A Nyström-Based Esprit Algorithm for DOA Estimation of Coherent Signals

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***Abstract*—Sample covariance matrix (SCM) and its eigenvalue decomposition (EVD) are needed in the conventional subspace based algorithm. Under normal circumstances, the computation burden is really heavy, especially for the estimation of the coherent signals which also needs to use the spatial smooth technology. In this paper, we firstly use the Nyström-Based method to obtain the signal subspace, which results in reducing the computational complexity greatly. To further reduction of the computational complexity under the completely coherent condition, we use an improved SVD method to calculate the equivalent forward and backward covariance matrices. Then we use the ESPRIT method to estimate coherent signals' direction of arrival (DOA), and we also compare the performance among the Nyström-Based algorithm, the SCM-EVD method and the classical space smooth algorithm. The simulation results show that this new method achieves almost the same accuracy with SCM-EVD based method and classical spatial smooth method with a much less computation burden.**

Keywords - Nyström; ESPRIT; coherent; DOA estimation

I. Introduction

Subspace based algorithm has been widely used for DOA estimation. It is well known that the subspace based method which depends on the decomposition of the observation space into signal subspace and noise subspace can provide high resolution DOA estimation with good accuracy. While the classical subspace based methods involve the estimation of the covariance matrix and eigen-decomposition, such as the MUSIC (Multiple Signal Classification)[1][2] and ESPRIT (Estimation Signal Parameter via Rotational Invariance Techniques) [3][4]. As a result, the classical subspace based methods are rather computationally expensive, especially for the case that the model's order number in those matrices is large. Recently, the method based on the Nyström Algorithm [5]-[7] simplifies the computation complexity, which use the block matrix method to find the signal subspace without the computation of SCM and its EVD. Coherent signals environment is common in reality, such as multi-path effect. The coherent signals make the rank of array spatial covariance matrix a loss and causes ineffectiveness in some super-resolution subspace algorithms such as MUSIC and ESPRIT. To deal with the coherent signals, expecially completely coherent signals [4], we need to do de-coherent processing to get the full rank sample covariance matrix. There are many methods to realize. For instance, forward spatial smoothing algorithm [3], backward spatial smoothing method. The improved SVD(singular value de-composition) is used for coherent signals to estimate the DOA [4], but it needs to do the spatial smoothing processing to calculate the covariance matrix. In this paper, we account for the complete coherent case, combining the Nyström-Based Algorithm with the improved SVD algorithm. In this way. it can greatly reduce the computational complexity. In addition, we have also compared it with the classical spatial smooth method and the SCM-EVD method[4]. The simulation results show that the method presented by this paper achieves almost the same accuracy with the SCM-EVD method and a little suboptimal to the classical spatial smooth methed.but it has less computation burden.

II. Problem Formulation

A. Data Model

Let us consider an uniform linear array (ULA) composed of M isotropic sensors. Assume that P $\left(P<M\right)$ narrow-band signals impinge upon the ULA from distinct direction $\theta_1,\theta_2 \cdots \theta_P$, The $M\times 1$ output vector of the array, which is corrupted by additive noise, at the k th snapshot can be expressed as

$$\mathbf{x}\left(k\right)=\sum_{i=1}^{P}\mathbf{a}\left(\theta_i\right)*s_i\left(k\right)+\mathbf{n}\left(k\right)\quad k=0,\cdots,N-1 \tag{1}$$

Where $s_i\left(k\right)$ is the scalar complex waveform referred to as the i th signal, $\mathbf{n}\left(k\right)\in\mathbb{C}^{M\times 1}$ is the complex noise vector. N and P denote the number of snapshots and the number of signals respectively. $\mathbf{a}\left(\theta_i\right)$ is the steering vector of the array toward direction θ_i and take the following form

$$\mathbf{a}\left(\theta_i\right)=\frac{1}{\sqrt{M}}\left[1,e^{j\varphi_i},\cdots,e^{j(M-1)\varphi_i}\right]^T \tag{2}$$

where $\varphi_i=(2\pi d\sin\theta_i)/\lambda$ in which $\theta_i\in\left(-\pi/2,\pi/2\right)$, d is inter-element spacing and λ is the wavelength. In matrix form, the received data becomes

