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Consistency in Artificial Chaotic Spiking Neurons

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Abstract—This paper studies response of the chaotic spiking neurons to spike-train inputs. Applying the inputs, chaotic behavior is changed into a variety of phenomena and we introduce an interesting phenomenon: applying some kind of random input, the circuit exhibits identical nonperiodic steady state response for various initial states. Such phenomena have been refered to as "Consistency". Presenting a simple test circuit, the consistency is confirmed experimentally.

1. Introduction

Integrate-and-fire neuron models (IFMs) have been studied in order to consider neuron functions and their engineering applications [1]-[8]. Applying spike-train input, the IFMs can exhibit periodic/nonperiodic responses whose analysis is basic to construct pulse-coupled networks (PCNs). The PCNs can exhibit rich synchronous/asynchronous phenomena and have several remarkable properties as compared with smooth coupled systems: faster transient to steady state, lower power consumption and flexible coding ability [9]. The PCNs have a variety of applications such as image segmentation [10] [11], associative memories [4], impulsive communications [5] [6], feature selectors [7] and address-event-representation [8].

This paper studies response of a chaotic spiking circuit (CSC) to spike-train inputs. The CSC can be regarded as a circuit model of spiking neurons [12] [18]. The CSC can be a building block of PCNs having a variety of synchronous patterns with applications. The CSC consists of two capacitors, one linear two-port voltage-controlled current source (2P-VCCS) and one impulsive switch [11]. If the input does not present one capacitor voltage repeats vibrate-and-fire dynamics and can output various chaotic/periodic spike-trains. Such behavior is impossible in usual autonomous IFMs that can not vibrate below the threshold [1] [10]. Applying an input, the CSC can output a variety of spike-trains. We consider an interesting phenomenon. That is "consistency": applying some kind of nonperiodic input, the CSC can exhibit identical steady state response for various initial values.

A simple test circuit is presented and the consistency is confirmed in the laboratory. We note that "consistency" have been studied mainly in complicated physical systems [16]. This paper gives the observation of the phenomenon in simple spiking neurons. It may be basic information to bridge between interesting nonlinear phenomena and PCNs.

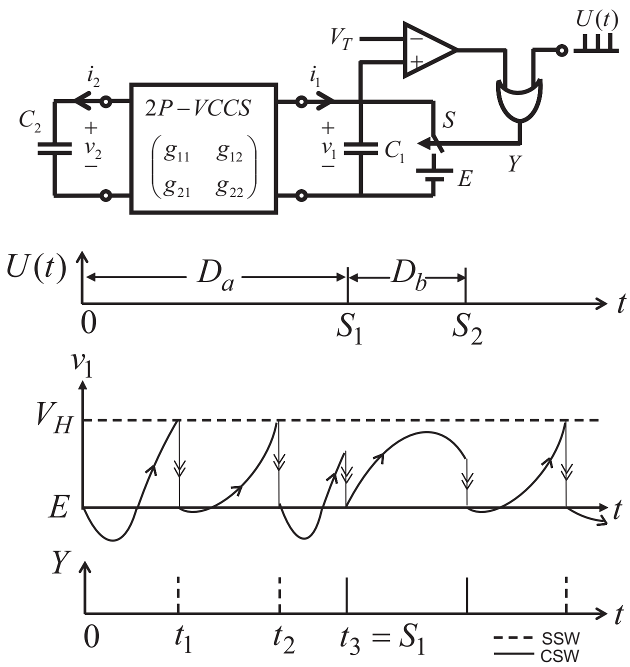

Figure 1: Chaotic spiking circuit and vibrate-and-fire behavior

2. Chaotic Spiking Circuit

Fig. 1 shows the CSC where the 2P-VCCS is characterized by

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

We assume that capacitor voltages v_1 and v_2 can vibrate if v_1 is below the threshold V_T and S is open. At the moment when v_1 reaches V_T, S is closed impulsively and v_1 is reset to the base E holding continuity of v_2: we call it self switching (SSW). $U(t)$ is a spike-train input with amplitude $V_H - V_L$. As n-th spike arrives at the time T_n, S is

