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Efficient Channel Estimation Scheme for Pulse-shaping OFDM Systems.

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Abstract—This paper proposes a low-complexity channel estimation sheme for the pulse-shaping OFDM systems in the highly mobile multipath environment. The estimation scheme is based on the tapped-delay-line model for the channel. The main idea behind our scheme is to approximate the fading process for each tap with finite terms of its Taylor series expansion. The proposed scheme also tracks the channel. We show that the channel can be estimated and tracked with reasonable mismatch error while using considerably short training period.

1. INTRODUCTION

It is well-known that, OFDM signal is robust against time-dispersion, caused by multipath propagation. That is, intersymbol interfence (ISI) is relatively low in the conventional OFDM signal. However, it is relatively sensitive to frequency dispersion caused by time-variation of the channel. Time-variations corrupt orthogonality of the subcarriers, thus intercarrier interference (ICI) occurs.

Recently, there has been increased interest in using OFDM systems in a mobile environment. They are considered to be promising candidates for the next generation mobile communications. The mobile environment is characterized by time-frequency dispersion. Signal distortion caused by the time-frequency dispersive channels depends crucially on the time-frequency localization of the pulse-shaping filter[1]. For instance, conventional OFDM systems that use rectangular-type pulse with guard interval can prevent ISI, but do not combat ICI. The design of time-frequency well-localized pulse-shaping OFDM filter is, therefore, an active area of research.

By introducing well-localized transmitter pulse-shapes, it is possible to reduce the ISI/ICIs [2]. The performance of such pulse-shaping OFDM systems in dispersive time-varying channels depend critically on the time-frequency localization of the transmitter and receiver filters. In this paper we consider Biorthogonal Frequency Division Multiplexing based on Offset QAM (BFDM/OQAM). BFDM systems relaxe orthogonality condition and uses linearly independent subcarriers. As a result they allows broader class of pulses, particularly the Gaussian transmitter pulse, which has the best time-frequency localization. Therefore, we can consider BFDM/OQAM as a generalization of the pulse-shaping OFDM.

The use of time-frequency well-localized transmitter pulses for BFDM/OQAM systems results in a decrease in ISI/ICI, and the ISI/ICI may be neglected in some practical cases [2]. However, in highly mobile environments ISI/ICIs increase and can no longer be neglected. A careful investigation of statistical properties of ISI/ICIs and proper equalization and estimation thereof are needed for further improvement of the systems [4].

Generally, the channel state information is needed for the construction of robust equalization. Previously proposed robust channel estimators neglect ISI/ICIs. In this paper, we consider ISI and ICI simultaneously. Large number of so-called channel parameters that explicitly related to ISI/ICI makes the channel estimation prohibitively complex [4]. In this paper we propose a estimation scheme that estimates implicit parameters using maximum likelihood criterion. Using this parameters we calculate the channel parameters and track them.

This paper is organized as follows. Section II introduces tapped-delay-line model for the typical mobile multipath wireless channel. Next, Section III briefly reviews the background for BFDM/OQAM system, analyze ISI/ICI and defines channel parameters. Then, Section IV presents the proposed estimation scheme. Finally, Section V discusses numerical study. Here we demostrate that that the channel can be estimated and tracked with reasonable mismatch error while using considerably short training period.

2. TAPPED-DELAY-LINE MODEL FOR THE MOBILE MULTIPATH CHANNEL

In this paper we consider mobile multipath channel. We make a standard assumption of wide-sense stationary uncorrelated scattering (WSSUS). Further we assume Rayleigh paths with Jakes' Doppler power spectrum. Maximum Doppler shift is f_d. If transmitted low-pass signal has bandwidth $W \gg f_d$, then we can consider low-pass frequency responce of the channel within the band $(-\frac{1}{2}W; \frac{1}{2}W)$. Then, it is easy to see that the time-varying impulse response $h(\tau; t)$ of the channel can be modelled as the tapped-delay-line [6]:

$$h(\tau; t) = \sum_{m=1}^{M} h_m \xi_m(t) \delta\left(\tau - \frac{m}{W}\right) \tag{1}$$

