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Reduction of Computational Complexity Using Interference Cancellation in Maximum Likelihood Receiver for SDM Systems

Takahiko Saba† and Jun Konishi†

† Dept. of Computer Science, Chiba Institute of Technology
2-17-1 Tsudanuma, Narashino, Chiba 275-0016 JAPAN
E-mail: saba@cs.it-chiba.ac.jp

Abstract—The maximum likelihood detection (MLD) algorithm is known to be the mathematically optimal approach for detecting signals of space division multiplexing (SDM) systems. However, the computational complexity of MLD increases exponentially according to the number of transmit antennas. This paper proposes the computational complexity reduction scheme exploiting the property of hybrid automatic repeat request (HARQ) for MLD-based receivers in SDM systems. In HARQ scheme, the incorrectly received packet are stored at the receiver. We propose to make use of this stored packet in signal detection. By subtracting the replica signal which is made from the stored packet from the received superposed signal, we can reduce the order of MLD calculation. Simulation results can show the proposed scheme greatly reduces the computational complexity and increase the throughput performance in the SNR region where the retransmissions frequently occur.

1. Introduction

A space-division multiplexing (SDM) system can increase its maximum throughput in proportion to the the number of transmit antennas. The maximum likelihood detection (MLD) algorithm is known to be the mathematically optimal approach for detecting SDM signals [1]. However, the computational complexity of the signal separation using MLD increases exponentially according to the modulation levels and the number of transmit antennas, especially in SDM systems using multi-carrier transmission such as orthogonal frequency division multiplexing (OFDM). Thus, the reduction of the signal separation complexity is an important subject in SDM systems.

Some methods have been proposed to reduce the computational complexity of MLD without reducing the detection performance [2],[3]. In [2], an approach that applies QR decomposition associated with the M–algorithm to MLD is proposed. Furthermore, the combination of MLD and space-filtering technique is proposed in [3]. In these schemes, however, there is a restriction that the number of receive antennas must be greater than that of transmit antennas because of the natures of QR decomposition and space-filtering. Therefore, we consider another way to reduce the computational complexity of MLD in this paper.

In general, hybrid automatic repeat request (HARQ) is essential for wireless packet communication to achieve reliable data transmission. In this paper, we propose to make use of this stored packet at signal separation to reduce the computational complexity of MLD-based receivers in SDM systems. By subtracting the replica signal which is made from the stored packet from the received superposed signal, we can virtually decrease the number of transmit antennas.

Our proposed scheme does not modify the structure of MLD itself. Therefore, even the formerly proposed complexity reduction schemes [2], [3] can be applied to our proposed scheme when the number of receive antennas is greater than that of transmit antennas. Simulation results can exhibit the proposed scheme greatly reduces the computational complexity of the system and increase the throughput performance in low carrier-to-noise power ratio (CNR) region where the retransmissions frequently occur.

2. System Model

We explain the SDM system considered in this paper. The system consists of N_t transmit (Tx) and N_r receive (Rx) antennas. Furthermore, we assume the system employs the basic Type-I HARQ. In practice, the incorrectly received packets are often stored at the receiver rather than discarded, and when the retransmitted packet is received, the information from both packets are combined (which is know as Chase combining) before being fed to the decoder of the error-correction code, which can increase the probability of successful decoding. However, MLD generally outputs the hard-decision symbols against each Tx antenna. Thus, Chase combining cannot directly be applied to the system [4]. The received superposed signal can be given by

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{v}, \tag{1}$$

where $\boldsymbol{H}$ is an N_r-by-N_t channel matrix, and $\boldsymbol{s}$ and $\boldsymbol{v}$ are N_r-by-1 vectors of transmit symbols and received noise, respectively.