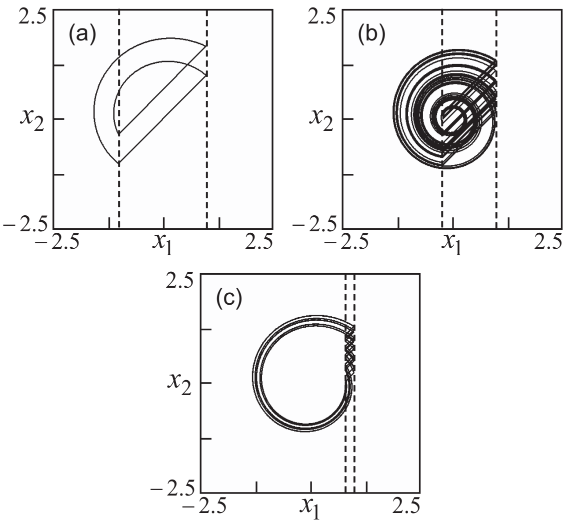

Figure 2: Chaos and periodic response for $\delta = 0.1$, $p = 1.0$. (a) Periodic response for $q = -1.0$, (b) Chaotic response for $q = -0.2$, (c) Chaotic response for $q = 0.8$.

closed and v_1 is reset: we call it compulsory switching (CSW). Repeating these vibrate-and-fire, the CSC can output various spike trains Y. The CSC has unstable complex characteristic roots $\delta\omega \pm j\omega$, $s_n = \omega T_n$, and the following dimensionless variables and parameters are used:

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11}/C_1 & g_{12}/C_1 \\ g_{21}/C_2 & g_{22}/C_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (2)$$

SSW: $(v_1(t^+), v_2(t^+)) = (E, v_2)$ if $v_1(t) = V_H$
CSW: $(v_1(t^+), v_2(t^+)) = (E, v_2)$ if $t = s_n$.

$$U(t) = \begin{cases} V_H & \text{at } t = S_n \\ V_L & \text{otherwise} \end{cases} \quad Y = \begin{cases} V_H & \text{if } v_1 = V_T \text{ or } t = S_n \\ V_L & \text{otherwise} \end{cases}$$

$$\tau = \omega t,\ \text{"}\cdot\text{"} \equiv \frac{d}{d\tau},\ \delta = \frac{1}{2\omega}\left(\frac{g_{11}}{C_1} + \frac{g_{22}}{C_2}\right), q = \frac{E}{V_T},$$
$$p = \frac{1}{2\omega}\left(\frac{g_{11}}{C_1} - \frac{g_{22}}{C_2}\right), x_1 = \frac{v_1}{V_T},\ x_2 = \frac{1}{V_T}\left(pv_1 + \frac{g_{12}}{\omega C_1}v_2\right), \quad (3)$$
$$u(\tau) = \frac{U(\frac{\tau}{\omega}) - V_L}{V_H - V_L}, \omega^2 = -\frac{g_{12}g_{21}}{C_1C_2} - \frac{1}{4}\left(\frac{g_{11}}{C_1} - \frac{g_{22}}{C_2}\right)^2 > 0.$$

Eq. (2) is transformed into Eq. (4).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \delta & 1 \\ -1 & \delta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ for } x_1 < 1 \text{ and } u = 0 \quad (4)$$

SSW: $(x_1(\tau^+), x_2(\tau^+)) = (q, x_2(\tau) - p(1-q))$ if $x_1(\tau) = 1$
CSW: $(x_1(\tau^+), x_2(\tau^+)) = (q, x_2(\tau) - p(x_1(\tau) - q))$ if $\tau = s_n$.

$$u(\tau) = \begin{cases} 1 & \text{at } \tau = s_n \\ 0 & \text{otherwise} \end{cases} \quad y = \begin{cases} 1 & \text{if } x_1 = 1 \text{ or } \tau = s_n \\ 0 & \text{otherwise} \end{cases}$$

Let t_n be n-th switching instant at which n-th output spike appears ($y(t_n) = 1$ in Fig. 1) by either SSW or CSW.