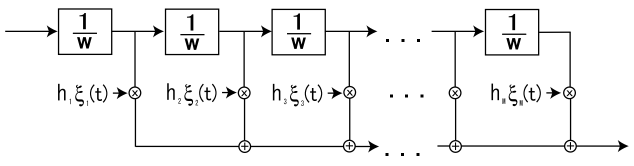

Figure 1: Tapped-delay line model of the channel

where h_m are complex valued tap weights, $\xi_m(t)$ are normalized low-pass fading processes, $\delta(\tau)$ is the Dirac delta-function. Using the assuption of the Jakes' model $\xi_m(t)$ are expressed as [5]:

$$\xi_m(t) = \sqrt{\frac{2}{L_m}} \sum_{l_m=1)}^{L_m} \exp\left(j2\pi f_d t \cos\alpha_{l_m} + \phi_{l_m}\right) \qquad (2)$$

where, α_{l_m} and ϕ_{l_m} are arrival angle and phase shift of the l_m-th path. These random variables are independent and uniformly distributed in $(-\pi;\pi)$. Autocorrelation of the fading process in (2) is shown to be [5]:

$$R_{\xi_m\xi_m}(\Delta t) = \mathbf{E}\{\xi_m(t+\Delta t)\xi_m(t)\} = 2J_0(2\pi f_d \Delta t) \qquad (3)$$

where, $\mathbf{E}(\cdot)$ stands for mathematical expectation, $J_0(\cdot)$ is 0-order Bessel function of the first kind. The tapped-delay-line model of the mobile multipath channel is shown in Figure 1.

3. BFDM/OQAM SYSTEM, ISI/ICI AND CHANNEL PARAMETERS

Here, we review some background for BFDM/OQAM, analyze the ISI/ICI for the system that occur in the channel described in Section II, and derive second-order statistics for the channel parameters.

3.1. Biorthogonal Basis

Let us define a set **G** that consists of pairs of the translations and the modulations of a real-valued transmitter pulse $g(t)$:

$$\mathbf{G} = \begin{cases} g^{\mathcal{R}}_{k,l}(t) = g(t-lT)e^{j2\pi Fkt} \\ g^{\mathcal{I}}_{k,l}(t) = g(t-lT+T/2)e^{j2\pi Fkt} \end{cases} \quad tj$$

where, T is the symbol period, F is subcarrier separation, $N \in \mathbb{N}$ is number of subcarriers and $k,l \in \mathbb{Z}$. From the Gabor theory, **G** may form a Riesz basis for a separable Hilbert space only if $TF \geq 1$. For data to be transmitted and received perfectly, in the absence of a channel, elements of **G** should be linearly independent (not necessarily be orthogonal). If the elements are orthogonal, the system is called OFDM/OQAM. For any Riesz basis **G**, there exists unique dual Riesz basis **W**, such that **G** and **W** are biorthognal [3]. **W** also consists of the pairs of translations

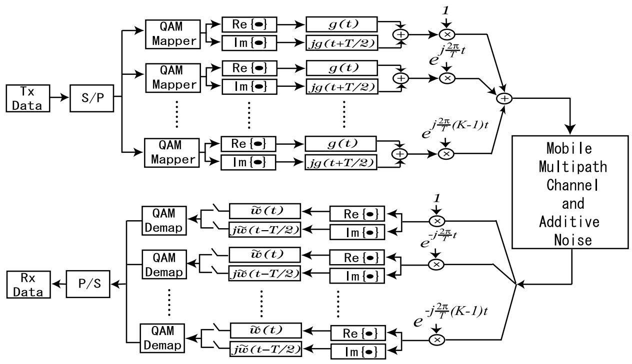

Figure 2: BFDM/OQAM system, $\tilde{w}(t) = w(-t)$

and modulations of some real-valued pulse $w(t)$:

$$\mathbf{W} = \begin{cases} w^{\mathcal{R}}_{k,l}(t) = w(t-lT)e^{-j2\pi Fkt} \\ w^{\mathcal{I}}_{k,l}(t) = w(t-lT+T/2)e^{-j2\pi Fkt} \end{cases}$$

where $k,l \in \mathbb{Z}$.

