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Multiple basins of consistency in a Mackey-Glass electronic circuit driven by colored noise

Hoipang Yip, Satoshi Sano, Atsushi Uchida, and Shigeru Yoshimori

†Department of Electronics and Computer Systems, Takushoku University
815-1 Tatemachi, Hachioji, Tokyo 193-0985, Japan
Email: k_sinbad@hotmail.com

Abstract– We experimentally observe consistency in a Mackey-Glass electronic circuit (MGC) repeatedly driven by a chaotic signal and a colored noise signal. We discovered that, multiple consistent response signal patterns are observed by driving the MGC with colored noise. To confirm our experimental observations, we use a numerical model for consistency of MGC. We found that consistency is obtained at large drive signals. More than one type of response waveform patterns are observed, which we call multiple basins of consistency.

1. Introduction

As for many nonlinear systems, consistent behavior is shown to a repeated input signal. Consistency indicates reproducibility of the output of a nonlinear system repeatedly driven by a certain signal. Consistency has been thought as a common concept seen in various interdisciplinary fields. An experimental observation of consistency has been reported in a solid-state microchip laser [1] so far. Some numerical simulations of consistency have been investigated [2, 3], however, no experimental observation of consistency has been reported in other nonlinear dynamical systems.

In this study, we experimentally observe consistency in a Mackey-Glass electronic circuit driven by a chaotic signal and a colored noise signal. We quantitatively investigate the degree of consistency when the amplitude of a drive signal is changed.

2. Mackey-Glass electronic circuit

Leukocyte productivity dynamics of human beings is as the origin of the Mackey-Glass model [4]. The model equation is shown as follows.

$$\frac{dx(t)}{dt} = \frac{acx(t-\tau)}{1+cx^{n}(t-\tau)} - bx(t) \qquad \text{...(1)}$$

This equation contains a delay of time τ and a nonlinear function, and generates a very complex chaos waveform with high dimensionality. This model is implemented in an electronic circuit [4]. The block diagram is shown in Fig. 1.

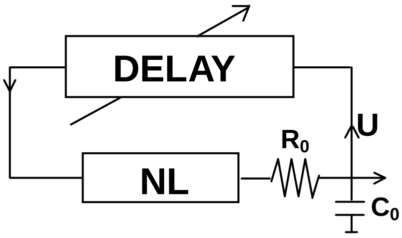

Fig.1 Block diagram of the Mackey-Glass electronic circuit

3. Experiment

3.1. Setup

We use a Mackey-Glass electronic circuit as a nonlinear dynamical system [5]. The circuit consists of two field effect transistors for the nonlinear part, three electronic amplifiers, a RC filter, and a delay line [4]. 50 pairs of capacitors (10 nF) and inductors (4.7 mH) are combined to generate time delay. The total delay time is 318 μs which corresponds to a half period of chaotic waveforms.

Figure 2 shows our experimental setup. We set the parameter values of the Mackey-Glass electronic circuit to generate chaos. A chaotic signal is detected by an oscilloscope and stored in a computer. The chaotic signal is reproduced by using an arbitrary function generator as a drive signal. The amplitude of the chaotic drive signal is adjusted by an amplifier and added with the feedback signal of the chaotic electronic circuit at an adder circuit. The mixed signal is sent to the nonlinear part of the electronic circuit. The voltage of the electronic circuit after the delay line is observed by a digital oscilloscope as a response waveform. The chaotic drive signal is repeatedly sent to the circuit and two response waveforms corresponding to the drive signal are detected and compared to observe consistency.

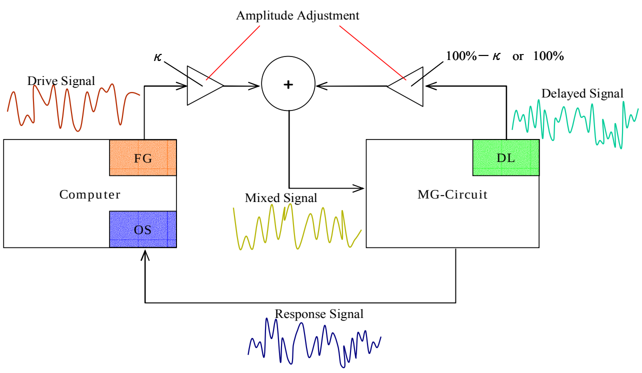

Fig.2 Experimental setup for consistency

3.2. Experiment with a chaotic drive signal

Figure 3 shows our experimental result of the temporal waveforms of a drive and two response signals and the correlation plot between the two responses. The two response waveforms are almost the same, i.e., the consistency exists. Linear cross correlation is obtained between the two response waveforms as shown in Fig. 3(b).

