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A Neuron Circuit Model with Smooth Nonlinear Output Function

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Abstract—We propose a novel neuron circuit model. The output function has smooth nonlinear characteristic such as the sigmoid function. It is confirmed that the input-output relation is smooth in the experimental neuron circuit. Moreover, the oscillatory neural network with cyclic coupling of the neuron circuit generates smooth limit cycle.

1. Introduction

To realize electronic circuits of neural networks, it is necessary to express the circuit model of the neuron which is component of the network. Several analog neuron circuits models which used operational amplifiers have been derived [1]. In these circuit models, by using the saturation characteristic and hysteresis characteristic of the operational amplifier, the output function of the neuron is actualized [2, 3]. Because the output function becomes peacewise linear, non-smooth points exist. The trajectory of the neural networks with the neuron circuit models becomes non-smooth curve.

In this paper, we propose a neuron circuit model with a smooth nonlinear output function. The neuron circuit is composed by some operational amplifiers, diodes, resistances, and a capacitance. The opposite multiple connection with two diodes is used in the part of the output function. The output function has smooth nonlinear characteristic such as the sigmoid function. It is expected neural networks with the proposed circuits generate smooth trajectories. Thereby, the neural networks can give smooth control signal for the controled system directly.

2. Circuit Design

2.1. Analog Neuron Model

An analog neuron model is described by

$$\tau_i \frac{dx_i}{dt} = -x_i + \sum_j \omega_{ij} y_j + I_i \tag{1}$$

$$y_i = \sigma(x_i) \tag{2}$$

where x_i, τ_i and I_i are the internal state, the time constant and the external input of i-th neuron, respectively. ω_{ij} is the coupling coefficient from j-th neuron to i-th neuron. σ is an output function. Therefore, y_i is the output state of i-th neuron. Eq.(1) and Eq.(2) are formed by following three parts:

1) Spatial summation characteristic,
2) First order delay characteristic,
3) Output function.

2.2. Spatial summation circuit

The spatial summation characteristic is actualized by an adder circuit of operational amplifiers. Figure 1 illustrates the spatial summation circuit. The input-output relation of the circuit is given by

$$x_i = \frac{R_{if}}{R_{i1}} y_1 + \ldots + \frac{R_{if}}{R_{in}} y_n + \frac{R_{if}}{R_{if}} I_i. \tag{3}$$

We put $\omega_{ij} = R_{if}/R_{ij}$, Eq. 3 can be represented by

$$x_i = \sum \omega_{ij} y_j + I_i. \tag{4}$$

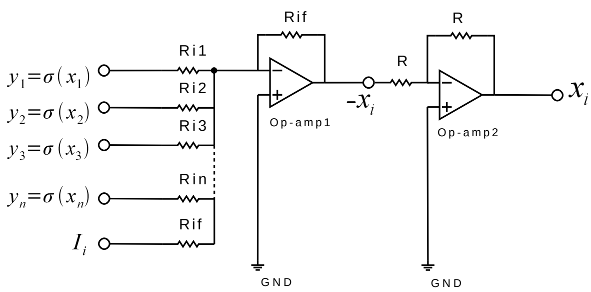


Figure 1: Spatial summation circuit. y_i corresponds the output voltage of other neuron. I_i is the external input. The reversal amplifier circuit by Op-amp 2 makes agree the phase of input and output.

2.3. First order delay circuit

The first order delay characteristic can be actualized the integral circuit with an operational amplifier, a resistance and a capacitance. Figure 2 shows a first order delay circuit. The circuit includes the spatial summation circuit in Fig. 1. The dynamics of the circuit is given by

$$CR_{if} \frac{dx_i}{dt} = -x_i + \sum_j \omega_{ij} y_j + I_i. \tag{5}$$

Eq.(5) equals Eq.(1), if $\tau_i = CR_{if}$.