G and **W** are dual basis iff the following biorthogonality conditions are satisfied:

$$\Re\{< g^{\mathcal{R}}_{k,l}(t), w^{\mathcal{R}}_{k',l'}(t) >\} = \delta_{k,k'}\delta_{l,l'} \qquad (4)$$

$$\Im\{< g^{\mathcal{R}}_{k,l}(t), w^{\mathcal{I}}_{k',l'}(t) >\} = 0 \qquad (5)$$

$$\Re\{< g^{\mathcal{I}}_{k,l}(t), w^{\mathcal{R}}_{k',l'}(t) >\} = 0 \qquad (6)$$

$$\Im\{< g^{\mathcal{I}}_{k,l}(t), w^{\mathcal{I}}_{k',l'}(t) >\} = \delta_{k,k'}\delta_{l,l'} \qquad (7)$$

where, $< \cdot >$ denotes $\mathbf{L}^2$ space inner product, $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote real and imaginary part, respectively, $\delta_{k,k'}$ denotes the Kronecker delta, and $k,l,k',l' \in \mathbb{Z}$. For more thorough discussion on this topic refer to [3].

3.2. BFDM/OQAM Systems, ISI/ICI, and the channel parameters

The baseband BFDM/OQAM signal can be expressed as:

$$s(t) = \sum_{k=0}^{K-1} \sum_{l=-\infty}^{\infty} \left\{ c^{\mathcal{R}}_{k,l} g^{\mathcal{R}}_{k,l}(t) + j c^{\mathcal{I}}_{k,l} g^{\mathcal{I}}_{k,l}(t) \right\} \qquad (8)$$

where, K is subcarrier number, and $c^{\mathcal{R}}_{k,l}$,$c^{\mathcal{I}}_{k,l}$ are real and imaginary parts of the transmitted symbols $c_{k,l}$, respectively. Transmitted signal propagates through mobile multipath channel described in Section 2. Received noisy signal may be written as:

$$r(t) = \sqrt{E_s} \sum_{m}^{M} h_m \xi_m(t) s(t - \frac{m}{W}) + n(t) \qquad (9)$$

where E_s is signal energy per channel use, and $n(t)$ is additive white Gaussian noise (AWGN) within signal bandwidth with the variance $N_0/2$ per complex dimension.

Demodulation performed at the receiver can be expressed as:

$$d^{\mathcal{R}}_{k',l'} = \Re\{< r(t), w^{\mathcal{R}}_{k',l'}(t) >\} = \quad (10)$$

$$= \int_{-\infty}^{\infty} \Re\left\{r(t)e^{-j2\pi Fk't}\right\} w(t - l'T)dt$$

$$d^{\mathcal{I}}_{k',l'} = \Im\{< r(t), w^{\mathcal{I}}_{k',l'}(t) >\} = \quad (11)$$

$$= \int_{-\infty}^{\infty} \Im\left\{r(t)e^{-j2\pi Fk't}\right\} w(t + \frac{T}{2} - l'T)dt$$

where, $d^{\mathcal{R}}_{k',l'}$,$d^{\mathcal{I}}_{k',l'}$ are the real and imaginary parts of received symbols $d_{k',l'}$, respectively. Block diagram of the BFDM/OQAM system is shown in Figure 2. $\tilde{w}(t) = w(-t)$ can be seen as a matched filter to transmitter pulse $g(t)$.