To investigate the parameter dependence, we change the ratio of the drive signal and the feedback signal (κ), and the presence of consistency is evaluated by using the cross correlation function of the two response signals. The result is shown in Fig. 4. Cross correlation is around 1 as the drive signal strength is increased. We found that consistency is obtained in MGC driven by a chaotic signal with large amplitude.

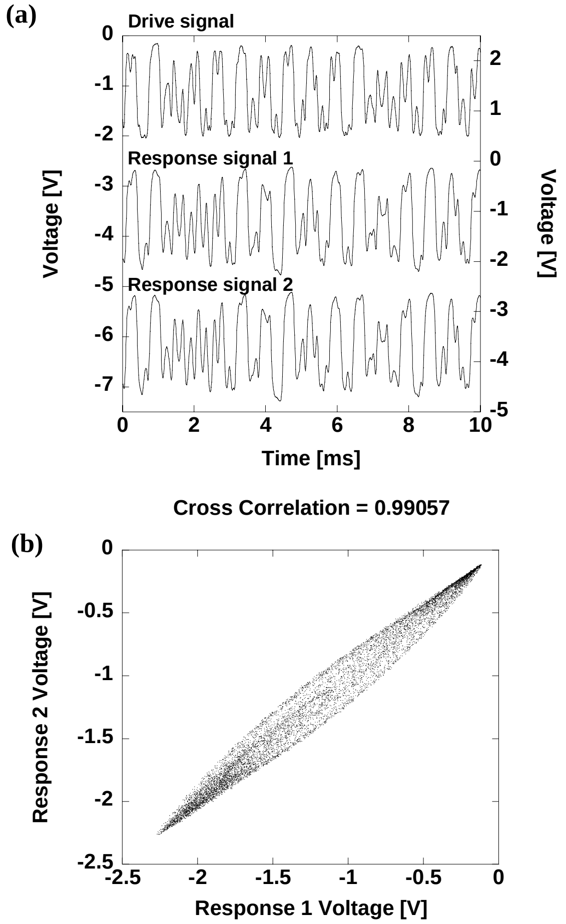

Fig.3 (a) Temporal waveforms and (b) cross correlation of MG circuit driven by chaos

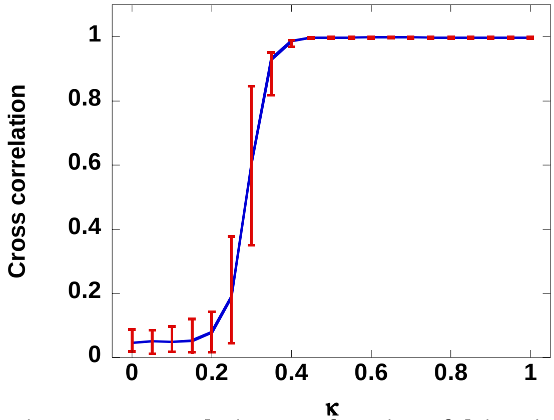

Fig.4 Cross correlation as a function of drive signal ratio with chaos drive signal.

3.3. Experiment with a colored-noise drive signal

Instead of a chaotic drive signal, we next use an exponentially correlated colored noise waveform generated using an Ornstein-Uhlenbeck process [6] with the inverse of a correlation time of 1.5 kHz as a drive signal. The colored noise signal is generated numerically and stored in a computer. The colored noise signal is reproduced by using a function generator. The amplitude of the colored noise signal is adjusted by an amplifier and added with the feedback signal of the chaotic electronic circuit at an adder circuit. The mixed signal is sent to the nonlinear part of the electronic circuit. The voltage of the electronic circuit after the delay line is observed by a digital oscilloscope. The drive signal is repeatedly sent to the circuit and several response waveforms corresponding to the drive signal are detected and compared to observe consistency.

