Application of Liao's ABC in 2-D FDFD Algorithm

YU Tongbin, ZHANG lei, ZHAO yanhui

Institute of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China
njtyytb@sina.cn

***Abstract*-Liao's absorbing boundary condition (ABC) has been widely applied in the early finite difference time domain (FDTD) method, due to its advantages of simple form, good absorption effect and easy programming. In this paper, the frequency domain form of Liao's ABC is derived and successfully applied in 2-D frequency domain finite difference (FDFD) algorithm. The numerical experiments validate that Liao's ABC still shows excellent absorption effect in 2-D FDFD algorithm as it does in the 2-D FDTD algorithm.**

I. INTRODUCTION

Due to the limitation of the computer capacity, appropriate boundary condition should be assigned to accurately simulate the electromagnetic process of free space at the boundaries of the domain when solving the open region electromagnetic problems. To solve this problem, many ABC were proposed by scholars, such as Mur ABC [1], Liao's ABC [2], PML [3,4], UPML [5] and CPML [6], and so on. PML is so far the most effective method which is widely used in many numerical calculations of electromagnetic such as EM compatibility problems [7-9], EM scattering problems [10-12]. In order to obtain perfect absorption effect, much more boundary cells must be set for PML, which greatly increases the computational consuming, especially in FDFD calculations.

Liao's ABC was derived by extrapolating wave function in time and space domain based on the Newton's backward differential polynomial. It owns advantages of simple form, easy implementation and better absorption effect. It can be directly used near the corners of a computational domain without any modification and can also meet the accuracy requirements for most of numerical calculations of engineering. However, lack of stability has restricted its application, especially for the higher-order Liao's ABC. Thus W. C. Chew and other scholars analyzed the reason of the instability of Liao's ABC, and stabilized Liao's ABC by adding small damping factor [13-15]. Lei Zhang and Tong-Bin Yu improved the stability of Liao's ABC remarkably by order-weighting method [16].

FDFD is one of the earliest methods for numerical calculation of electromagnetic, whose theory is simple. But it needs to solve large matrix equation in calculating electromagnetic problem. Therefore, it is particularly important for FDFD algorithm to save the computing space. We usually apply the Mur ABC in FDFD calculations, though its absorbing effect is poor. In this paper, Liao's ABC is introduced into the FDFD algorithm, and improves the accuracy of FDFD by easy implementation.

II. THEORY

The FDTD method is well known for the solution of the wave equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi(x,y;t)=0 \qquad (1)$$

Where ϕ is scalar field, which is computed on a rectangular finite-difference grid with grid spacing Δs and time step Δt: $\phi(m\Delta s, n\Delta s; l\Delta t)=\phi^l_{m,n-1}$ .Using central differencing in time and space domain, the wave equation is approximated as:

$$\frac{\phi^l_{m+1,n}-2\phi^l_{m,n}+\phi^l_{m-1,n}}{\Delta s^2}+\frac{\phi^l_{m,n+1}-2\phi^l_{m,n}+\phi^l_{m,n-1}}{\Delta s^2}$$
$$-\frac{1}{c^2}\frac{\phi^{l+1}_{m,n}-2\phi^l_{m,n}+\phi^{l-1}_{m,n}}{\Delta t^2}=0 \qquad (2)$$

At the boundary of the finite computational domain, an ABC must be used to model radiation in a free space.

For a "right" boundary at $x=x_{max}$, Liao's ABC gives the updated boundary field $\phi(x_{\max}, y_j, t+\Delta t)$ in terms of field values at previous times lying along a straight line perpendicular to the boundary:

$$\phi(x_{\max}, y_j, t+\Delta t)=\sum_{i=1}^{N}(-1)^{i+1}C_i^N$$
$$\phi(x_{\max}-i\alpha c\Delta t, y_j, t-(i-1)\Delta t) \qquad (3)$$

Where C_i^N is the binomial coefficient $N!/[i!(N-i)!]$, N is the order of the boundary condition, i is the space step, $\alpha c\Delta t$ is the space sample interval. If we set $\alpha c\Delta t$ equal to Δs, the sample interval just coincides with the grid, so the form of Liao's ABC is very simple, the updated boundary field of Liao's ABC is given as:

$$\phi(x_{\max}, y_j, t+\Delta t) = \sum_{i=1}^{N} (-1)^{i+1} C_i^N \phi(x_{\max-i}, y_j, t-(i-1)\Delta t) \quad (4)$$

The Liao's ABC in FDTD algorithm can be seen as a discrete time system. This feature meets the demands of time discrete transform in signal and system course. Therefore it can be taken Z-transforming, and be transformed into frequency domain. We present the Liao's ABC in frequency domain as following:

$$\phi(x_{\max}, y_j) = \sum_{i=1}^{N} (-1)^{i+1} C_i^N \phi(x_{\max-i}, y_j) * \mathrm{e}^{(-ij\omega\Delta t)} \quad (5)$$

In the application of Liao's ABC in FDFD algorithm, it should be paid attention to the selection of time interval Δt, which is not as the one in FDTD algorithm, for it needn't take the stability into account. In the simulation, we can get better results when we set the time interval $\Delta t = \Delta s / c$. This problem will be mentioned again in the following analysis.

