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Control of Spatiotemporal Intermittency by Parametric Resonance

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Abstract—We investigate the control of spatially extended chaotic state in an electroconvective systems of a nematic liquid crystal. The spatiotemporal intermittency (STI) is a coexistent state of laminar (*defect lattice* : DL) and turbulent domains. The controlling STI that turbulent state changes to DL can achieved via slowly varying amplitude modulation (AM) of the applied ac voltage. This suppression of intermittent characters is caused by a resonance between the oscillation of lattices and the slow AM. We calculated correlation functions for the binarized time series measured at a local point which represents transition dynamics between turbulent and DL. It has been found that the correlation functions show power-law decay and their indices becomes smaller when AM is applied. This result suggests that not only fraction of the turbulence but also fluctuations are suppressed by the resonant controlling.

1. Introduction

Chaos in real systems often reduces efficiencies of various devices in engineering systems [1] as well as providing renovate ideas in interdisciplinary research field such as biology [2] and information science [3]. “Controlling chaos” by using small perturbations applied to the chaotic system has therefore attracted broad interests of scientists [4]. It can be realized by applying delayed-feedback signal [5] or periodically external (either additive or parametric) force to the system [6]. The verification of the achievement of controlling chaos has been implemented by using the measures such as Lyapunov exponent, attractor dimension, autocorrelation function and shift of a bifurcation point to chaos not only in theoretical models [6], but also in experimental systems such as the bistable magnetoelastic system [7], the electric circuit [8] and the laser system [9].

Recently extension of controlling chaos to spatiotemporal chaos (STC) has been tried [10]. Controlling STC is an interesting problem because there are great needs for application in various systems such as plasma system [11], turbulence [12] and electrochemical reaction [13]. However the understanding about the controlling STC is not yet sufficient for applications. In a real system the controlling STC involves difficulty due to a large number of degrees of freedom. It is very important to examine realization of the controlling STC in experiments based on theoretical results obtained in the nonlinear physics.

Spatiotemporal intermittency (STI) is one of the typical examples of STC, where ordered and disordered states coexist as a dynamical domain structure. STI is observed in various systems such as a coupled map lattice [14], a partial differential equation [15], a viscous fingering [16] and a convective system [17], and has been investigated mainly with the viewpoints of statistical physics [15, 17, 18].

In an electroconvective system of a planar-oriented nematic liquid crystal an ordered pattern called defect lattice (DL), which is a lattice structure of defects embedded in a roll pattern, is observed [19]. In DL, the turbulent domains appear locally with increasing the control parameter (applied voltage). Thus DL is a typical example of 2-dimensional STI which consists of DL and turbulence respectively corresponding to the ordered state and the disordered state [20]. We have found that the turbulent domains in STI are changed to the lattice state by the application of slowly modulated fields of the applied voltage. It is suggested that STI could be controlled by the parametric modulation.

As mentioned above the detailed investigation of STI has been performed with the viewpoints of statistical physics. Temporal behavior such as time correlation function and duration time of one state brings us important information with respect to transition between laminar and turbulence in STI [21]. Therefore by research of the controlling STI from the view point of statistical mechanics, it can be expected to obtain more detailed information on the controlling STI. In the present paper we report on statistical properties of temporal behavior of STI under the parametric modulation.

2. Experiment

The experimental setup was similar to one reported previously [22]. The horizontal size of the system was $383d \times 383d$ where the distance d between the electrodes was 26.1 μm. The nematic liquid crystal *p*-methoxybenzilidene-*p’*-*n*-butylaniline (MBBA) was aligned planarly in $x - y$ plane, and its initial director is defined to be parallel to the x-axis. The experimental temperature was controlled at 30.00 ± 0.02°C. The dielectric constant and the electric conductivity of the sample cell measured perpendicularly to the director of the liquid crystal were 5.0 and $3.7 \times 10^{-7}\Omega^{-1}\mathrm{m}^{-1}$, respectively.

An electroconvective pattern can be observed by polarized light transmitted through the sample, since distortion of the director due to the convection makes spatial modulation of refractive index for extraordinary light. The image data were taken by a charge coupled device (CCD) camera (HAMAMATSU C4880-80) mounted on a polarizing microscope and stored on a hard disk of a computer for further analysis. Self-made programs and the software ImageJ were used for the image analysis.