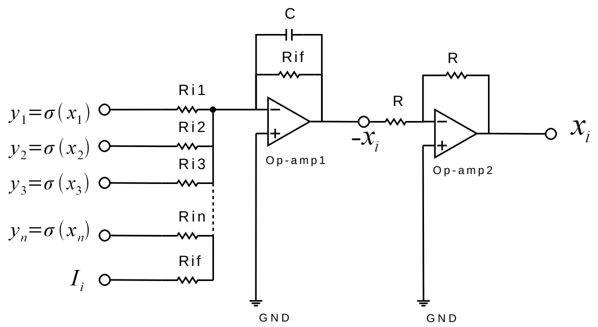


Figure 2: First order delay circuit. The circuit is constructed by Op-amp 1 with a resistance (R_f) and a capacitance (C). The reversal amplifier circuit by Op-amp 2 makes agree the phase of input and output.

2.4. Output function circuit

We realize a smooth nonlinear output function such as the sigmoid function. The sigmoid function has three characteristics: an S-shaped curve, a monotonous increase and a boundedness.

We design the output function circuit using of the characteristic of the diode. Figure 3(a) shows the output function circuit. It is actualized by connecting a resistance (R_f) and two diodes (D_1, D_2) in opposite connection. The reversal amplifier circuit by Op-amp 2 makes agree the phase of input and output. Figure 3(b) shows the input-output relation of the output function circuit calculated by the circuit simulator QUCS. We can observe S-shaped curve and monotonous increase by this figure. But, the boundedness of the output function circuit is not guaranteed exactly.

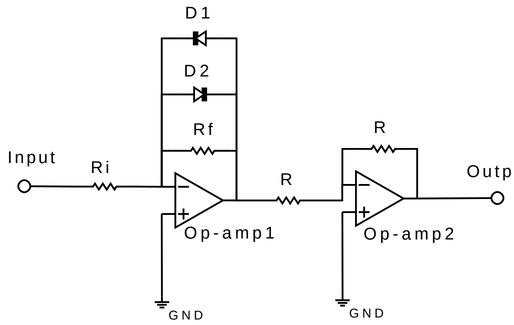
2.5. Analog Neuron Circuit

The proposing analog neuron circuit is constructed with a spatial summation circuit, a first order delay circuit and an output function circuit. The circuit diagram is shown in Figure 4. Figure 4(a) is the excitatory neuron circuit. This type neuron outputs same sign value for the input value. Figure 4(b) is the inhibitory neuron circuit. This type neuron outputs opposite sign value for the input value. The difference between the excitatory neuron circuit and the inhibitory neuron circuit is only reversal or non-reversal amplifier circuits constructed with Op-amp 4 in Figure 4.

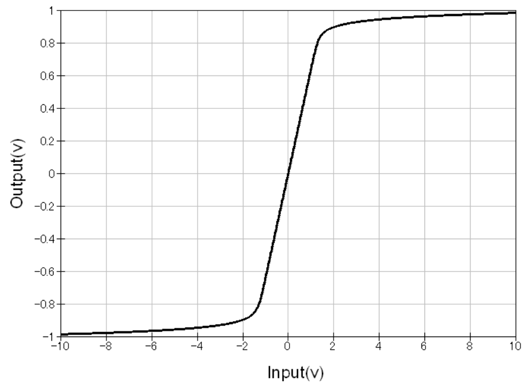
3. Production of Circuit

3.1. Excitatory Neuron Characteristic

We make the experimental circuit of the exitatory neuron model in Figure 4(a). The operational amplifiers and diodes are TL084 and 1S88, respectively. The properties



(a) Circuit diagram.



(b) Input-output relation.

