q \frac{d}{dt}\tilde{\boldsymbol{z}}(t) \quad = \quad \boldsymbol{W}_2 \mathcal{B} \boldsymbol{u}(t), \tag{13}$$

where

$$\tilde{\boldsymbol{\Lambda}} = \begin{bmatrix} \lambda_{s,1} & & & & & & & & \\ & \ddots & & & & & & \boldsymbol{0} & \\ & & \lambda_{s,m} & & & & & & \\ & & & 2\mathrm{Re}\lambda_{c,1} & -2\mathrm{Im}\lambda_{c,1} & & & & \\ & & & 2\mathrm{Im}\lambda_{c,1} & 2\mathrm{Re}\lambda_{c,1} & & & & \\ & & & & & \ddots & & & \\ & \boldsymbol{0} & & & & & 2\mathrm{Re}\lambda_{c,n} & -2\mathrm{Im}\lambda_{c,n} \\ & & & & & & 2\mathrm{Im}\lambda_{c,n} & 2\mathrm{Re}\lambda_{c,n} \end{bmatrix},$$

$$\begin{aligned} \tilde{\boldsymbol{z}} &= \left[z_{s,1} \ \ldots \ z_{s,m} \ \ \mathrm{Re}z_{c,1} \ \ \mathrm{Im}\bar{z}_{c,1} \ \cdots \ \mathrm{Re}z_{c,n} \ \ \mathrm{Im}\bar{z}_{c,n}\right]^T, \\ \boldsymbol{W}_2 &= \left[\boldsymbol{w}_{s,1} \ \ldots \ \boldsymbol{w}_{s,m} \ \ 2\mathrm{Re}\boldsymbol{w}_{c,1} \ \ 2\mathrm{Im}\boldsymbol{w}_{c,1} \right. \\ &\qquad \left. \ldots \ 2\mathrm{Re}\boldsymbol{w}_{c,n} \ \ 2\mathrm{Im}\boldsymbol{w}_{c,n}\right]^T. \end{aligned}$$

On the other hand, (9) is rewritten by the real equation:

$$\boldsymbol{y}(t) \quad = \quad \mathcal{B}^T \boldsymbol{W}_3 \tilde{\boldsymbol{z}}(t), \tag{14}$$

where

$$\begin{aligned} \boldsymbol{W}_3 \quad = \quad & \left[\boldsymbol{p}_{s,1} \ \ \ldots \ \boldsymbol{p}_{s,m} \ \ 2\mathrm{Re}\boldsymbol{p}_{c,1} \ \ -2\mathrm{Im}\boldsymbol{p}_{c,1} \right. \\ & \left. \ldots \ 2\mathrm{Re}\boldsymbol{p}_{c,n} \ \ -2\mathrm{Im}\boldsymbol{p}_{c,n}\right]. \end{aligned}$$

Furthermore, the reduced-order impedance matrix of (10) can be written by

$$\boldsymbol{H}_q(s) \quad = \quad \mathcal{B}^T \boldsymbol{W}_3 \left(\boldsymbol{I} + s\tilde{\boldsymbol{\Lambda}}_q\right)^{-1} \boldsymbol{W}_2 \mathcal{B}. \tag{15}$$

A network with the impedance (15) is passive, since (15) is equivalent to (10). The matrix $\boldsymbol{I} + s\tilde{\boldsymbol{\Lambda}}_q$ is not only sparse but also has at most 2×2 block submatrices on the diagonal. Hence, the matrix is easily invertible, and the frequency-domain analysis using (15) is efficiently carried out. Furthermore, equations of (13) and (14) are real ones, thus, the reduced-order circuit can be simulated on SPICE-like simulators [15].

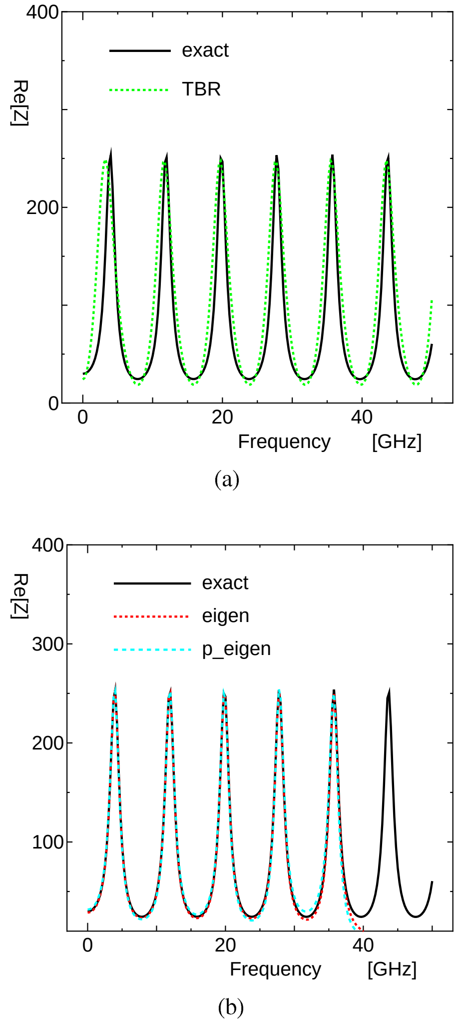

Fig. 2. Frequency responses in Example 1. (a)Results obtained from TBR. (b)Frequency responses of non-passive/passive eigenspace methods.

