Numerical Analysis of Electromagnetic Bandgap Structures of Arbitrary Shapes

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Abstract

Electromagnetic bandgap structure consisting of an arbitrary number of cylinders are considered. In this paper, the scattering properties from this multilayered structure consisted of arbitrary shaped cylinders are analyzed. The method is used to study large varieties of configurations and structures in respect to varying shapes, sizes and material compositions. Practical applications are illustrated showing filtering characteristic for E-polarized wave.

1. Introduction

The analysis of a multilayered periodic structure composed of cylindrical, metallo-dielectric objects has been presented by many authors [1]. When the shape of the scatterer is the circular cylinder or sphere, the eigen-function expansion method can be used in order to examine their scattering properties. These structures possess a stop band in their reflection characteristics and therefore are called electromagnetic bandgap (EBG) structures in the microwave wavelength range or photonic bandgap (PBG) structures in the infrared wavelength range. The constructive elements of these structures are comparable in size to the operation wavelength and may be composed of different media. Recently, EBGs and PBGs are of great interest due to their extraordinary properties and potential applications such as filters [2].

In this article, the scattering properties from multilayered structures consisted of periodic arrays of deformed dielectric cylinders are examined by integral equation formulation and solved by the Method of Moment (MoM) [3]. For the structures considered in this article, each layer is consisted of one or more elements arranged accordingly without touching each other in the unit cell. Within the unit cell of a period, each scatterer undergoes multiple scattering between adjacent scatterers. The matrix elements obtained by the MoM contains the scattering parameters of all scatterers within the unit cell, which depend on the scatterer size, shape, composition and orientation, but not on the nature of the incident. Scattering properties show useful results for the design of selection and stopband filters for optical frequency region applications.

2. Formulation

The periodic structure under investigation is composed of an array of uniformly spaced 2-D deformed cylinders illuminated by a E-polarized plane wave. The geometry is assumed to be infinite in the z-direction and therefore, the field quantities and permittivity vary in x and y as shown in Fig. 1. The periodic arrays are immersed in homogeneous media forming composite material media separated from the free space medium. The composite media of each array contained an arbitrary shape cylinder infinitely arranged along the direction of periodicity d. The scattering elements assumed to have a constant permeability and an inhomogeneous relative permittivities $\varepsilon_{rv}(x, y)$, ε_{rbv} is the relative permittivity of the homogeneous region outside the scattering elements for the v-th region, where ($v = 1, 2, \cdots, V$), h is both the layers separation and layers thickness and V is the number of layers. The incident electric field is propagating in free space with its $\mathbf{k}$ vector on the xy plane at an angle of incidence θ_i with respect to the y axis and $\mathbf{r}$ is the position vector. The wave vector is given by $\mathbf{k} = \hat{x}k_x + \hat{y}k_y$ where $k_x = -k_0 \sin\theta_i$ and $k_y = -k_0 \cos\theta_i$ are the wave constants in the x and y direction respective. The wave number k_0 in a homogeneous free space region is defined by $k_0^2 = \omega^2\mu_0\varepsilon_0$. The $\exp(j\omega t)$ time variation is suppressed. As

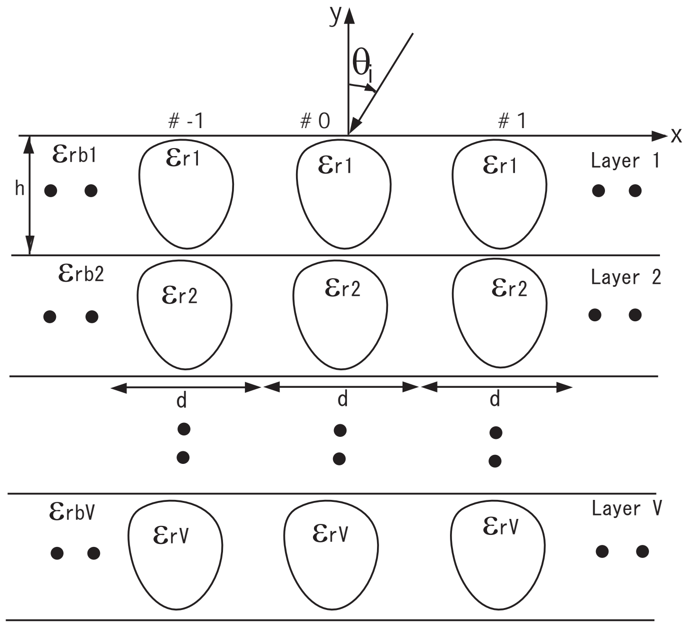

Figure 1: Goemetry for electromagnetic bandgap structure of arbitrary shapes.