An ac voltage $V_{\mathrm{ex}}(t) = \sqrt{2}V(1 + W)\cos(2\pi f_{\mathrm{ac}}t)$ was applied to the sample in the z-direction using a digital synthesizer (NF1946). For standard experiments of electroconvection W is zero. In the present experiment, the amplitude modulation (AM) of the external voltage $W = a_{\mathrm{m}}\cos(2\pi F_{\mathrm{m}}t)$ is applied. f_{ac} was set to $0.65f_{\mathrm{c}}$, where f_{c} is a critical frequency and usual convection is observed below f_{c}. F_{m} is in the order of 1 Hz which is much smaller than f_{ac}. Hereafter we use the normalized voltage $\varepsilon = (V^2 - V_{\mathrm{c}}^2)/V_{\mathrm{c}}^2$, where V_{c} is the threshold voltage of convection, as a control parameter.

3. Results and Discussions

In the planar nematic system, first a stationary straight convective pattern appears beyond an onset voltage V_{c}. For a slightly higher ε defect chaos state appears, where the defects nucleate, glide and annihilate randomly in space and time. With increasing ε, the numerous defects align regularly, and DL emerges. The DL is formed by interaction between convective modes and azimuthal rotation modes of the director [19]. The interaction also induces oscillation of the lattice, because it is regarded as a kind of activator-inhibitor one [22, 23]. Since the units of the lattice are coupled through the elastic interaction of director, they compose a coupled oscillator system. Though the oscillation arises intermittently in space and time, the frequency of the oscillation of the DL is almost constant at a fixed value of ε. It can be regarded therefore as an intrinsic frequency f_0 of the DL oscillation. f_0 linearly depends on ε [22].

When ε is further increased, the lattice locally collapses and changes into turbulent state as shown in Fig 1. This state corresponds to STI where the DL domains (laminar state) and the turbulent one (turbulent state) coexist. The turbulent domain fluctuates with changing of its shape and size. In order to extract quantitative information of the intermittent state, we adopted the local spectrum method which was also used in the previous research [20]. By the method, the whole area is divided into DL state and turbulent one depending on the magnitude of the local spectrum component for the wavenumber corresponding to DL. We made binalized image by assigning 1 to turbulent domains and 0 to DL. Then the area fraction S_{T} of the total turbulent domains can be obtained from the binalized image. As

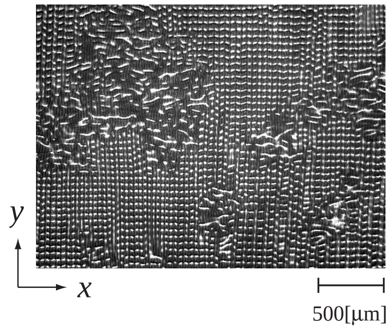

Figure 1: Spatiotemporal intermittency formed in DL. The image size is $109d \times 82d$ and $\varepsilon = 0.80$.

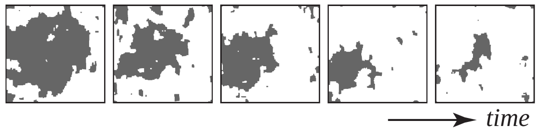

Figure 2: Temporal change of a binarized image in the process of the controlling STI with $F_{\mathrm{m}} = f_0$ and $a_{\mathrm{m}} = 0.04$. The black and white regions indicate turbulent domain and DL one, respectively. The time interval of images is 25 min and the observed area is $130d \times 131d$.

ε is further increased, S_{T} increases, and finally the turbulent domains cover the whole system. Thus the transition process is characterized by S_{T}.

When the AM was applied to the system exhibiting the STI as a parametric force, the size of the turbulent domain decreases by formation of lattices [24]. This represents a realization of controlling STC. Especially, it should be noted that spatial structure can be reproduced by temporal modulation of the external field. The formation of lattices occurs mainly at boundaries between the turbulent domain and DL domain, that is, nucleation of lattice state in turbulent domain is not dominant in this process as seen in Fig. 2.

We measured the area fraction S_{DL} (= $100 - S_{\mathrm{T}}$) of DL in the controlled state with changing the F_{m} [24]. Since the resulting curve of $S_{\mathrm{DL}}(F_{\mathrm{m}})$ has a peak at $F_{\mathrm{m}} \cong f_0$, it is regarded as a kind of resonance curve and $F_{\mathrm{m}} = f_0$ is the optimal frequency for the controlling STI. We also confirmed that the oscillation change from intermittent to coherent by applying AM . From these observations it can be concluded that this suppression of intermittent characters is due to parametric resonance of the oscillation of lattices by the AM. This occurs through the mediation of the elastic interaction of the director. Hereafter F_{m} is fixed to f_0 through all measurements.