The MLD algorithm evaluates Euclidean distance between the received symbol vector $\boldsymbol{r}$ and replicated symbol vector $\hat{\boldsymbol{H}}\boldsymbol{s}$. Here, $\boldsymbol{s}$ represents all the possible candidates of the transmitted symbols. The estimate vector $\hat{\boldsymbol{s}}$ is derived as the vector that minimizes this distance. $\hat{\boldsymbol{s}}$ is given by

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s}} ||\boldsymbol{r} - \hat{\boldsymbol{H}}\boldsymbol{s}||^2, \tag{2}$$

where $\hat{\boldsymbol{H}}$ is the estimated $\boldsymbol{H}$, and $||\cdot||^2$ denotes Euclidean norm.

In this paper, we consider the numbers of both complex additions and complex multiplications as measures of the computational complexity. The numbers of additions and multiplications of Euclidean norm in Eq. (2) can be given by

$$N_{add} = N_r N_t + N_r - 1, \tag{3}$$

$$N_{mul} = N_r N_t + N_r, \tag{4}$$

In SDM systems, this calculation is carried out against all the possible symbol candidates. The number of possible signals $\boldsymbol{s}$, that is, the number of metric computations is given by

$$N = M^{N_t}, \tag{5}$$

where M is the number of constellation points. Thus, the numbers of both complex additions and complex multiplications of MLD can be given by

$$N_{add} = M^{N_t}(N_r N_t + N_r - 1), \tag{6}$$

$$N_{mul} = M^{N_t}(N_r N_t + N_r). \tag{7}$$

Since MLD algorithm requires exhaustive search through all the possible candidates $\boldsymbol{s}$, its computational complexity grows exponentially according to M and N_t.

3. Proposed Receiver

Fig. 1 shows a structure of the proposed receiver. In the system, transmissions can be classified into three cases: (1) all the N_t transmitted packets are new; (2) $(N_t - 1)$ or N_t packets are retransmitted; (3) n $(N_t - 1 > n)$ packets are retransmitted. When all the packets in transmission are new (Case (1)), the receiver separates the N_t packets using MLD algorithm. The separated packets are demodulated, and corrected errors through FEC decoding. The receiver detects decoding errors using cyclic redundancy check (CRC). When errors are detected in the packets, the retransmission of the packets is requested. The decoded erroneous packets are stored in a buffer. In Case (1), the computational complexity is the same as that of the conventional MLD receiver.

Next, we consider Case (2) where $(N_t - 1)$ or N_t packets are retransmitted. In this case, the receiver can make $(N_t - 1)$ replica signals from the stored packets at each Rx antenna. By subtracting the $(N_t - 1)$ replicas from the received signal at each antenna, the received symbol vector

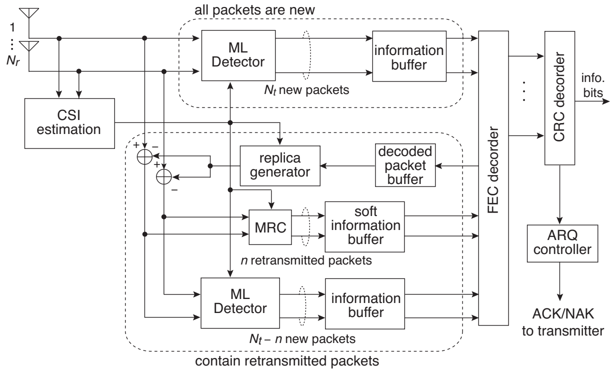

Figure 1: Proposed receiver.

from the jth Tx antenna can be separated from the received signal without using MLD. This can be expressed as

$$\boldsymbol{r}_j = \boldsymbol{r} - \hat{\boldsymbol{H}}^{-j}\tilde{\boldsymbol{s}}^{-j}, \tag{8}$$

where j is an integer between 1 and N_t, $\hat{\boldsymbol{H}}^{-j}$ is the reduced $\hat{\boldsymbol{H}}$ by removing jth column, $\tilde{\boldsymbol{s}}$ is the stored symbol vector, and $\tilde{\boldsymbol{s}}^{-j}$ is the reduced $\tilde{\boldsymbol{s}}$ by removing jth entry. The resultant $\boldsymbol{r}_j$ can be demodulated using maximum ratio combining (MRC). Therefore, diversity gain is also obtained. The numbers of both complex additions and complex multiplications during the operations of replica generation, cancellation, and MRC can be expressed by