Figure 3: Output function circuit. (a) illustlates circuit diagram and (b) shows the input-output relation of the circuit by circuit simulator.

of a capacitance and resistances in the excitatory neuron is

$$C = 0.1[\mu\text{F}],$$
$$R_1 = R_2 = R_4 = R_5 = R_6 = 1[\text{k}\Omega],$$
$$R_3 = 500[\Omega], R_7 = 3[\text{k}\Omega].$$

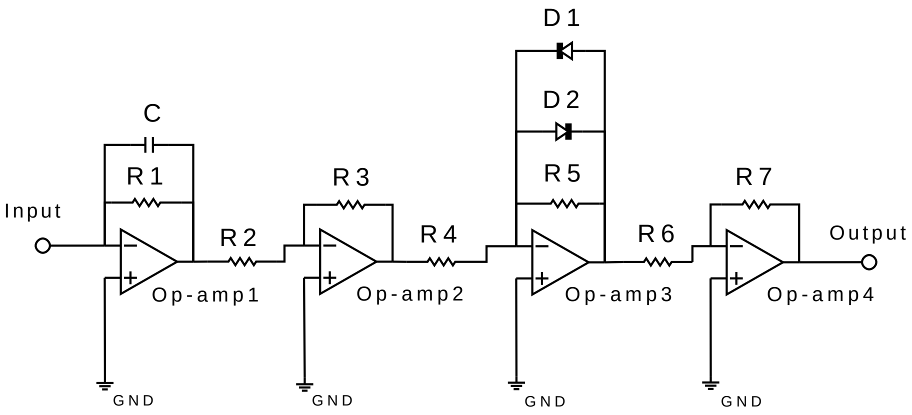
Figure 5(a) is the input-output characteristic of the excitatory neuron circuit. This figure is illustrated by an oscilloscope when sawtooth waveform signal is inputted into the circuit. We can observe smooth S-shaped curve and monotonous increase.

3.2. Inhibitory Neuron Characteristic

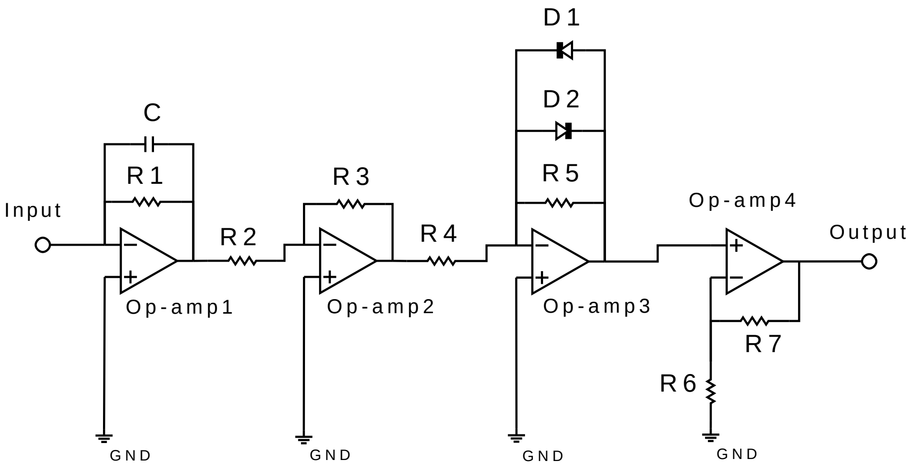
We make the experimental circuit of the inhibitory neuron model in Figure 4(b). The operational amplifiers and diodes are TL084 and 1S88, respectively. The properties of a capacitance and resistances in the excitatory neuron is

$$C = 0.1[\mu\text{F}],$$
$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1[\text{k}\Omega],$$
$$R_7 = 2[\text{k}\Omega].$$

Figure 5(b) is the input-output characteristic of the inhibitory neuron circuit. This figure is illustrated by an oscilloscope when sawtooth waveform signal is inputted into

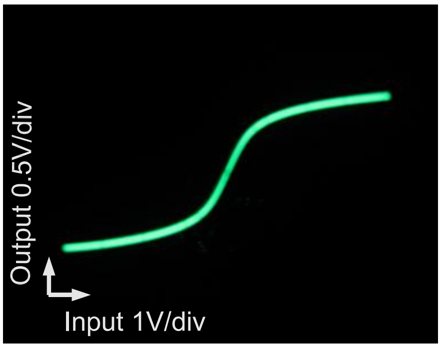


(a) Excitatory neuron circuit.