It takes several hours to complete controlling ($S_{\mathrm{T}} \cong 0$) from an initial value (e.g. $S_{\mathrm{T}} \cong 90\%$) in the present experiment. This controlling time is much longer than those

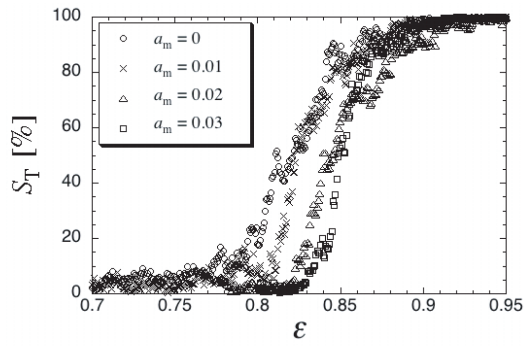

Figure 3: Transitions to turbulence with AM for several values of a_m. AM was set to $F_m = f_0$ and the applied voltage was increased with rate $r = 0.05$ mV/s. The observed image size is $130d \times 123d$.

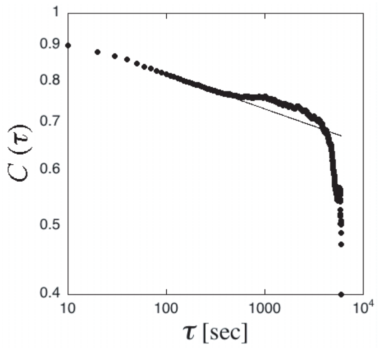

Figure 4: Log-log plot of an avaraged correlation function of a binarized time series at a local point of a STI system. ε was set to 0.90 where mean value of S_T is 50%, and AM with $a_m = 0.02$ was applied. Solid line denotes result of the fitting to $C(\tau) \sim \tau^{-\mu}$ for $t < 5$ min, which gives $\mu = 0.05$.

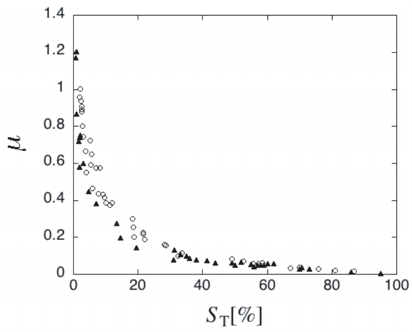

Figure 5: Ratio of the power indices of the power-law decay in the correlation functions. ∘ and ▲ indicates the indices for the case without AM and with AM, respectively.

in the controlling chaos experimentally realized in [7, 8, 9] and characteristic times of the system (e.g., director relaxation time and f_0^{-1}). It was also confirmed that the controlling time becomes shorter as a_m increases below 0.04.

Transitions to turbulence via the STI with the AM for several values of a_m is shown in Fig. 3. When the AM is applied to the system the bifurcation point, where a turbulent domain appears, shifts from the original bifurcation point ε_0^* to a higher value ε^* of ε. From the results of the shifts $\varepsilon^* - \varepsilon_0^*$ measured for various a_m it is also clarified that the shift linearly increases for the increase of a_m, and there is no threshold for a_m.

Autocorrelation functions of the binalized time series at a local point in stationary state were measured to know statistical properties as mentioned above. Even under the AM, the system can go into a stationary state with finite S_T for fixed ε and a_m. In the case without (with) the AM we waited for 1 hour (3 hours) in order to acquire data in stationary stochastic process. One correlation function was averaged over 81 correlation functions obtained from spatially different locations. Since the distance among locations is longer than 10 times of the unit length of the lattice, statistically independent data were obtained. The correlation function exhibits power-law decay in short time range (~ 10 min) as shown in Fig. 4. It is known that correlation function for the two states Poisson process shows exponential decay. The origin of the power-law decay of correlation functions, which may be caused by strongly correlated fluctuation and frequently appears in nonequilibrium systems, is still open problem. The detailed research of the peculiar property of the correlation function for the STI will be discussed in elsewhere.

The index of the power-law decay at least held at $t < 5$ min. Fig. 5 shows the S_T dependence of the indices for the cases with AM with $a_m = 0.02$ and without AM. A similar trend, that is, the indices decrease as S_T increases, is obviously shown for both cases. As shown in Fig. 5, the power indices decrease due to the application of AM. The ratio of the power index for the case with AM to that for the case without AM is nearly constant whose mean value is 0.79 ± 0.14 independent of S_T. In the presence of AM each state of DL and turbulence is sustained for longer time in comparison with no AM case, which suggests that fluctuations of the system are suppressed by AM.

By observation we may interpret the results as follows. In the case with the AM, since all lattice units are entrained into the frequency of the AM, transition from DL state to turbulent one becomes more difficult to arise than the case without AM. Consequently, it is also difficult for the turbulent state to change to the DL one, because S_T is almost kept. This is a reason why the correlation function obeys a power law with smaller value of the index under AM.