IV. Examples

A. Example 1

The driving point impedance of RLC ladder network with 100 RLC sections ($R = 0.1[\Omega]$, $L = 5.0 \times 10^{-10}[H]$, $C = 8.0 \times 10^{-15}[F]$) and the terminating resister ($20[\Omega]$) was calculated by using some methods. Figure 1 shows the results obtained from the eigenspace method ('eigen'), PRIMA [5] ('PRIMA'), and TBR [13] ('TBR'), and the exact response ('exact'), where the order of all the macromodels is 11. The results obtained from the eigenspace method is almost identical to the exact response up to 5-th peak parts. Since the original eigenvalues are 1 single (real) and 5 complex conjugate pairs, each peak is corresponding to a complex eigenvalue. However, the PRIMA model cannot capture the 5 peak parts completely and has a large erroneous peak around $30[GHz]$.

Figure 1 shows that TBR is not suitable for this problem. Even though the order of macromodel is taken as 60, the frequency-response obtained from TBR does not match the exact response as shown in Fig. 2(a).

Using the guaranteed passive eigenspace method provided in Sect. II, we computed the response ('p_eigen'). The response is almost identical to non-passive case ('eigen').

B. Example 2

The driving point impedance of RLC ladder network with 100 RLC sections ($R = 1.0[\Omega]$, $L = 1.0 \times 10^{-9}[H]$, $C = 5.0 \times 10^{-12}[F]$) and the terminating resister ($20[\Omega]$), was calculated. Although the circuit structure is the same as Example 1, the frequency response is very smooth different from the exact response of Fig. 2. Therefore, this example is a good choice to estimate performance of each method. Figure 3(a) shows the results obtained from the passivity-preserving eigenspace method, PRIMA, TBR, the guaranteed passive gramian-based projection method [10] ('sym'), and the approximated gramian ('smith') is used instead of solving the generalized Lyapunov equation [9]. The order of the macromodels is 30 except for TBR. The truncated order of TBR is 5, which means that TBR is the most efficient of all the methods.

We can see that the frequency responses obtained from the methods using projection matrix have erroneous peak parts. To remove them, the eigenspace method was used. Figure 3(b) shows the low pass filtered results for 'PRIMA', 'sym', and 'smith' of Fig. 3(a), where 2 complex conjugate pairs of poles with up to the second largest imaginary parts are eliminated for 'PRIMA' and 'smith', and 5 complex conjugate pairs are eliminated for 'sym'. We can see that unwanted peak parts of Fig. 3(b) are removed. It should be noted that passivity of the reduced circuits is guaranteed after low pass filtering.

V. Conclusions

The sparse and passive reduced-order modeling of RLC networks has been presented, where a part of eigenvalues and eigenvectors is used. This method provides more accurate model than PRIMA and TBR to a class of problems. Since this method is based on the eigen decomposition, the application is limited to a small problem. However, it is shown that this method is effective for sparsification and low pass filtering after reduction using the Krylov subspace methods [5]-[7] and the guaranteed passive gramian-based method [10]. Therefore, the proposed method is also available for post-processing the reduced-order models obtained from these MOR methods.

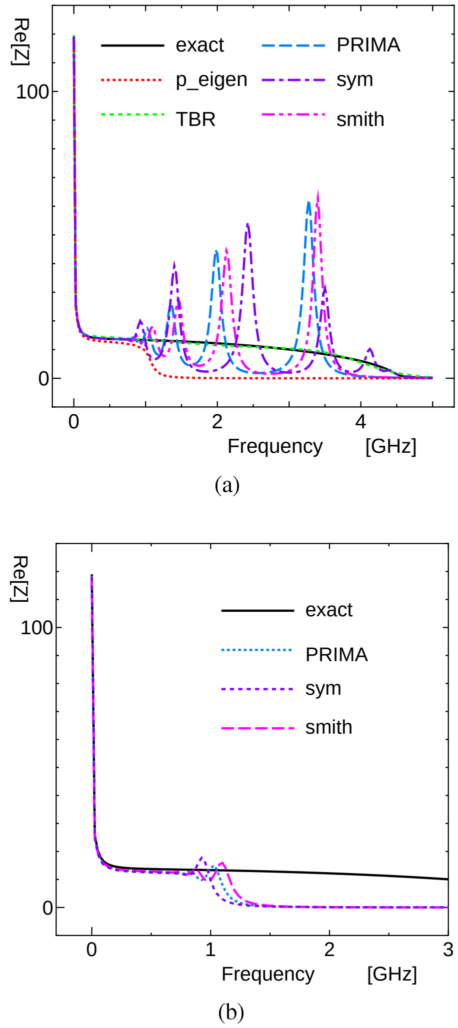

Fig. 3. Frequency responses in Example 2. (a)Results obtained from the methods using projection matrix. (b)Low pass filtered results.

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