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$$\mathbf{x}(k)=\mathbf{A}(\theta)\mathbf{s}(k)+\mathbf{n}(k) \quad k=0,1\cdots,N-1 \tag{3}$$

Where

$$\mathbf{A}(\theta)=\left[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_p)\right] \tag{4}$$

$$\mathbf{s}(k)=\left[s_1(k),s_2(k),\cdots,s_p(k)\right] \tag{5}$$

are the $M\times P$ steering matrix and the $P\times 1$ complex signal vector, respectively. Furthermore, the background noise uncorrelated with the signal is modeled as a zero-mean Gaussian complex random process , which is both stationary spatially and temporally.

Then the sample covariance matrix can be calculated as

$$\mathbf{R_X}=E[\mathbf{XX}^H] \tag{6}$$

Where $\mathbf{X}=\left[\mathbf{x}(1),\mathbf{x}(2),\cdots,\mathbf{x}(N)\right]$ is a $M\times N$ data matrix, and the superscript $(\bullet)^T$ and $(\bullet)^H$ denote the transpose and conjugate transpose, respectively.

B. Nyström Method For Matrix Approximation

As we know the $\mathbf{X}$ is a $M\times N$, we can divide the data into following form

$$\mathbf{X}=\begin{bmatrix}\mathbf{X}_1\\ \mathbf{X}_2\end{bmatrix} \tag{7}$$

$\mathbf{X}_1$ is a $P\times N$ matrix. then the covariance matrix can be partitioned as

$$\mathbf{R_X}=\begin{bmatrix}\mathbf{R_{11}} & \mathbf{R_{12}}\\ \mathbf{R_{21}} & \mathbf{R_{22}}\end{bmatrix} \tag{8}$$

where $\mathbf{R_{11}}=E\left[\mathbf{X_1X_1^H}\right],\mathbf{R_{12}}=E\left[\mathbf{X_1X_2^H}\right],\mathbf{R_{21}}=E\left[\mathbf{X_2X_1^H}\right],\mathbf{R_{22}}=E\left[\mathbf{X_2X_2^H}\right]$. Since the covariance matrix is a symmetric matrix, we approximate the covariance matrix by Nyström Method[6][7]. Then we obtain

$$\mathbf{R_{NCE}}=\begin{bmatrix}\mathbf{R_{11}} & \mathbf{R_{12}}\\ \mathbf{R_{12}^H} & \mathbf{R_{12}^H R_{11}^{-1} R_{12}}\end{bmatrix} \tag{9}$$

the matrix $\mathbf{R_{NCE}}$ is the approximation of the covariance $\mathbf{R_X}$,and we can use the $\mathbf{R_{NCE}}$ to estimate the signal subspace. Here we give the result directly

$$\mathbf{R_{NCE}}=\mathbf{U_S \Lambda U_S^H} \tag{10}$$

Where

$$\mathbf{U_S}=\mathbf{FU_F\Lambda_F^{-1/2}} \tag{11}$$

and

$$\mathbf{F}=\begin{bmatrix}\mathbf{R_{11}}\\ \mathbf{R_{12}^H}\end{bmatrix}\mathbf{R_{11}^{-1/2}} \tag{12}$$

$$\mathbf{F^H F}=\mathbf{U_F\Lambda_F U_F^H} \tag{13}$$

The Nyström-based method avoids the calculation of the SCM and its SVD. In particular, it only needs to compute $\mathbf{R_{11}}$ and $\mathbf{R_{12}}$. since the estimation accuracy and computation complicity has been evaluated in [7]. The computational complexity of SCM-SVD method is $o\left(M^3\right)+o\left(M^2N\right)$, but the computational complexity of the proposed Nystrom method is $o\left(P^2N\right)+o\left((M-P)NP\right)+o\left(MP^2\right)$. Usually the number of sensors and the number of signals is very small. So by this method we can reduce much complexity, here we give out the MUSIC spectrum based on this method for DOA estimation Fig 1.

