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Wave Phenomena in Cellular Neural Networks Using Two Kinds of Template Sets

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Abstract—In this study, we propose cellular neural networks using two kinds of template sets. The computer simulations show that the proposed system can generate similar phenomena which is observed from the two-layer cellular neural networks. Investigating these phenomena contributes to understand complex systems and to apply them to engineering systems.

1. Introduction

Cellular Neural Network (CNN) [1]-[3] is one kind of neural networks. The main characteristic is the local connection. There have been many studies on CNN and many kinds of CNN have been proposed. One of them is two-layer CNN. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern [4], active wave propagation [5] and so on. Investigating these phenomena contributes to understand complex systems and to apply them to engineering systems.

In this study, we propose CNN using two kinds of template sets. One of the aim of this study is a reproduce of active wave propagation phenomena observed from coupled two-layer CNN. Our final aim is investigation of a new class of complex systems using this CNN. The proposed system consists of two types of cells which have different template sets. These cells are places as checkered. The computer simulations show that the proposed CNN can generate similar phenomena which is observed from the two-layer CNN.

2. CNN Using Two Kinds of Template Sets

Figure 1 shows a system model in this study. We assume that the system has a two-dimensional M by N array structure. Each cell in the array is denoted as $c(i, j)$, where (i, j) is the position of the cell, where $1 \leq i \leq M$ and $1 \leq j \leq N$. The coupling radius is assumed to be one in this study. In this proposed CNN, we use two kinds of template sets. Cells having one template set are called as cell α and the other are called as cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i + j$ is an even number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\alpha + \sum_{c(k,l)} A_\alpha(i, j; k, l) y_{kl} + \sum_{c(k,l)} B_\alpha(i, j; k, l) u_{kl} \tag{1}$$

2: The case that $i + j$ is an odd number.

$$\frac{x_{ij}}{dt} = -x_{ij} + I_\beta + \sum_{c(k,l)} A_\beta(i, j; k, l) y_{kl} + \sum_{c(k,l)} B_\beta(i, j; k, l) u_{kl} \tag{2}$$

$A_{\{\alpha\beta\}}(i, j; k, l) y_{kl}$, $B_{\{\alpha\beta\}}(i, j; k, l) u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively.

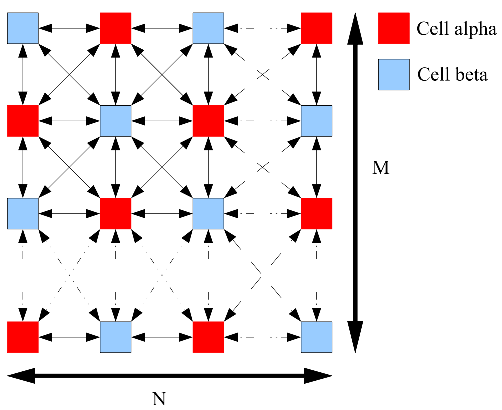

Figure 1: Structure of CNN using two kinds of template sets.

The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \tag{3}$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

The variables u and y are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j1) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \tag{5}$$

This proposed system is more complex than the normal CNN. This system has a peculiar characteristic in order to investigate a new class of coupled oscillatory systems. Namely, a pair of cell α and cell β are needed for a simple oscillation. However, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators.

3. Computer Simulations

First, we try to reproduce some phenomena observed from mutually coupled two-layer CNN [5]. The CNN array consists of 50×50 cells with a zero-fixed boundary condition.

3.1. Stripe Line (Checkerboard)

Figure 2 shows simulation results using the following template set.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -0.2 & 0 & -0.2 \\ 0 & 1.8 & -1 \\ -0.2 & 0 & -0.2 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} 0.1 & 0 & 0.1 \\ 1 & 0.3 & 0 \\ 0.1 & 0 & 0.1 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0, \tag{6}$$

We can observe a stripe line formation. In the case of the two-layer CNN, a checkerboard pattern formation is observed using the following template sets.