Figure 5 shows the temporal waveforms of the drive and four corresponding response signals detected at different times. The temporal waveforms of Response 1 and 3 are almost identical, i.e., consistency is observed. Consistency is also observed between Response 2 and 4. However, Response 1 and 2 (also Response 3 and 4) are different waveforms. Figure 6 shows the cross correlation between two of the four response waveforms. It is found that linear cross correlation is obtained between Response 1 and 3, and between Response 2 and 4. However, the other combinations of the four response waveforms are not consistent. Therefore, it is found that there exist two types of consistent response waveforms.

To investigate the quantitative characteristics of consistency, we introduce a measure of consistency *C* as the average cross-correlation between responses for a set of response system initial conditions,

$$C = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1(j>i)}^{N} \left[\frac{< (x(t;x_i(0)) - \bar{x}_i)(x(t;x_j(0)) - \bar{x}_j) >}{\sigma_i \sigma_j} \right] \quad \text{... (2)}$$

where $x(t;X_i(0))$ and $x(t;X_j(0))$ are variables *x(t)* starting from different initial conditions of $X_i(0)$ and $X_j(0)$,

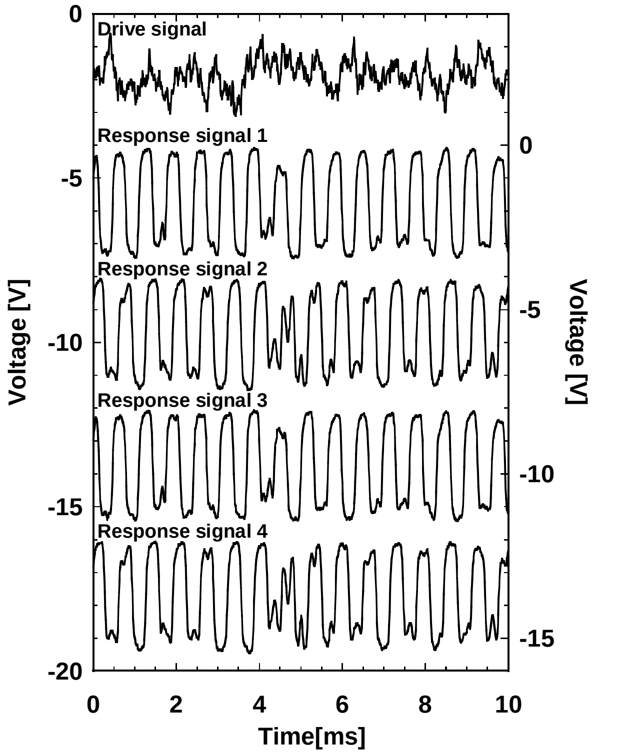

Fig.5 Temporal waveforms of MGC driven by a colored-noise signal with cut-off frequency of 1.5 kHz.

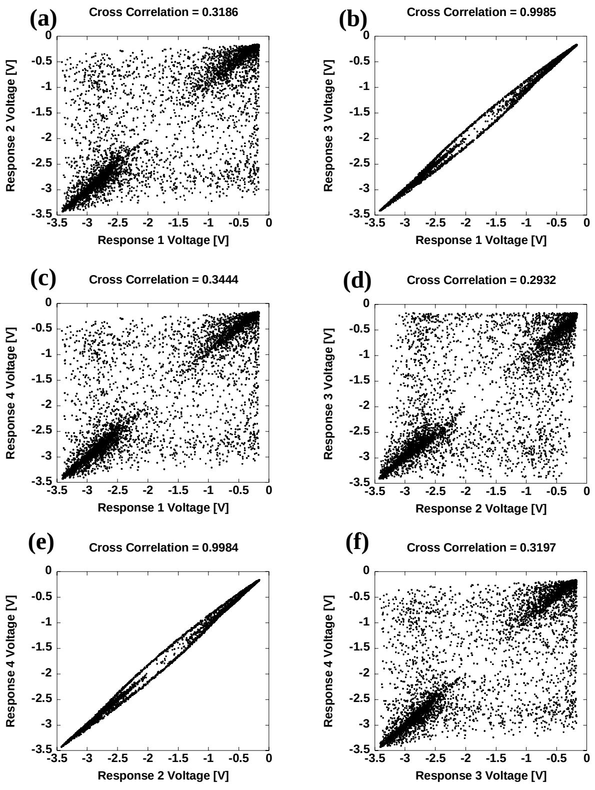

Fig.6 Correlation plots comparing
(a) Response 1 and 2 (b) Response 1 and 3, (c) Response 1 and 4,
(d) Response 2 and 3, (e) Response 2 and 4, (f) Response 3 and 4.

