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Numerical solutions of non-linear integral-differential equations for ferromagnetism

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In 1935 Landau first described a magnetization pattern in a single crystal of a ferromagnetic body. Since then the understanding of this complex problem has increased with increases in computing power. The difficulties are most evident in treating the long-range dipole-dipole interactions for which the energy is a six-fold integral over the ferromagnetic body. The dipole-dipole interaction acts to suppress divergences of the magnetization $\boldsymbol{M} = M_s\boldsymbol{m}$, where M_s is the fixed magnitude of the magnetization and $\boldsymbol{m}$ is a unit vector. Non-linearities arise necessarily because the three components of the magnetization are constrained by the unit vector. Divergences of M can appear internally as magnetic volume charge and at the interfaces as magnetic surface charge. Primarily it is the fields from the surface charge that drive the magnetic configurations into non-uniform patterns at the expense of all other energies that would be minimized by a uniform magnetization in a preferred direction. If a magnetic field H_z is applied along the preferred $\boldsymbol{z}$ direction, the Zeeman energy density $-m_z M_s H_z$, the anisotropy energy density $-Km_z^2$ and the exchange energy density $A(\mathrm{grad}\,\boldsymbol{m})^2$ are all minimized by $\boldsymbol{m} = \hat{\boldsymbol{z}}$, but in a finite singly-connected body the dipole-dipole interaction is not minimized. In the doubly-connected geometry of a toroidal shell, the dipole-dipole energy can be taken into account fully by imposing the absence of magnetic charge as a constraint. The micromagnetic integral-differential equations of Landau and Lifshitz can then be solved analytically and serve as a check on computer simulations. The simplest singly connected body is an ellipsoid for which the magnetostatic energies of the fully magnetized body have been known for more than a century. The processes by which magnetization patterns in ellipsoids changes under the influence of applied magnetic fields are revealed by solving the Landau-Lifshitz equations numerically for submicron-sized bodies. The ellipsoid supports magnetic configurations in which a vortex-like structure undergoes transformations in response to applied fields. Analytical descriptions of the gyrations and transformations of these structures remain a mathematical challenge. There are even more severe challenges in computing for the mostly neglected problem of including the magnetoelastic properties of ferromagnetic bodies. The talk, using mostly pictures, will touch on the properties of toroidal shells, general ellipsoids, and a problem where magnetoelasticity is essential to understanding of magnetization processes.