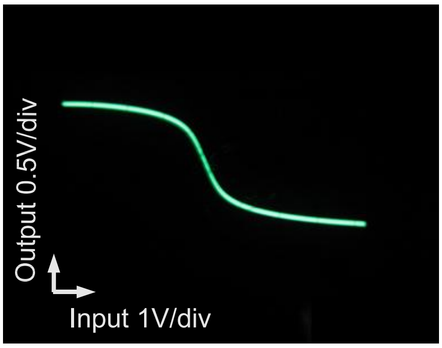


(b) Inhibitory neuron circuit.

Figure 4: Neuron circuit models, (a) is an excitatory neuron circuit and (b) is an inhibitory neuron circuit. The neuron models are constructed with a spatial summation and first order delay circuit of Op-amp 1 and with an output function circuit of Op-amp 3. An Op-amp 4 of excitatory neuron is a reversal amplifier circuit. On the other hand, an Op-amp 4 of inhibitory neuron is a non-reversal amplifier circuit.



(a) Excitatory neuron characteristic



(b) Inhibitory neuron characteristic

Figure 5: The input-output characteristics of experimental circuits. Characteristic of excitatory neuron (a) and characteristic of inhibitory neuron (b) are observed smooth S-shaped curve.

the circuit. We can observe smooth curve and monotonous decrease.

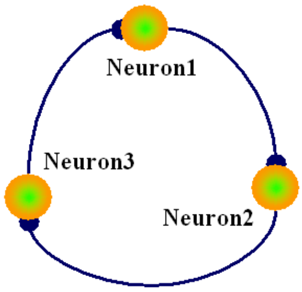


Figure 6: Oscillatory neural network with cyclic coupled three inhibitory neurons. The network is constructed with the inhibitory neuron circuit of Fig. 4(b).

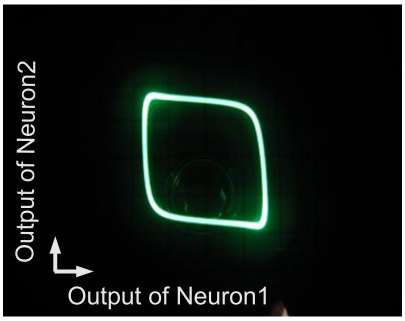
4. Neural network with neuron circuits

We confirm that neural networks can be constructed by the proposing neuron circuits. The neural network with cyclic coupling neurons is known as simple oscillatory system [4, 5]. Figure 6 illustrates the neural network with cyclic coupling neurons. The network is constructed with the inhibitory neuron circuit of Fig. 4(b). Figure 7 shows the experimental results of oscillatory neural network. Figure 7(a) illustrates the phase portrait in the output plane of Neuron 1 and Neuron 2.The smooth limit cycle is observed. Figure 7(b) shows the waveform in the output of Neuron 1. It is confirmed that the trajectory of neural network with the neuron circuit model becomes smooth curve.

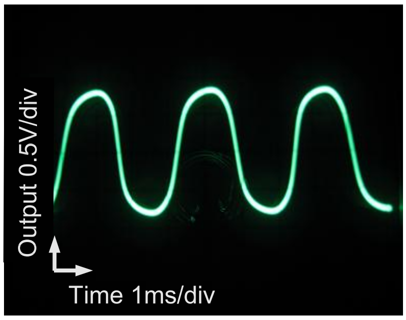
5. Conclusion

We proposed a novel neuron circuit model with a smooth nonlinear output function. The neuron circuit is composed by four operational amplifiers, two diodes, seven resistances, and a capacitance. The characteristics of the proposed model are confirmed to make the experimental circuit. The experimental result shows that the neuron circuit has a smooth non-linear function. Moreover, the smooth oscillatory state is observed in the network with the neuron circuit.

The future problems are to construct the several neural networks with the proposed neuron circuit and to analyze the networks. Also, it is known that a sigmoid characteristic circuit is constituted by FET [6]. We will propose the neuron circuit model with FET.



(a) Phase portrait



(b) Waveform

Figure 7: The experimental results of the oscillatory neural network. (a) is the phase portrait in the output plane of Neuron 1 and Neuron 2. (b) is the waveform in the output of Neuron 1.

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