the plane wave illuminates the array structure, the scattered field at any point of the structure is obtained using the result of scattering by each array [3]. The total electric field $E(\mathbf{r})$ is given as follows:

$$E(\mathbf{r}) = E_{inc}(\mathbf{r}) - j\frac{k_0^2}{4}\sum_{l=-\infty}^{\infty}\int_{S_{0v}} H_0^{(2)}(k_0\rho_l^v)E(\mathbf{r}'_v)\exp(-jk_x ld)(\varepsilon_r(\mathbf{r}'_v)-1)dS'_{0v} \quad (1)$$

where S_{0v} is the surface cross sectional domain of the unit cell for the v-th region, ε_{rv} is the relative permittivity constant inside the v-th cylinder. $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind. $E(\mathbf{r}'_v)$ the the unknown total electric field of the v-th region and ρ_l^v is the distance between the observation point $\mathbf{r}$ and the source points $\mathbf{r}'_v$ of the reference region. The reference regions are divided into N number of cells, and the total electric field and relative permittivities are assumed to be constant on each cell. The integration is perform on ecah cell of the reference regions using Richmond approximation [7]. To obtain the matrix equations for Eq. (1) by Method of Methods (MoM), the pulse function as the basic expansion function is used with a point matching technique in each cell. Applying the linear relation on each layered array, we obtained a matrix form composed of N equations.

$$\sum_{n=1}^{N} C_{mn}E_n = E_m^i \qquad m = 1,\cdots,N \quad (2)$$

The evaluation of the matrix coeffcients constitutes the time consuming evaluation of the periodic Green's function but this overcomes by the use of lattice sum techniques [4] and Poisson's summation formulation [6]. E_n is the total electric fields of cell n position on the reference region of the composite material. E_m^i is the incidence field at cell m on the reference region. The total unknown E_n can be solved by any standard iterative method or by matrix inversion of the matrix coefficients. In this study, the GMRES solver is used as the iterative scheme. Once the total electric field is calculated, the scattering properties can be obtained from the layered structure.

3. Numerical Results

Numerical calculations are carried out for detailed examination of the scattering properties for various shaped multilayered periodic structures. The results are shown for the frequency range that satisfied the condition $d/\lambda_0 < 1$ at normal incidence and $\lambda_0 = 1.55\mu m$. For normal incident angle, only the fundamental Floquet mode $l = 0$ is propagating in this range.

The characteristics are plotted for the power reflection coefficient as a function of normalized wavelength for the fundamental space harmonic, $\varepsilon_{rb} = 1.0$ for all layers. A numerical example for six layered structure consisted of identical dielectric circular cylinders with radius $a = 0.3d$ and layers separation $h = 0.7d$ is shown in Fig. 2. The dotted line shows a good agreement between result from the present approach to that published in [5] for the relative permittivity $\varepsilon_r = 2.0$. The solid line shows the result for the structure with relative permittivity $\varepsilon_r = 2.5$. These results show several resonance properties indicating the multiple scattering process as a result of the coupling of propagating and evanescent space harmonics between layers.

Another example is a five layered structure consisted of rounded square cylindrical arrays obtained by apodizing a circular cylinder of radius $a = 0.3d$. The result shown in Fig. 3 is for arrays with relative permittivities $\varepsilon_{r1} = \varepsilon_{r3} = \varepsilon_{r5} = 2.0$ and $\varepsilon_{r2} = \varepsilon_{r4} = 2.25$. The modeling relation used to obtain this result is indicated by the inset for $b = 0.7a$ and $h = 0.7a$. The result also shows clear resonance peaks.

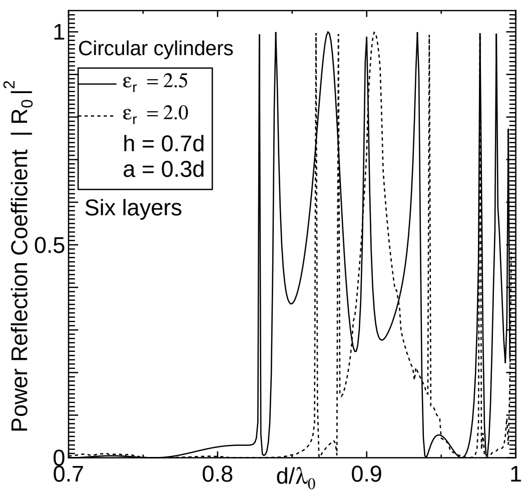

Figure 2: Six layered structure consisted of a circular cylinder with radius $a = 0.3d$ and layers seperation $h = 0.7d$.