2.1. CSC without spike-train input

First we consider the case of CSC without spike-train input ($u(\tau) = 0$). In this case, Eq. (4) is characterized by three parameters: δ, p and q. When S is open ($x_1(\tau) < 1$), the exact piecewise solution is given by

$$\begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} = e^{\delta\tau}\begin{bmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \quad (5)$$

If input does not present then CSW does not exist and CSC is an autonomous system. The CSC exhibits chaotic/periodic phenomena for parameter q as shown in Fig. 2 (a), (b) and (c). In the following parts, we consider response of chaos in Fig. 2 (b) ($\delta = 0.1$, $p = 1.0$, $q = -0.2$) to nonperiodic input.

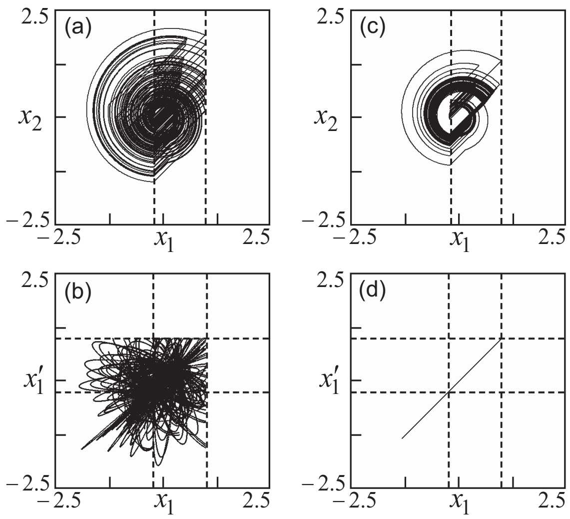

Figure 3: Typical responses to nonperiodic input ($\delta = 0.1, p = 1.0, q = -0.2$). (a) & (b) Chaotic response and its dependence on initial state for $d_a = 3.3, d_b = 5.0$. (c) & (d) "Consistency" and its dependence on initial state for $d_a = 3.3, d_b = 3.8$. (x_1, x_2) and (x_1', x_2') systems have the same parameters but different initial values: $(x_1(0), x_2(0)) = (-0.2, 0)$ and $(x_1'(0), x_2'(0)) = (-0.2, 0.5)$.

3. Consistency

Here we consider an random input such that two spike intervals d_a and d_b appear randomly. Note that the case of periodic inputs have discussed in [18]. For simplicity we consider the case where the appearing rate of d_a and d_b are $d_b/(d_a + d_b)$ and $d_a/(d_a + d_b)$, respectively. In such a case the CSC usually exhibits chaotic responses which are sensitive to initial state as shown in Fig. 3 (a) and (b). However, in some parameter range, we have confirmed interesting response as shown in Fig. 3 (c) and (d): the random input causes non-periodic response that is identical for various initial values. Fig. 4 illustrates measuring system of this phenomenon: when a common nonperiodic input is applied to two systems which have different initial states and the same parameter values of (δ, p, q), the both systems exhibits identical nonperiodic phenomena in the steady state

(Fig. 3 (d)). This is ”consistency”: a kind of chaotic synchronous phenomena such that a system exhibits identical steady state response to nonperiodic input for various initial values.

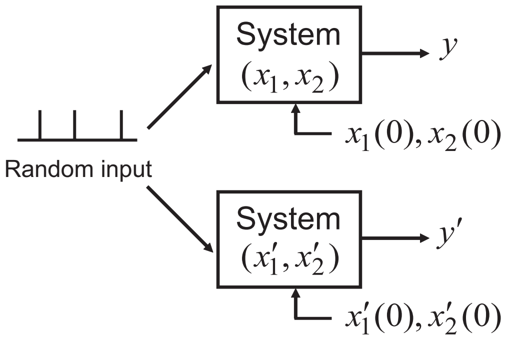

Figure 4: System setup to measure ”consistency”.