$$N_{add} = N_t(N_r N_t + M - 1), \tag{9}$$

$$N_{mul} = N_t(N_r N_t + M). \tag{10}$$

Finally, when n packets are retransmitted (Case (3)), the receiver can make n replica packets from the stored packets at each Rx antenna. These replicas are subtracted from the received signal as

$$\boldsymbol{r}_{\bar{\boldsymbol{n}}} = \boldsymbol{r} - \hat{\boldsymbol{H}}^{-\boldsymbol{n}}\tilde{\boldsymbol{s}}^{-\boldsymbol{n}}, \tag{11}$$

where $\boldsymbol{n}$ is a set of n integers between 1 and N_t, $\bar{\boldsymbol{n}}$ is a complementary set of $\boldsymbol{n}$, $\hat{\boldsymbol{H}}^{-\boldsymbol{n}}$ is the reduced $\hat{\boldsymbol{H}}$ by removing n columns, and $\tilde{\boldsymbol{s}}^{-\boldsymbol{n}}$ is the reduced $\tilde{\boldsymbol{s}}$ by removing n entries. The remainder signal is separated into each packet using MLD as

$$\hat{\boldsymbol{s}}_{\bar{\boldsymbol{n}}} = \arg\min_{\boldsymbol{s}_{\bar{\boldsymbol{n}}}} ||\boldsymbol{r}_{\bar{\boldsymbol{n}}} - \hat{\boldsymbol{H}}^{-\bar{\boldsymbol{n}}}\tilde{\boldsymbol{s}}^{-\bar{\boldsymbol{n}}}||^2. \tag{12}$$

Since the subtraction operation in Eq. (11) virtually decreases the order of MLD calculation, the computational complexity is greatly reduced. The computational complexity of Eq. (12) is given by

$$N'_{add} = M^{(N_t - n)}(N_r N_t - nN_r + N_r - 1), \tag{13}$$

$$N'_{mul} = M^{(N_t - n)}(N_r N_t - nN_r + N_r), \tag{14}$$

After the signal separation, the receiver decodes the $(N_t - n)$ packets and makes the replica signals from the decoded packets. Then, for each of n retransmitted packets, the receiver subtracts the $(N_t - 1)$ replicas from the received signal. The reminder packet is decoded after MRC. Thus, the receiver can decode n retransmitted packets with more diversity gain. The numbers of both complex additions and complex multiplications during the operations of replica generation, cancellation and MRC including the calculation in Eq. (11) can be expressed by

$$N'_{add} = n(N_r N_t + N_r + M - 1), \tag{15}$$

$$N'_{mul} = n(N_r N_t + N_r + M). \tag{16}$$

In this paper, the average number of complex floating point operations (FPOs) is used to compare the complexities of systems. Here, it is assumed that one complex addition needs two complex FPOs, and one complex multiplications needs six complex FPOs. Table 1 summarizes the number of FPOs in the proposed receiver.

In the proposed receiver, the number of FPOs are reduced according to the number of retransmitted packets. Furthermore, decoding of retransmitted packets benefits by diversity gain. However, in the actual system, stored packets contain errors. Therefore, the replica subtraction operation cannot provide satisfactory performance because the precision of replicas are not enough. This causes the residual interference components. This also affects the succeeding MLD performance. Thus, we also consider the case where only MLD is used in Case (3).

4. Performance Evaluation

In this section, we evaluate the proposed system and compare it with the conventional SDM/HARQ system using only MLD by computer simulation. In this paper, we evaluate the system throughput performance; the throughput η is defined as [4]

$$\eta = R_b \times \frac{N_{suc}}{N_{trans}}, \tag{17}$$

where R_b represents the actual information bit rate, and N_{trans} and N_{suc} represent the total number of transmitted packets and the successfully received packets, respectively.