Passing through the mobile multipath channel, the transmitted signal loses its biorthogonality, causing ISI/ICI in the received signal. Within the *coherence time* of the channel, received symbols in (10) and (11) can be approximated as:

$$d^{\mathcal{R}}_{k,n} = \sum_{i,j} \left\{h^{k,\mathcal{R}}_{i,j,\mathcal{R}} c^{\mathcal{R}}_{k\ominus i,n-j} + h^{k,\mathcal{R}}_{i,j,\mathcal{I}} c^{\mathcal{I}}_{k\ominus i,n-j}\right\} + n^{\mathcal{R}}_{k,n} \quad (12)$$

$$d^{\mathcal{I}}_{k,n} = \sum_{i,j} \left\{h^{k,\mathcal{I}}_{i,j,\mathcal{R}} c^{\mathcal{R}}_{k\ominus i,n-j} + h^{k,\mathcal{I}}_{i,j,\mathcal{I}} c^{\mathcal{I}}_{k\ominus i,n-j}\right\} + n^{\mathcal{I}}_{k,n} \quad (13)$$

where, $\ominus$ is a modulo-K subtraction, $n^{\mathcal{R}}_{k,n}$ and $n^{\mathcal{I}}_{k,n}$ are noise components. $h^{k,\mathcal{R}}_{i,j,\mathcal{R}}$, $h^{k,\mathcal{R}}_{i,j,\mathcal{I}}$, $h^{k,\mathcal{I}}_{i,j,\mathcal{R}}$ and $h^{k,\mathcal{I}}_{i,j,\mathcal{I}}$ are equivalent channel parameters or simply, *channel parameters*. The equivalent channel includes transmitter and receiver filters, as well as the physical medium, i.e. the time-frequency dispersive channel. Except the case $k \neq i$ they present ISI/ICI caused by the channel. Note that even when $k = i$, they show an interference between real and imaginary parts of the same symbol. Channel parameters verify:

$$h^{k,\mathcal{R}}_{i,j,\mathcal{R}} = \sqrt{E_s} \int_{-\infty}^{\infty} \Re\left\{\sum_{m}^{M} h_m g(t - jT - \frac{m}{FK}) \cdot \xi_m(t) e^{j2\pi Fi(t-\tau_m)} e^{-j2\pi Fkt}\right\} w(t)dt \quad (14)$$

$h^{k',l',\mathcal{I}}_{k,l,\mathcal{R}}$, $h^{k',l',\mathcal{R}}_{k,l,\mathcal{I}}$ and $h^{k',l',\mathcal{I}}_{k,l,\mathcal{I}}$ can be expressed in the same way.

4. PROPOSED CHANNEL ESTIMATOR

Due to a large number of the channel parameters involved in (12) and (13) the direct estimation is prohibitively complex [4]. Therefore we estimate h_m and $\xi_m(t)$ and calculate the channel parameters using (14). The power spectrum of fading process $\xi_m(t)$ is in the band $(-f_d; f_d)$. We observe here that $f_d = f_c\frac{v}{c}700Hz$ at the vehicle speed $v = 300km/s$ and carrierfrequency $f_c = 2.5GHz$. It turns out to be rather slow process comparing with the transmitted signal ($T \approx 3.2$). Thus, we approximate $\xi_m(t)$ with first two terms of its Taylor series expansion within in the neighbourhood of the estimation time instant. That is $\xi_m(t) \approx \xi_m(0) + \xi'_m(0)t$. At each estimation step we estimate a new parameter $\beta_m = h_m\xi_m(0)$ and track $\xi_m(t)$ with linear interpolation between the estimates.

