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Abstract—Recently, a new method was proposed by the present authors to construct a complex network based on a deterministic model from any given time series data. In this new method, a weighted directed network embodying time structure of the corresponding scalar time series is constructed. We apply the method to classify several models and investigate their network topology.

1. Introduction

Understanding the complex features of various dynamical systems in the real world continues to be a crucial challenge across physical, natural, and social sciences. For better understanding of such complicated interactions, it is useful to first transform the system into a new frame of reference. Recently, a new method was proposed to construct a complex network based on a deterministic model from a time series [1]. In this method, a weighted directed network embodying time structure of corresponding scalar time series is constructed. Walker et al. applied a simplified version of this method to collate the predictive properties of individual granular sensors in a series of biaxial compression tests and showed the effectiveness of the method [2].

In the analysis of a time dependent phenomenon produced by a complicated system, we usually start by simply observing the time series data of the phenomenon, expecting the data as a full manifestation of the nature of the phenomenon. Then we build a model from the time series. We often encounter the cases in which the models built from several time series have the same basis functions with slightly different parameters. In such cases, it is not easy to distinguish these model from one another. If such distinction could be possible, it would give us deeper understanding of the nature of these phenomena. We consider that the new network construction method proposed by the present authors can be applied to make this distinction, or the classification of time series [1].

In this paper we first review the method of constructing a complex network based on a deterministic model structure from a given time series, and then apply the method to four models. Based on the results, we investigate the relationship between their network topologies and the nature of the time series.

2. Methodology

The method described here is composed of two steps: (i) building a Reduced Auto-Regressive (RAR) model from a given time series and (ii) constructing a complex network from the RAR model.

2.1. Building an RAR model

Given a time series $\{x_t\}_{t=1}^{n}$ of n observations, an RAR model with the largest time delay w can be expressed by

$$x(t) = a_0 + a_1 x(t-l_1) + a_2 x(t-l_2) + \cdots + a_w x(t-l_w) + \varepsilon(t), \quad (1)$$

where a_i ($i = 0, 1, 2 \ldots, w$) are unknown parameters, and $\varepsilon(t)$ is assumed to be unknown independent and identically distributed random variables, which are interpreted as fitting errors. The parameters a_i are chosen to minimize the sum of squares of the fitting errors [3].

Among the various information criteria used to find the best (optimal) model, we employ the description length (DL) proposed by Rissanen [4]. The DL formula is

$$\mathrm{DL}(k) = n \ln \frac{\mathbf{e}^T \mathbf{e}}{n} + k \ln n, \quad (2)$$

where n is the number of data points, k is the model size and $\mathbf{e}$ is the fitting errors.

2.2. Constructing a network

After building an RAR model, we transform the model into a directed network (i) by representing each term $x(t)$ at time t by a node labelled by the time and (ii) by drawing an arrow directed from a node $x(t-i)$ to the node $x(t)$, where the time delay term $x(t-i)$ appears in the RAR model for the expression of $x(t)$. This arrow represents the influence of $x(t-i)$ on $x(t)$ with time delay i. We interpret the absolute value of the parameter a_i as the "influence" of $x(t-i)$ on $x(t)$ and transform the influence as a "distance" between nodes $x(t)$ and $x(t-i)$ using a_i on the network space; the larger the absolute value of a_i, the shorter the distance between $x(t)$ and $x(t-i)$. Although there may be several ways to introduce an appropriate "distance," we introduce the following simple distance based on elementary linear algebra [1].

Equation (1) can be interpreted as the scalar product of the coefficient vector

$$\vec{a} \equiv (a_1, a_2, \ldots, a_w)\,, \tag{3}$$

and the set of linearly independent "unit vectors", $(x(t-l_1), x(t-l_2), \ldots, x(t-l_w))$, where the constant parameter a_0 and $\varepsilon(t)$ are not used because these contain no time information. By this interpretation, we introduce the "angle" θ_i between the directions of $x(t)$ and $x(t-i)$ as

$$\theta_i \equiv \arccos\left(\frac{a_i}{\sqrt{a_1^2 + a_2^2 + \cdots + a_w^2}}\right). \tag{4}$$

The distance we introduce should have following properties . Firstly, when vectors $x(t)$ and $x(t-i)$ are in the same direction, the angle θ_i becomes 0 or π and the distance d_i should be 0. We expect the analyticity of the "distance" around $\theta = 0$ and put $d_i \approx \theta_i$ in this case. Secondly, when the vectors $x(t)$ and $x(t-i)$ are perpendicular ($a_i = 0$), the angle θ_i becomes $\pi/2$ and the distance d_i should be infinity. Thus d_i should be inversely proportional to $\cos\theta_i$. Finally, the distance must always be a positive real number. Hence, we define the distance d_i between the nodes $x(t)$ and $x(t-i)$ as

$$d_i \equiv |\tan\theta_i|\,. \tag{5}$$

According to Eq. (1), the nodes contained in a model are directly connected to $x(t)$. We refer the distance calculated from parameters in the model, Eq. (5), as the direct distance (*DD*). When we take into account time evolution of dynamical system, a pair of nodes can be connected indirectly via some other nodes and we can consider the sum of all distances through the path as the indirect distance (*ID*) between these two nodes. In a network we sometimes find a path with a shorter *ID* than the *DD*. In such a case, we can consider that the indirect path that gives the shortest length is the path on which the information is passed through most effectively and that these two nodes are essentially connected through the shortest *ID* path. We treat the collection of the paths that have the shortest length for any given node pairs, namely the minimum spanning tree, as the network of the system. The network constructed in this way reveals the underlying hierarchical structure of the linear model and enables us to know whether the influence of a term may come through other terms. For more details see [1].