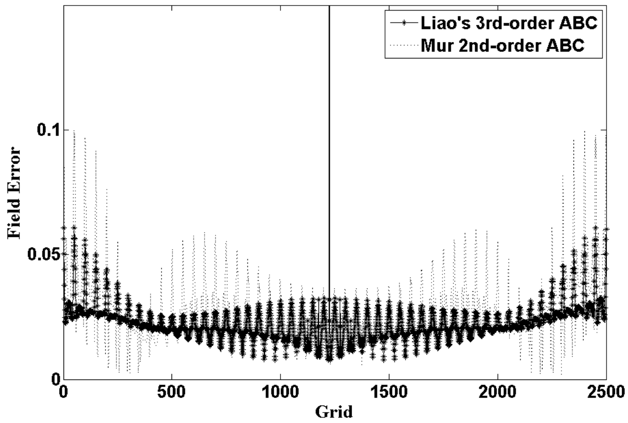
III. NUMERICAL VALIDATION

In order to validate the calculation accuracy of Liao's ABC in FDFD, we compare the field error computed by Liao's ABC with the one computed by Mur ABC in the cases of single frequency. The global error computed by Liao's ABC is compared with the one computed by Mur ABC in the band ranging from 0.1GHz to 30GHz, as it is done in [17]. In the numerical experiments, we take the TM wave as the example, and calculate the radiation field of infinite line source in free space. The size of the calculating region is (50*50) cells. The line source is set at the center of the calculating region (25*25). The time interval is set as dt = dx/c. The analytical solution of the radiation field of line source in free space is as following:

$$E_z(r,\omega) = \frac{\omega\mu}{4} I H_0^{(2)}(kr) \quad (6)$$

Where, $H_0^{(2)}(\cdot)$ is the second class zero-order Hankel function.

Figure.1 shows the relative field error of the numerical solution calculated by Liao's third-order ABC, Mur second-order ABC on contrast with the analytical solution. It can be seen that the maximum calculation error of Liao's third-order ABC at the boundaries is 6%, and the one of Mur second-order ABC is 10%. In the computing region, the maximum calculation error of Liao's third-order ABC is 3%, and the one of Mur ABC is 5%. Though the field calculated by Mur ABC is more accurate at local region, it is not as accurate as the ones by Liao's third-order ABC at most of the region, especially at the boundaries.



Figrue1. The radiation field error of the line source in free space calculated by Liao's third-order ABC and the Mur ABC (f=300MHz, δ=λ/20, δ is the grid size)

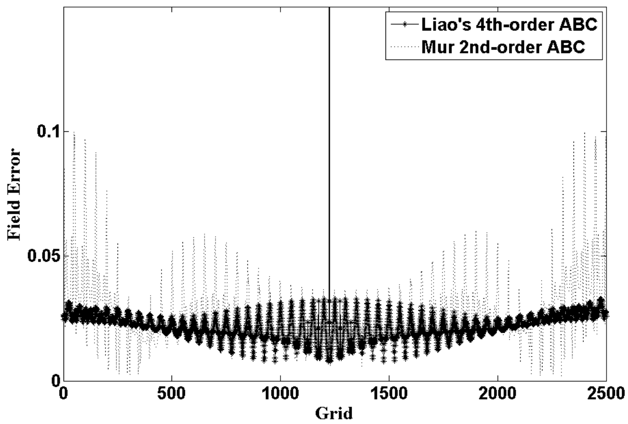


Figure.2 The radiation field error of the line source in free space calculated by Liao's fourth-order ABC and the Mur ABC (f=300MHz, δ=λ/20)

Figure 2 shows the relative field error of the numerical solution calculated by Liao's fourth-order ABC, Mur second-order ABC on contrast with the analytical solution. It can be seen that the maximum error of Liao's fourth-order ABC is 2.5% in the entire computational domain, which declines remarkably at the boundaries compared with the one of Liao's third-order ABC. The maximum error of Mur second-order ABC is 10% at the boundaries and 5% in the computed region. The field values error of Mur ABC is slightly better in local region, but is larger than Liao's fourth-order ABC in most of region.

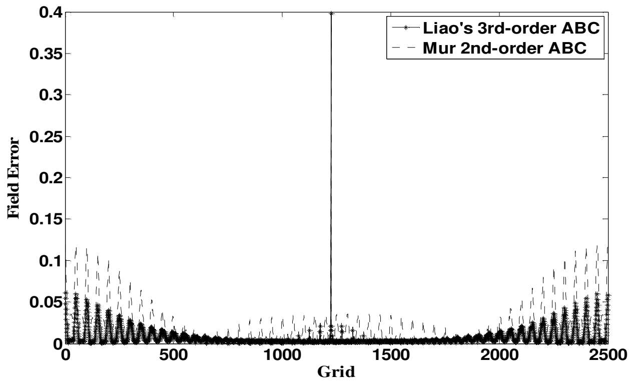
In order to analyze the impact of the grid size to the absorption effect of Liao's ABC and Mur ABC, we calculate the field error in small grid of λ/40.