$$\boldsymbol{A}_1 = \begin{pmatrix} 0 & -0.2 & 0 \\ -0.2 & 1.8 & -0.2 \\ 0 & -0.2 & 0 \end{pmatrix}, \quad \boldsymbol{A}_2 = \begin{pmatrix} 0 & 0.1 & 0 \\ 0.1 & 0.3 & 0.1 \\ 0 & 0.1 & 0 \end{pmatrix},$$

$$\boldsymbol{C}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{C}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0, \tag{7}$$

In this template set, all corners of matrices are set to zero. By utilizing this characteristic, Eq. (6) can be equivalent to Eq. (7). As a results, cell β corresponds to the second layer cell. Thus, a checkerboard pattern formation in the two-layer CNN becomes a stripe line pattern formation. In the following subsections, the same method is used.

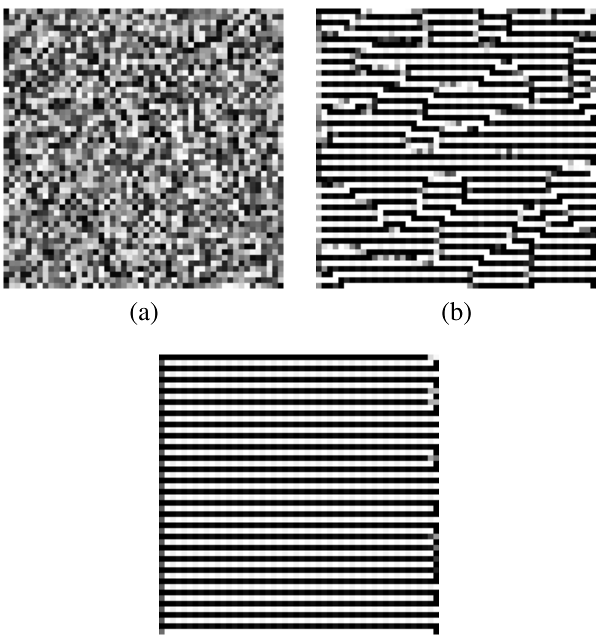

Figure 2: Stripe line pattern formation. (a) Initial state. (b) Transient state. (c) Stable state.

3.2. Stripe

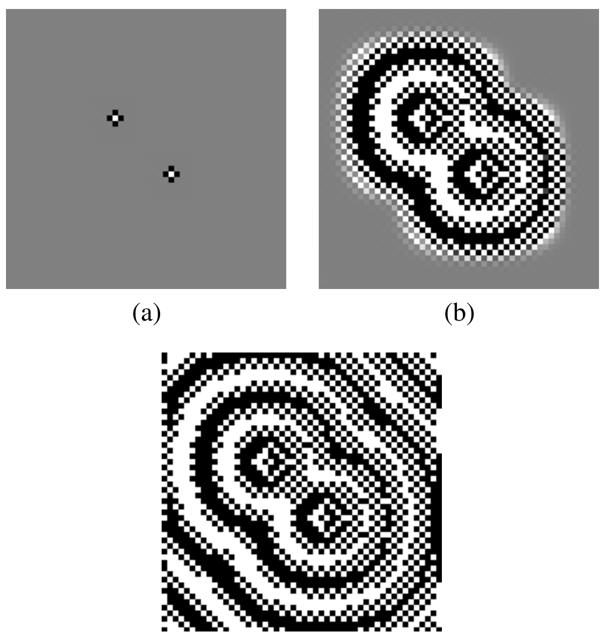

Figure 3: Stripe pattern formation. (a) Initial state. (b) Transient state. (c) Stable state.

Figure 3 shows simulation results using the following template set.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 0.1 & 0 & 0.1 \\ 0 & 2.2 & -1 \\ 0.1 & 0 & 0.1 \end{pmatrix}, \ \boldsymbol{A}_\beta = \begin{pmatrix} 0.01 & 0 & 0.01 \\ 1 & 2.56 & 0 \\ 0.01 & 0 & 0.01 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = 0, \ I_\beta = 0, \tag{8}$$