[1]We understand that the "distance" introduced here depends on the nature of the system, and other distances (e.g. inverse) may be justified in some situations. However, we consider that this distance has broad range of applicability because it reflects the overall balance of the size of parameters in the model. We note that the proposed method is independent of the definition of the "distance"

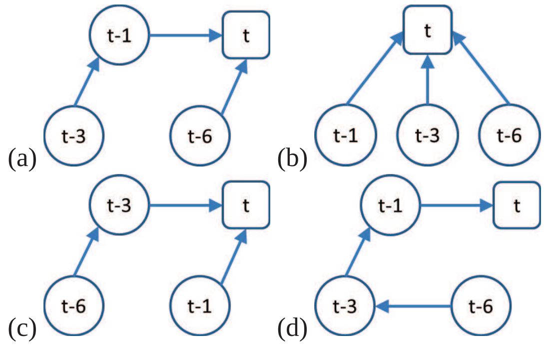

Figure 1: (Colour online) The optimal path network of four cases of the model, Eq. (6): (a) case (i) $a_1 = 0.722$, $a_3 = -0.391$, $a_6 = 0.223$, (b) case (ii) $a_1 = 0.713$, $a_3 = -0.394$, $a_6 = 0.219$, (c) case (iii) $a_1 = 0.711$, $a_3 = -0.395$, $a_6 = 0.213$, and (d) case (iv) $a_1 = 0.712$, $a_3 = -0.389$, $a_6 = 0.212$.

3. Classification of models

We demonstrate the application of our algorithm for the classification of models. We take the following RAR model,

$$x(t) = a_1\, x(t-1) + a_3\, x(t-3) + a_6\, x(t-6), \tag{6}$$

with the following four cases: (i) $a_1 = 0.722$, $a_3 = -0.391$, $a_6 = 0.223$, (ii) $a_1 = 0.713$, $a_3 = -0.394$, $a_6 = 0.219$, (iii) $a_1 = 0.711$, $a_3 = -0.395$, $a_6 = 0.213$, and (iv) $a_1 = 0.712$, $a_3 = -0.389$, $a_6 = 0.212$. These four cases are not identical and not extremely different. By applying our method, however, we can clearly see the network topologies built from these four models are quite different.

For the case (i) the coefficient vector is

$$\vec{a} = (0.722,\ -0.391,\ 0.223)\,, \tag{7}$$

and using the $\vec{a}$ we obtain the distance between the nodes, the direct distances (*DD*s)

$$\vec{d} = (0.623,\ 1.933,\ 3.682)\,. \tag{8}$$

The distance between nodes represents the magnitude of influence from the other nodes. According to the model, the nodes $x(t-1)$, $x(t-3)$ and $x(t-6)$ are directly connected with the node $x(t)$. However, there are cases where the distance of the directly connected path is not always the shortest. The optimal (shortest) path from $x(t-3)$ to $x(t)$ is the one via $x(t-1)$. The direct distance (*DD*) from $x(t-3)$ to $x(t)$ is 1.933, the indirect distance (*ID*) from $x(t-3)$ to $x(t)$ is the summation, $0.623 + 0.623 + 0.623 = 1.870$. The *ID* is shorter than the *DD*. Hence, we conclude that the most significant influence of the term $x(t-3)$ to $x(t)$ is not direct but comes through the term $x(t-1)$. On the other hand, the optimal (shortest) path from $x(t-6)$ to $x(t)$ is *DD*. Figure 1(a) shows the optimal path network of case (i) based on the result. Figure 1(a) also shows that the node $x(t)$ is directly connected with $x(t-1)$ and $x(t-6)$, and the connection from $x(t-3)$ is indirect via $x(t-1)$. This is an outcome of the global structure of interrelation and hierarchy between terms of the model, Eq. (6) with parameters of case (i), and the interplay between the sizes of the parameters and the network topology reveals the time structure. Other three cases, (ii), (iii), and (iv), are analysed in the same way. The direct distances of case (ii) is (0.632, 1.893, 3.720), the of case (iii) is (0.631, 1.879, 3.819), and that of case (iv) is (0.622, 1.910, 3.827). In Figure 1 (a)-(d), we show the network structures corresponding to the four models, from (i) to (iv). Note the distinctive topologies in these network structures. It should be emphasized again that we cannot extract this information by simply examining models with parameters of four cases.

4. Summary

We applied our new network construction algorithm with time structure based on the deterministic model structure from given time series as an useful model classification scheme. We showed that the networks constructed by our method from indistinguishable models have distinctive network topologies that can be easily classified.

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