4.1. Maximum-likelihood Estimation

As a training waveform for the estimation we employ single-carrier waveform with bandwidth $W = FK$. This makes the training period considerably shorter and simplifies computational complexity. We assume that a pulse carrying the training sequence is a sinc pulse, i.e. $g(t) = \sin(\pi t/T)/(\pi t/T)$. Then it is to check that the data sequence at the output of the matched filter would expressed as:

$$r_n = \sum_{m=1}^{M} \beta_m s_{n-m} + n_n \quad (15)$$

where s_{n-m} are the training sequence, β_m are parameters to be estimated, n_n are AWGN components. If we observe M values of r_n then we have $\vec{r} = \mathbf{S}\vec{\beta}$, where $\vec{r} = [r_1, ..., r_M]^T$, $\vec{\beta} = [\beta_1, ..., \beta_M]^T$, and $\mathbf{S}$ is a Toeplitz matrix that consisted of s_{n-m}. We use maximum-likelihood method to estimate $\vec{\beta}$. The estimates can be written as:

$$\max_{\vec{\beta}} \arg P(\vec{r}, \vec{\beta})P(\vec{\beta}) \quad (16)$$

where, $P()$ stands for probability distribution function. We observe that βm are independent Gaussian random variables with the variance ($\sigma = 2J_0(0)\sigma_{h_m} = \sigma_{h_m}$), where σ_{h_m} is the variance of h_m tap. Then (16) can be rewritten as:

$$P(\vec{\beta}) = \frac{1}{\sqrt{2\pi}^M} \exp\left(-\frac{1}{2}\vec{\beta}^T diag(\sigma_{h_1}, ..., \sigma_{h_M})\vec{\beta}\right)$$

$$P(\vec{r}, \vec{\beta}) = \frac{1}{\sqrt{\pi N_0}^M} \exp\left(-\frac{1}{N_0} \mid \vec{r} - \mathbf{S}\vec{\beta} \mid^2\right) \quad (17)$$

where, N_0 is variance of the noise component n_n, From (17) it is easy to see that log-likelihood cost function would be simply a M-dimensional elliptic function. The detail solution will be omitted. Once $\vec{\beta}$ is estimated the channel parameters are calculated by (14). $\vec{\beta}$ is linearly tracked between estimation steps.

4.2. Mismatch Analysis

As measure to evaluated the mismatch of the proposed estimator we choose a normalized total mean square error(MSE). The normalized MSE at subcarrier k will be defined as:

$$MSE(k) = \sum_{i,j} \mathbf{E}\{(1 - \frac{\hat{h}^{k,\mathcal{R}}_{i,j,\mathcal{R}}}{h^{k,\mathcal{R}}_{i,j,\mathcal{R}}})^2 + (1 - \frac{\hat{h}^{k,\mathcal{I}}_{i,j,\mathcal{R}}}{h^{k,\mathcal{I}}_{i,j,\mathcal{R}}})^2 + (1 - \frac{\hat{h}^{k,\mathcal{R}}_{i,j,\mathcal{I}}}{h^{k,\mathcal{R}}_{i,j,\mathcal{I}}})^2 + (1 - \frac{\hat{h}^{k,\mathcal{I}}_{i,j,\mathcal{I}}}{h^{k,\mathcal{I}}_{i,j,\mathcal{I}}})^2\} \quad (18)$$

where, $\hat{h}^{k,\mathcal{I}}_{i,j,\mathcal{R}}$, $\hat{h}^{k,\mathcal{R}}_{i,j,\mathcal{R}}$, $\hat{h}^{k,\mathcal{R}}_{i,j,\mathcal{I}}$, and $\hat{h}^{k,\mathcal{I}}_{i,j,\mathcal{I}}$ are the estimates of the channel parameters.

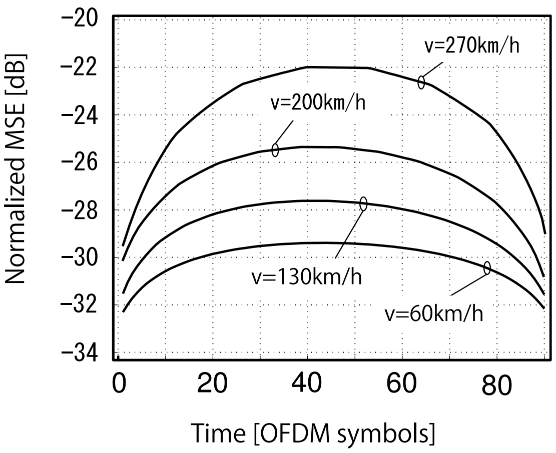

Figure 3: The normalized MSE of channel estimators versus time, at SNR=30dB. The channel is estimated at timeslot 1 and 90 and linearly interpolated in between