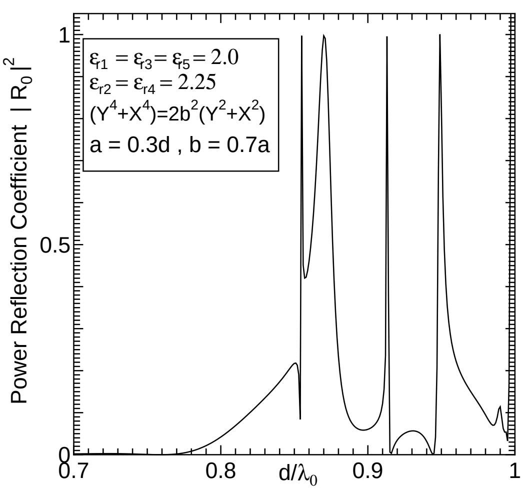

Figure 3: Five layered structure consisted of rounded square cylinders with radius $a = 0.3d$ and $h = 0.7d$.

3.1 Filter Applications

Electromagnetic bandgap filters application is discussed with various useful results showing high pass and stop band filtering characteristics. The numerical results for power reflection coefficient in decibel as a function of the normalized wavelength with design wavelength λ_0 assumed at $1.55\mu m$.

The first filter application is the formation of stop band filter and the filter characteristic for six layered structure is shown in Fig. 4. These arrays are consisted of elliptical cylinders with major axis $a = 0.3d$, minor axis $b = 0.7a$ and $h = 0.7d$. The relative permittivity constant $\varepsilon_r = 5.0$ is assumed to be the same in all layers. The stop bands are formed at two difference locations within the frequnecy range and complete resonance properties are observed between frequencies of stop bands.

The switching characteristic for six layered structure consisted of circular cylinder arrays with parameters $a = 0.15d$, $h = 0.5d$ is shown in Fig. 5. A very sharp upper-cutoff characteristics are obtained and the cutoff frequency is controllable by varying the periodicity d of the array. The side bands obtained are lower than -10[dB].

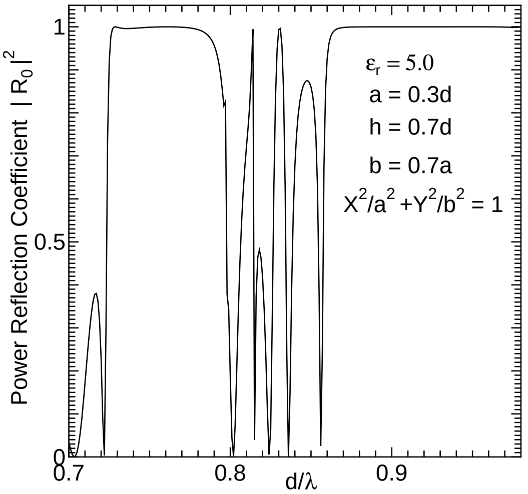

Figure 4: Filter characteristic for six layered structure consisted of an elliptical cylinder array with major axis $a = 0.3d$, minor axis $b = 0.7a$ and $h = 0.7d$.

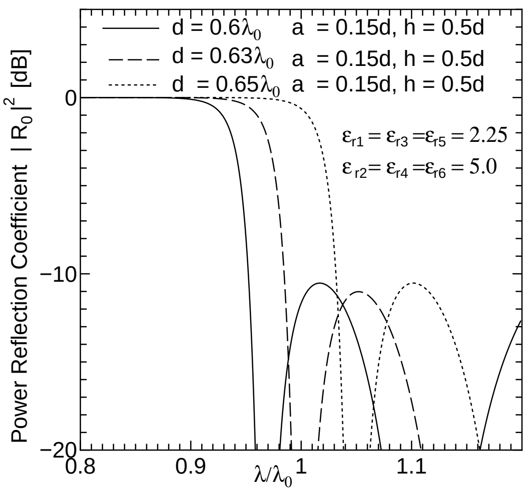

Figure 5: Switching characteristic for six layered structure consisted of a circular cylinder array with parameters: $a = 0.15d$, $h = 0.5d$.

4. Conclusion

Analysis for electromagnetic bandgap structures consisted of arbitrary shapes are studied. The problem is a two-dimensional scattering from periodic arrays of composite materials located in free space. The effects of the shape, material and structural arrangement of the scatterers are investigated. The periodic structure consisted of circular, elliptical and rounded square cylinders show that the dielectric material and structural arrangement of the parameters strongly affects both the width of stop bands formation and resonance properties of the structure. Practical applications for electromagnetic bandgap filter is designed for six layered structures with complete stop bands and switching characteristics within certain frequency range. Improvement in the efficiency of this method to analyze large number of layers is still ongoing subject to future consideration.

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