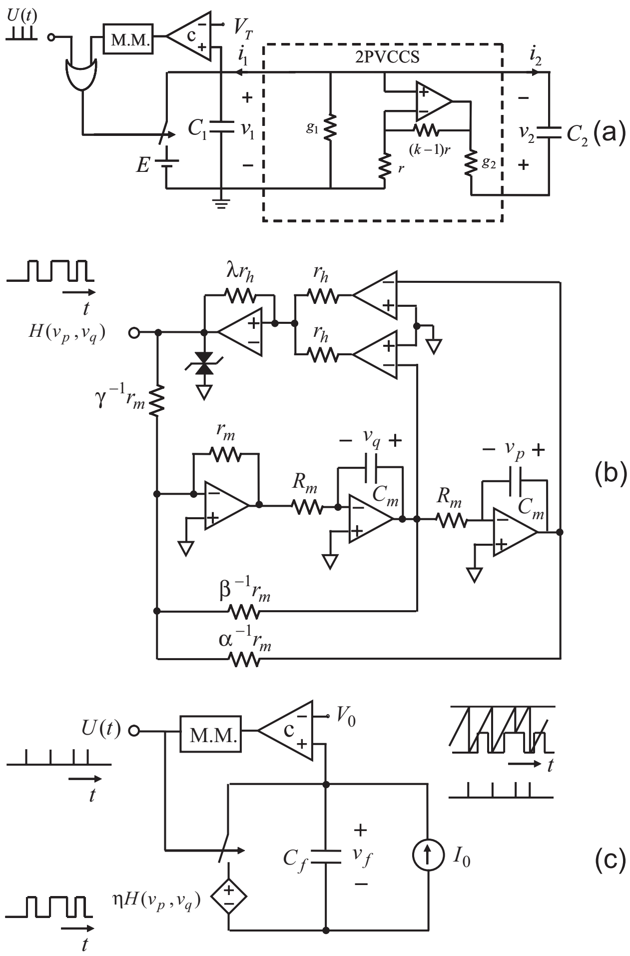

Figure 5: A nonperiodic spike-train input signal generator. (a) The CSC based on Wien bridge oscillator. (b) MPL ($C_m \simeq 0.068\ \mu$F, $R_m \simeq 10$ kΩ, $E \simeq 2.2$ V, $\alpha^{-1} \simeq 1.0$, $\beta^{-1} \simeq 4.6$, $\gamma^{-1} \simeq 3.0$). (c) IFC ($C_f \simeq 10$ nF, $I_0 \simeq 40\ \mu$A).

4. Laboratory experiments

For the laboratory experiments, we have fabricated a simple test circuit. The Wien bridge oscillator in Fig. 5 (a) is used in the CSC. The coefficients in Eq. (2) are $\frac{g_{11}}{C_1} = \frac{(k-1)g_2 - g_1}{C_1}$, $\frac{g_{12}}{C_1} = \frac{-g_2}{C_1}$, $\frac{g_{21}}{C_2} = \frac{(k-1)g_2}{C_2}$ and $\frac{g_{22}}{C_2} = \frac{-g_2}{C_2}$. If the input does not present, the circuit exhibits chaotic phenomena as shown in Fig. 6. It corresponds to Fig. 2 (b). In order to generate random input signal, we have used manifold piecewise linear chaos generator (MPL[17]). The differential equation of this system is described by

$$\ddot{v}_p - \beta \dot{v}_p + \alpha v_p = \begin{cases} \gamma E & \text{(A)} \\ -\gamma E & \text{(B)} \end{cases} \tag{6}$$

where $\cdot \equiv \frac{d}{d\tau'}$ and $\tau' = \frac{t}{R_m C_m}$. The right-hand side is switched from (A) to (B) ((B) to (A)) if $v_q = 0$ and $v_p \leq 0$ ($v_q = 0$ and $v_p > 0$). Fig. 7 shows the chaotic attractor observed in MPL. The MPL outputs a nonperiodic binary signal H. It is known that the output alternates E and $-E$ at the rate of 50 % [17]. This nonperiodic signal is applied to the integrate-and-fire circuit (IFC) as shown in Fig. 5 (c). In the IFC, the capacitor voltage v_f is integrated below the threshold voltage V_0. As v_f reaches V_0, it is reset to the base voltage ηH instantaneously. Since H is E or $-E$, the IFC outputs spike-train $U(t)$ consisting of two inter-spike-intervals. Applying $U(t)$ to the CSC, we have observed ”consistency” as shown in Fig. 8 (d).