We can observe a stripe pattern formation. In the case of the two-layer CNN, the boundary between white zone and black zone is very clear. However, the boundary between white zone and black zone is not clear for cells β. The corresponding template set in the two-layer CNN is shown as follows:

$$\boldsymbol{A}_1 = \begin{pmatrix} 0 & 0.1 & 0 \\ 0.1 & 2.2 & 0.1 \\ 0 & 0.1 & 0 \end{pmatrix}, \ \boldsymbol{A}_2 = \begin{pmatrix} 0 & 0.01 & 0 \\ 0.01 & 2.56 & 0.01 \\ 0 & 0.01 & 0 \end{pmatrix},$$

$$\boldsymbol{C}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \boldsymbol{C}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = 0, \ I_\beta = 0. \tag{9}$$

3.3. Active Propagation Phenomena

Figure 4 shows simulation results using the following template set.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 0.1 & 0 & 0.1 \\ 0 & 1.1 & -1 \\ 0.1 & 0 & 0.1 \end{pmatrix}, \ \boldsymbol{A}_\beta = \begin{pmatrix} -0.01 & 0 & -0.01 \\ 1 & 1.04 & 0 \\ -0.01 & 0 & -0.01 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = 0, \ I_\beta = 0, \tag{10}$$

Two concentric circular waves are generated. Their wave fronts propagate in all directions through the network from their initialized positions shown as Figs. 4(a)-(g). When the waves hit the end of CNN, the waves reflect shown as Figs. 4(g)-(l). After that the pattern becomes complex pattern as Figs. 4(m)-(t). In the case of the two-layer CNN, waves do not reflect at the end of CNN. We consider that the difference of the wave reflection comes from the fact that the cell α and the cell β on the edge of the proposed CNN can not be the same as the pairs of non-edge cells. Detailed investigation of the reason is our future research. The corresponding template set in the two layer CNN is shown as follows:

$$\boldsymbol{A}_1 = \begin{pmatrix} 0 & 0.1 & 0 \\ 0.1 & 1.1 & 0.1 \\ 0 & 0.1 & 0 \end{pmatrix}, \ \boldsymbol{A}_2 = \begin{pmatrix} 0 & -0.01 & 0 \\ -0.01 & 1.04 & -0.01 \\ 0 & -0.01 & 0 \end{pmatrix},$$

$$\boldsymbol{C}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ \boldsymbol{C}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \ \boldsymbol{B}_\beta = 0, \ I_\alpha = 0, \ I_\beta = 0. \tag{11}$$

4. Conclusions

We have proposed CNN using two kinds of template sets. In the proposed CNN, we could observe similar phenomena to the two-layer CNN. Investigating more complex phenomena which cannot be seen from the two-layer CNN is our future research.

References

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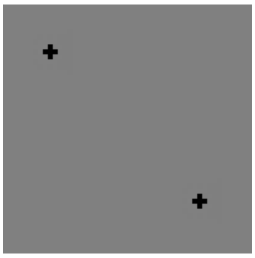
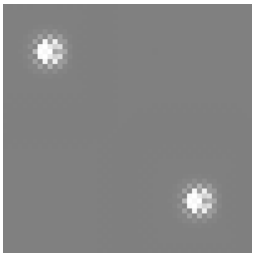
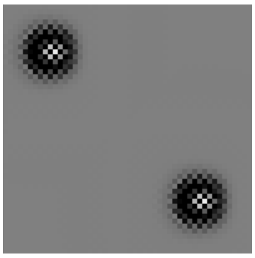
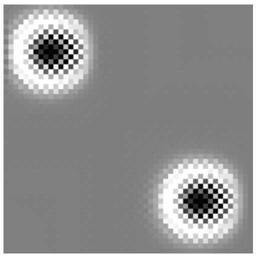
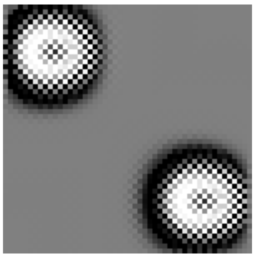
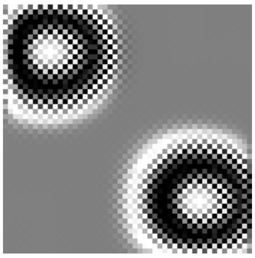

(a) initial state.