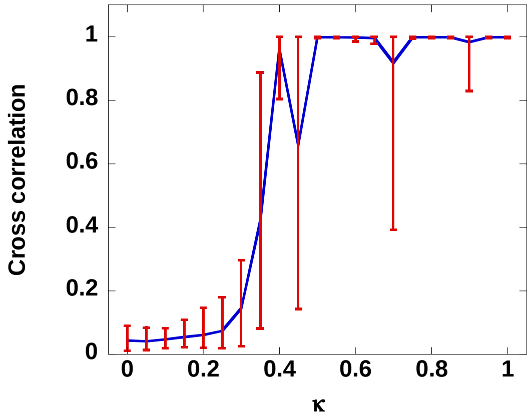

Fig.7 Cross correlation as a function of drive signal ratio with colored-noise drive signal.

$\bar{x}_i$ and $\bar{x}_j$ are the mean values of the two time series of $x(t; X_i(0))$ and $x(t; X_j(0))$, and σ_i and σ_j are the standard deviations of the two time series. The angle brackets denote time averaging.

Figure 7 shows the average consistency C for the four response waveforms as the drive signal strength κ is increased. The error bars indicate the maximum and minimum values of the cross correlation between two response waveforms. The average values of C approaches 1 as the drive signal strength is increased. The error bars also become small. However, there are some regions where the maximum value is 1 and large error bars exist. This is the region of multiple basins of consistency, because response waveforms are clustering and show large C for the corresponding pairs of waveforms and small C for different pairs. The region of multiple basins of consistency is widely observed at intermediate strengths of the drive signal. This result shows that there are multiple basins of consistency depending on the initial conditions. The concept of multiple basins of consistency is similar to the multi-stability where more than one attractor exists. We only found multiple basins of consistency in the Mackey-Glass electronic circuit driven by a colored noise signal, but not by a chaotic signal.

4. Numerical simulation

To confirm our experimental observations, we use a numerical model for consistency of Mackey-Glass electronic circuit. The equation of Mackey-Glass model is modified with a saturation effect because of the limitation of electronic amplifiers in the circuit.

We numerically calculated temporal waveforms and cross correlations. Figure 8 shows our numerical result of C as a function of the strength of the colored-noise drive signal. The regions of multiple basins of consistency are observed, as seen in Fig. 7. Therefore our numerical results agree well with our experimental observation.

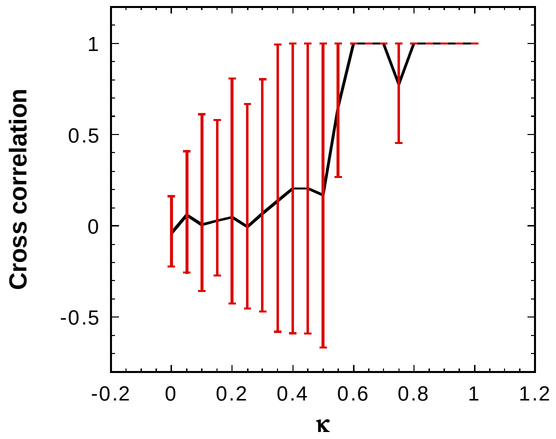

Fig.8 Cross correlation as a function of drive signal ratio obtained from numerical simulation

5. Conclusion

We experimentally and numerically observed multiple basins of consistency in the Mackey-Glass electronic circuit driven repeatedly by a chaotic signal and a colored-noise signal. We found that consistency is obtained at large drive signals. More than one type of response waveform patterns are observed, which is called multiple basins of consistency. The concept of multiple basins of consistency may be a general concept that can be observed in many nonlinear dynamical systems.

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