4. Conclusion

In this paper, we have reported on temporal behavior of the resonant control of STI in the defect lattice in an electroconvective system of a nematic liquid crystal. The controlling STI that turbulent state changes to the lattice can be achieved by slow modulation of the applied ac voltage with amplitudes of several percents. It is revealed that the correlation times of the turbulent state become longer by applying the modulation. These results imply that the fluctuations of the system are suppressed by the slow modulation of the control parameter. It could be expected that the results are general features of the controlling STC and it is desired especially in the engineering field.

Acknowledgments

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References

[1] F. C. Moon, *Chaotic Vibrations. An Introduction for Applied Scientists and Engineers*, John Wiley (New York, 1987)

[2] S. J. Schiff, K. Jerger, D. H. Duong, T. Chang, M. L. Spano and W. L. Ditto, Nature (London) **370**, 615 (1994).

[3] G. D. Vanwiggeren and R. Roy, Science **279**, 1198 (1998).

[4] V. V. Alekseev and A. Y. Loskutov, Sov. Phys. Dokl. **32**, 270 (1987). W. Yang, M. Ding, A. J. Mandell and E. Ott, Phys. Rev. E **51**, 102 (1995). T. Kapitaniak, *Controlling Chaos: Theoretical and Practical Methods in Non-linear Dynamics*, Academic Press, (1996). C. U. Choe, K. Höhne, H. Benner and Y. S. Kivshar, Phys. Rev. E **72**, 036206 (2005).

[5] E. Ott, C. Grebogi and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990). A. G. Balanov, N. B. Janson and E. Schöll, Physica **199D**, 1 (2004).

[6] R. Lima and M. Pettini, Int. J. Bifurcation Chaos Appl. Sci. Eng. **8**, 1675 (1998). K. A. Mirus and J. C. Sprott, Phys. Rev. E **59**, 5313 (1999).

[7] L. Fronzoni, M. Giocondo and M. Pettini, Phys. Rev. A **43**, 6483 (1991).

[8] Y. C. Lai, A. Kandangath, S. Krishnamoorthy, J. A. Gaudet and A. P. S. de Moura, Phys. Rev. Lett. **94**, 214101 (2005).

[9] R. Meucci, W. Gadomski, M. Ciofini and F. T. Arecchi, Phys. Rev. E **49**, R2528 (1994).

[10] S. Wu, K. He and Z. Huang, Phys. Lett. A **260**, 345 (1999). M. Jiang, X. Wang, Q. Ouyang and H. Zhang, Phys. Rev. E **69**, 056202 (2004). A. L. Fradkov, R. J. Evans and B. R. Andrievsky, Phil. Trans. R. Soc. A **364**, 2279 (2006).

[11] T. Fukuyama, R. Kozakov, H. Testrich and C. Wilke, Phys. Rev. Lett. **96**, 024101 (2006).

[12] P. Moin and T. Bewley, Appl. Mech. Rev. **47**, S3 (1994). R. Rathnasingham and K. S. Breuer, J. Fluid. Mech. **495**, 209 (2003).

[13] P. Parmananda, B. J. Green and J. L. Hudson, Phys. Rev. E **65**, 035202 (2002).

[14] H. Chaté, Physica D **32**, 409 (1988). Z. Jabeen and N. Gupte, Phys. Rev. E **74**, 016210 (2006).

[15] H. Chaté and P. Manneville, Phys. Rev. Lett. **58**, 112 (1987).

[16] S. Michalland, M. Rabaud and Y. Couder, Europhys. Lett. **22**, 17 (1993).

[17] S. Ciliberto and P. Bigazzi, Phys. Rev. Lett. **60**, 286 (1988). F. Daviaud, M. Bonetti and M. Dubois, Phys. Rev. A **42**, 3388 (1990).

[18] M. M. Degen, I. Mutabazi and C. D. Andereck, Phys. Rev. E **53**, 3495 (1996).

[19] S. Nasuno, O. Sasaki and S. Kai, Phys. Rev. A **46**, 4954 (1992). N. Oikawa, Y. Hidaka and S. Kai, Phys. Rev. E **70**, 066204 (2004).

[20] N. Oikawa, Y. Hidaka and S. Kai, Prog. Theor. Phys. Suppl. **161**, 320 (2006).

[21] Y. Pomeau, Physica D **23**, 3 (1986).

[22] N. Oikawa, Y. Hidaka and S. Kai, J. Phys. Soc. Jpn. **73**, 2917 (2004).

[23] E. Plaut and W. Pesch Phys. Rev. E **59**, 1747 (1999).

[24] N. Oikawa, Y. Hidaka and S. Kai, in preparation.