Figure 3 shows the relative field error of the numerical solution calculated by Liao's third-order ABC, Mur second-order ABC on contrast with the analytical solution in small grid. It can be seen that the absorption effect of Liao's third-order ABC is very good, which is better than the one of Mur second-order ABC. The calculated error of Liao's ABC is no more than 6% in the entire computing region and is less than 1% at the central region. However, the calculated error of Mur ABC is more than 12% at the boundaries and 3% in the center region, which is poor compared with the ones calculated in large grid.

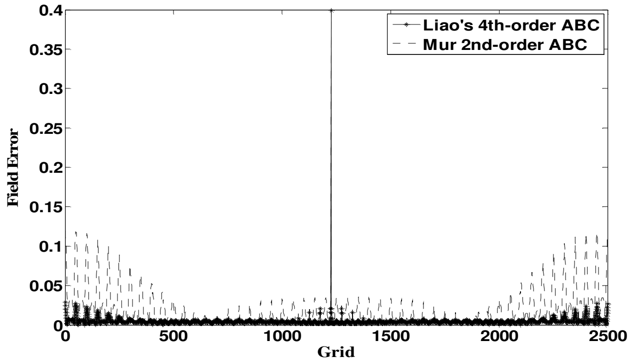
Figure 4 shows the relative error field of the numerical solution calculated by Liao's fourth-order ABC and Mur second-order ABC on contrast with the analytical solution in small grid. It can be seen that the calculation error of Liao's fourth-order ABC is very small, which has obvious advantage compared with Mur second-order ABC, The calculation error of Liao's ABC is no more than 3% in the entire calculating region and less than 1% at most of region, which can meet the accuracy requirements in majority of engineering calculations. However, the calculation error of Mur second-order ABC is more than 12% at the boundaries and 3% in the center region. Comparing figure1, figure2 with figure 3, figure 4, we can find out that the calculation accuracy of Liao's ABC becomes better. However, the one of Mur ABC becomes poor when calculating by using small grid.

What is given above is the case of single-frequency field values error, the following gives the global error calculated by Liao's ABC and Mur ABC in the band ranging from 0.1GHz to 30GHz, which is used to analyze the absorption effect of Liao's ABC.

Figure 5 shows the global error calculated by Liao's ABC and Mur second-order ABC in the band ranging from 0.1GHz-30GHz. The region size is (50*50) cells. It can be seen that the global error calculated by Liao's second, third, fourth-order ABC decrease by 4%, 8%, 12% comparing with the one calculated by Mur second-order ABC. The Liao's ABC shows good absorption effect.



Figrue3. The radiation field error of the line source in free space calculated by Liao's third-order ABC and the Mur ABC (f=300MHz, δ=λ/40)



Figrue4. The radiation field error of the line source in free space calculated by Liao's fourth-order ABC and the Mur ABC (f=300MHz, δ=λ/40)

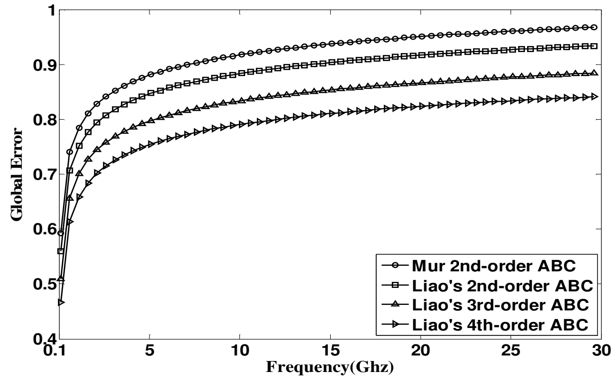


Figure 5 The global error of Liao's ABC and Mur ABC (f=300MHz,δ=λ/20)

Liao's second-order ABC and Liao's third-order ABC is smaller, but the Liao's fourth-order ABC shows better absorption effect. The global error calculated by Liao's second, third, fourth-order ABC in small grid decrease by 27%, 30%, 57% compared with the one calculated by Mur second-order ABC, which is better than the one calculated in large grid. So the advantage of Liao's ABC in absorption effect is more obvious compared with the one in large grid.

IV. CONCLUTION

In this paper, the form of Liao's ABC in frequency domain is deduced, and applied in the 2-D FDFD algorithm. Liao's ABC shows better absorption effect compared with Mur ABC in 2-D FDFD algorithm, without increasing the amount of storage and the complication of implementation..

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