Table 2 shows the common simulation parameters. The decoder of the convolutional code is basically a hard decision Viterbi decoder because MLD outputs the hard decision symbols. However, a soft decision Viterbi decoder can be used in our proposed receiver when the cancellation is performed.

First, the throughput performance of the proposed receiver which employs cancellation in Case (3) is compared with that of the conventional SDM system in Fig.2. From Fig.2, the throughput performance of the proposed receiver is better than that of the conventional MLD receiver when the CNR range is below 12 [dB]. This is because the proposed receiver can employ MRC in detecting symbols.

Table 2: Simulation parameters.

Modulation scheme	QPSK
Symbol rate	32 [ksym/s]
Packet length	500 [symbol]
FEC	Convolutional code
Code rate	$1/2$
Constraint length	9
Error detection code	16-bit CRC–CCITT
# of antennas	$(N_t, N_r) = (3, 2)$
Channel Model	Flat Rayleigh fading
Channel estimation	ideal
Normalized Doppler freq.	$F_d T = 0.001$
Max. # of retransmissions	20

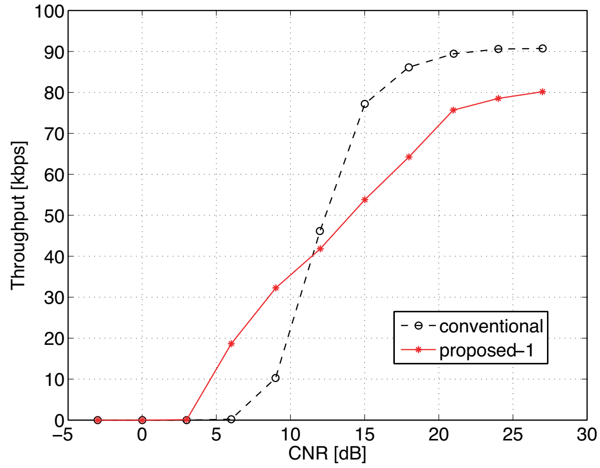

Figure 2: Throughput performance of proposed receiver (w/ cancellation in Case(3)).

On the other hand, the performance of the proposed receiver is inferior to that of the conventional receiver in the CNR region over 12 [dB]. Since the replica signals do not always have satisfactory precision, this degrades the detection performance. This effect is prominent when the number of new packets in transmission is two (Case (3)). Therefore, we consider to use MLD only in such a case.

Fig. 3 displays the throughput performance of the proposed receiver without employing cancellation in Case (3). From Fig. 3, the throughput performance of the proposed receiver is almost the same as that of the conventional receiver when the CNR range exceeds 15 [dB] because MLD surpasses in detection performance.

Finally, we investigates the average number of FPOs required for processing one packet. Fig. 4 illustrates the average numbers of FPOs of all systems. From Fig. 4, the average number of FPOs of proposed receiver is greatly reduced, especially in lower CNR range.

Table 1: The number of FPOs required for decoding N_t packets.

Case	# of new packets	FPOs in MLD	FPOs in symbol cancellation
(1)	N_t	$8M^{N_t}(N_rN_t + N_r) - 2M^{N_t}$	—
(2)	$N_t - n$	$8M^{(N_t-n)}(N_rN_t - nN_r + N_r) - 2M^{(N_t-n)}$	$8n(N_rN_t + N_r + M) - 2n$
(3)	0 or 1	—	$8N_t(N_rN_t + M) - 2N_t$

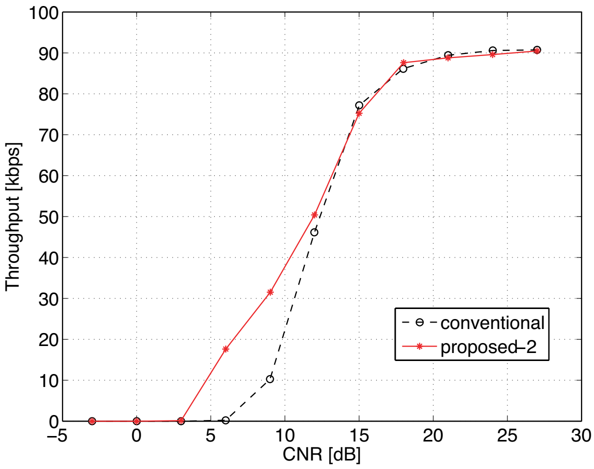

Figure 3: Throughput performance of proposed receiver (w/o cancellation in Case(3)).