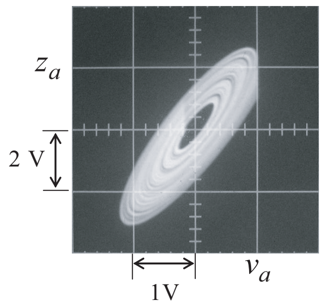

Figure 6: Laboratory measurements ($1/g_1 \simeq 40$ kΩ, $1/g_2 \simeq 50$ kΩ, $C_1 = C_2 = C \simeq 2.0$ nF, $k \simeq 3.5$, $V_T \simeq 1.0$ V, $E \simeq 0.2$ V; $\delta \simeq 0.1, p \simeq 1.0, q \simeq -0.2$). Chaotic phenomena without spike-train inputs.

5. Conclusions

We have studied response of chaotic spiking circuit with spike-train input. The CSC exhibits a variety of responses and we consider ”consistency”: applying some kind of random input, the circuit exhibits identical nonperiodic steady state response for various initial states. We have confirmed ”consistency” phenomena experimentally by presenting a simple test circuit. Future problems include analysis of bifurcation phenomena for another parameter and applications to signal processing by PCNs and to system identification from spike-trains.

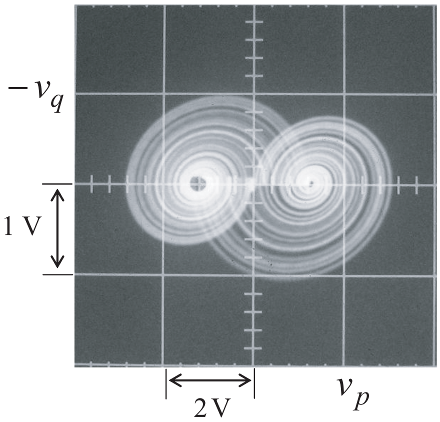

Figure 7: Laboratory measurements ($C_m \simeq 0.068\ \mu$F, $R_m \simeq 10$ kΩ, $E \simeq 2.2$ V, $\alpha^{-1} \simeq 1.0$, $\beta^{-1} \simeq 4.6$, $\gamma^{-1} \simeq 3.0$). Chaotic attractor observed in MPL.

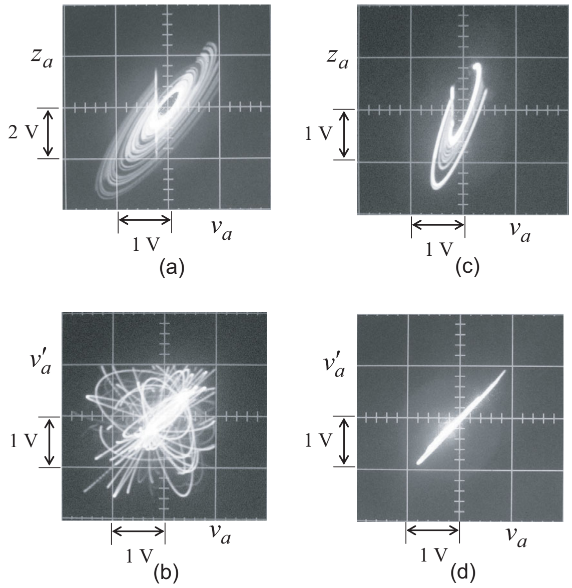

Figure 8: Laboratory measurements ($1/g_1 \simeq 40$ kΩ, $1/g_2 \simeq 50$ kΩ, $C_1 = C_2 = C \simeq 2.0$ nF, $k \simeq 3.5$, $V_T \simeq 1.0$ V, $E \simeq 0.2$ V; $\delta \simeq 0.1, p \simeq 1.0, q \simeq -0.2$). (a) and (b): Chaotic phenomena for $T_A \simeq 300\ \mu$s and $T_B \simeq 450\ \mu$s ($V_0 \simeq 1.50$ V, $\eta \simeq 0.14$, $d_a \simeq 3.3, d_b \simeq 5.0$). (c) and (d): Consistency phenomena for $T_A \simeq 300\ \mu$s and $T_B \simeq 340\ \mu$s ($V_0 \simeq 1.28$ V, $\eta \simeq 0.036$, $d_a \simeq 3.3, d_b \simeq 3.8$). (v_a, z_a) and (v'_a, z'_a) are states of the first and second CSCs, respectively.

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