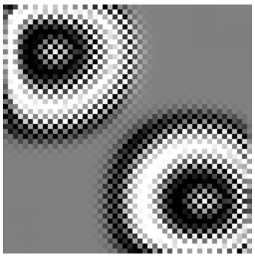

(b) 1000 iteration.

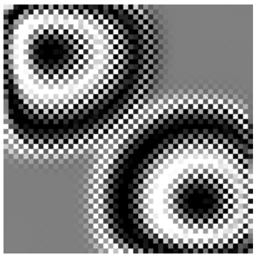

(c) 2000 iteration.

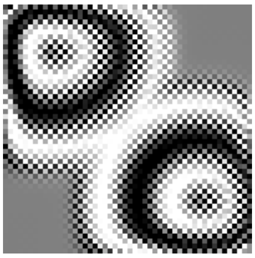

(d) 3000 iteration.

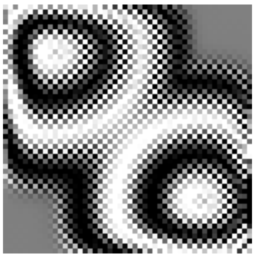

(e) 4000 iteration.

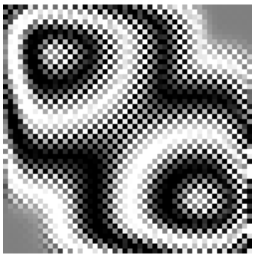

(f) 5000 iteration.

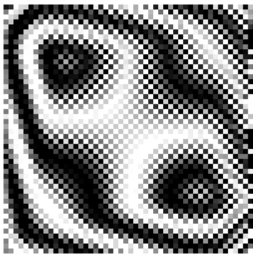

(g) 6000 iteration.


(h) 7000 iteration.

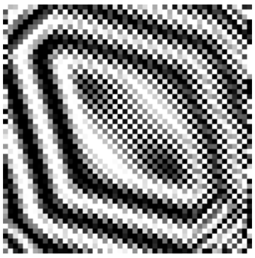

(i) 8000 iteration.

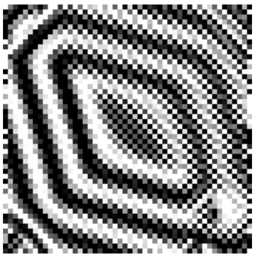

(j) 9000 iteration.

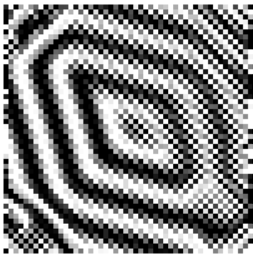

(k) 10000 iteration.

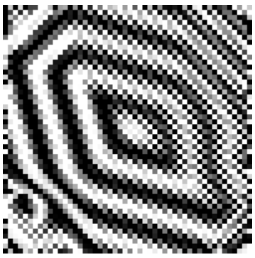

(l) 15000 iteration.

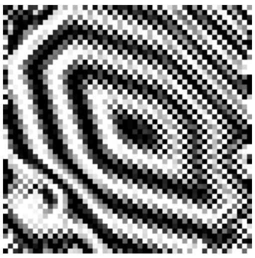

(m) 20000 iteration.

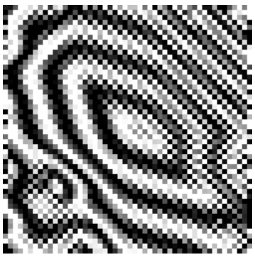

(n) 25000 iteration.


(o) 30000 iteration.


(p) 35000 iteration.


(q) 40000 iteration.


(r) 45000 iteration.


(s) 50000 iteration.

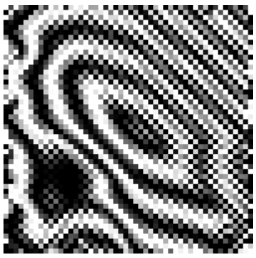

(t) 55000 iteration.

Figure 4: Active Propagation Phenomena. (a) Initial state. (b)-(t) Snapshots obtained at different iterations.