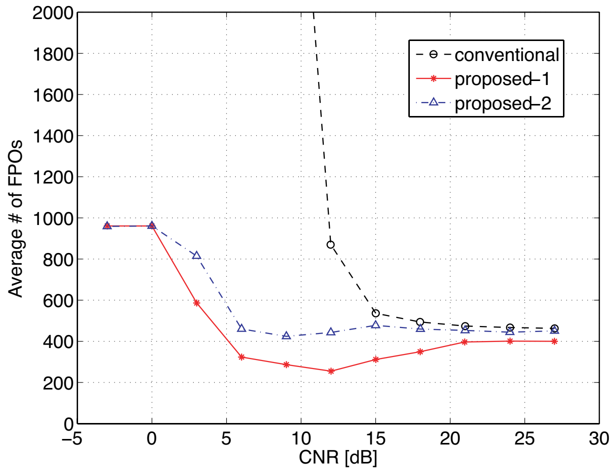

Figure 4: Average number of FPOs required for processing one packet.

The reason that the number of FPOs reaches a ceiling in the CNR region below 0 [dB] is the number of retransmissions reaches its maximum number, and the transmission is not succeeded. This is obvious from Figs. 2 and 3. In this region, the number of FPOs of the conventional MLD receiver reaches about 9200.

Furthermore, the proposed receiver which uses only MLD in Case (3) still indicates smaller number of FPOs than the conventional receiver. This is because Case (2) transmission occasionally occurs even in a high CNR region. Moreover, it is worthy of notice that the proposed scheme can be used with other complexity reduced MLDs because we do not modify the structure of MLD itself. Therefore, the proposed scheme is quite effective to reduce the complexity of MLD-based SDM receivers.

5. Conclusion

We have proposed a computational complexity reduction scheme which exploits the property of HARQ scheme for MLD-based SDM receivers. By subtracting the replica signal which is made from the stored packet from the received superposed signal, we can reduce the order of MLD calculation. From the simulation results, the proposed scheme can greatly reduce the computational complexity of MLD without reducing the throughput performance. Furthermore, since we do not modify the structure of MLD itself, the proposed scheme can be used with other complexity reduced MLD schemes. Therefore, the proposed scheme could become a realistic solution to improve the transmission quality with low computational complexity.

References

[1] R. van. Nee, A. van. Zelst, and G. Awater, “Maximum likelihood decoding in a space division multiplexing system,” *Proc. IEEE VTC 2000-Spring*, May 2000.

[2] H. Kawai, K. Higuchi, N. Maeda, M. Sawahashi, T. Ito, Y. Kakura, A. Ushirokawa, and H. Seki, “Likelihood Function for QRM–MLD Suitable for Soft-Decision Turbo Decoding and Its Performance for OFCDM MIMO Multiplexing in Multipath Fading Channel,” *IEICE Trans. Commun.*, vol. E88–B, no. 1, pp. 47–57, Jan. 2005.

[3] T. Fujita, A. Ohta, T. Onizawa, and T. Sugiyama, “A Reduced-Complexity Signal Detection Scheme Employing ZF and K-Best Algorithms for OFDM/SDM,” *IEICE Trans. Commun.*, vol. E88–B, no. 1, pp. 66–75, Jan. 2005.

[4] N. Miki, H. Atarashi, S. Abeta, and M. Sawahashi, “Comparison of Hybrid ARQ Schemes and Optimization of Key Parameters for High-Speed Packet Transmission in W-CDMA Forward Link,” *IEICE Trans. Fundamentals*, vol. E84–A, no. 7, pp. 1681–1690, July 2001.