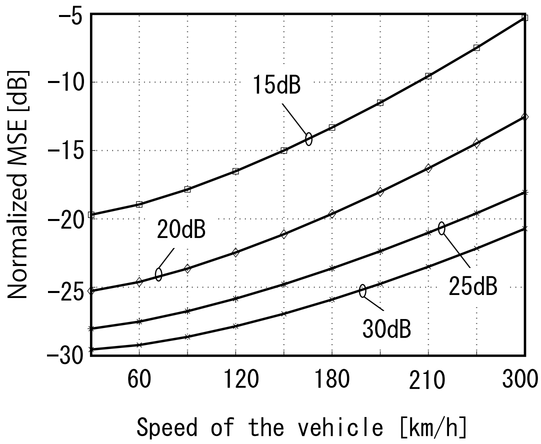

Figure 4: The normalized MSE of channel estimators at timeslot 45 versus speed, at different values of the additive noise

5. Numerical Simulation

The normalized MSE performance of the proposed estimator is evaluated by computer simulation. We set $k = 30$ in (18). The simulation BFDM/OQAM system is constructed at maximal spectral efficiency, i.e. $TF = 1$. Digital mapping is 16QAM. No guard interval is used and symbol period is $T = 3.2\mu s$. The dispersive time-varying channel is the Rayleigh fading channel with Jakes Doppler Spectrum as described in Section II. Maximum Doppler shift of the channel is $f_d \in [0Hz, 700Hz]$, which corresponds to vehicle speed of $v \in [0km/h, 300km/h]$, at carrier frequency $f_c = 2.5GHz$. The channel has 200 paths and exponentially decaying multipath intensity profile. Root-mean-square delay is $\tau_0 = 0.5\mu s$. 500 simulation runs are averaged to plot the total MSE.

Fig. 3. shows the normalized MSE versus SNR, at different values of the speed of vehicle. Here, signal to noise ration (SNR) is $30dB$.The channel is estimated at timeslot 1 and 90 and linearly interpolated in between. We can observe that the normalized MSE is symmetrical yielding maximum at center, namely at timeslot 45.

Fig.4 shows the normalized MSE versus the speed, at different values of the noise.

6. Conclusion

In this paper we have proposed an efficient estimators for pulse-shaping OFDM systems in the dispersive time-varying channels. The estimator based on the tapped-delay-line model of the channel. Simulation results shows that the estimator can track the channel with reasonable mismatch error, e.g. 20dB at speed 180km/h and SNR=20dB, while using considerable short training period, namely 5 timeslots.

References

[1] T.Strohmer, and S.Beaver, "Optimal OFDM design for time-frequency dispersive channels," IEEE Trans. Commun., vol. 51, no. 7, pp. 1111–1122, Jul. 2003.

[2] G.D.Schafhuber, and F.Hlawatsch, "Pulse-shaping OFDM/BFDM systems for time-varying channels: ISI/ICI analysis, optimal pulse design, and efficient implementation," Proc. PIMRC, vol. 3, pp. 1012–1016, Sept. 2002.

[3] H.G.Feichtinger, and T.Strohmer, *Gabor Analysis and Algorithms, Theory and Applications*, Birkhaüser, 1998.

[4] B.Mongol, T.Yamazato, H. Okada, and M.Katayama, "MIMO Zero-Forcing Equalizer for BFDM/OQAM Systems in Time-Frequency Dispersive Channels", IEICE Trans. Fund. vol E89A no 11, pp. 3114-3122, Nov. 2006.

[5] W.C.Jakes, *Microwave Mobile Communications*, IEEE Press, 1994.

[6] J.G.Proakis, *Digital Communications*, McGraw-Hill, Fourth Ed.,.

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