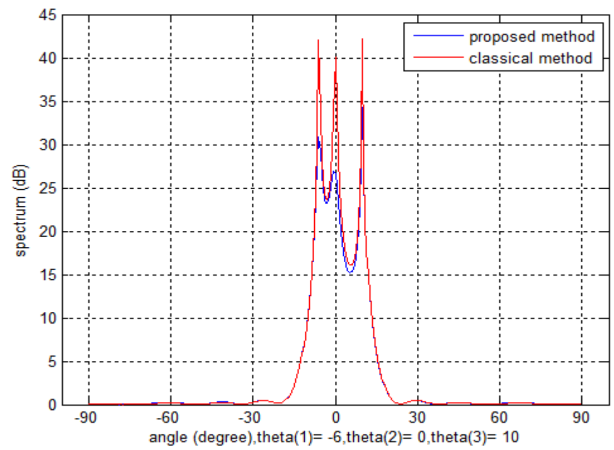

Fig.1 MUSIC based on the Nyström Method and the classical method

In the simulation the SNR(signal to noise ratio) is 20dB, from the Fig 1 we see that Nyström-based method almost has the same performance as the classical SCM-EVD based method.

III. Completely Coherent Signals Model and Improved SVD Algorithm

When the sources are completely coherent, namely the sources differ only in complex constant, we can express it as

$$s_i(t)=\alpha_i s_1(t) \qquad i=2,3\cdots P \tag{14}$$

The rank of sources covariance matrix is one. After the eigenvalue decomposition of $\mathbf{R_X}$, the dimension of the signal subspace is less than the rank of the array manifold $\mathbf{A}(\boldsymbol{\theta})$, which leads to the consequence that the steering vector is no longer orthogonal to the noise subspace and makes failure of subspace algorithm. Recently a method based on the eigenvector of the maximum eigenvalue [4] has been represented. We assume the noise is temporal white and the noise covariance matrix $\mathbf{R_N}$ is full-rank. As the steering vectors span the signal subspace, so we have linear representation form as:

$$\mathbf{R_N e_k}=\sum_{n=1}^{P}\alpha_k(n)\mathbf{a}(\theta_n) \tag{15}$$

Where $\mathbf{e_k}\ (1\le k\le K)$ is an eigenvector of the received signal covariance matrix (the first K eigenvectors corresponding to first K eigenvalue in decreasing order), $\alpha_k(n)$ is a linear combination factor. When the noise covariance matrix is an identity matrix, the above equation (15) is simplified as:

$$\mathbf{e}_k = \sum_{n=1}^{P} \alpha_k(n)\mathbf{a}(\theta_n), \quad 1 \le k \le K \tag{16}$$

For completely coherent case, namely $K = 1$, the above equation is reduced to:

$$\mathbf{e}_1 = \sum_{n=1}^{P} \alpha_1(n)\mathbf{a}(\theta_n) \tag{17}$$

It indicates that the largest eigenvector of the largest eigenvalue contains all the signal information. So the vector $\mathbf{e}_1$ can be used to reconstruct a equivalence covariance matrix $\mathbf{Y}$, it can be constructed as:

$$\mathbf{Y}_f = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1g} \\ e_{12} & e_{13} & \cdots & e_{1,g+1} \\ \vdots & \vdots & \ddots & \vdots \\ e_{1m} & e_{1m+1} & \cdots & e_{1M} \end{bmatrix} \tag{18}$$

Where $m = M - g + 1, m > P, g > P$.

As we known, the performance of the forward and backward method is more better than the only forward method, which can also be used to solve the coherent situation. So in this paper we also use the similar way, the backward matrix is :

$$\mathbf{Y}_b = \mathbf{J}_m \mathbf{Y}^* \mathbf{J}_g \tag{19}$$

Where $\mathbf{J}_m$ (m -dimension) and $\mathbf{J}_g$ (g -dimension) are anti-diagonal eye matrices, $(\bullet)^*$ denotes the conjugate operation, then we obtain the equivalence covariance matrix:

$$\mathbf{Y} = \frac{1}{2}[\mathbf{Y}_f + \mathbf{Y}_b] \tag{20}$$

Then we do SVD of the factorization of $\mathbf{Y}$ to obtain the signal subspace. And we can use the ESPRIT algorithm to estimate the DOA of the sources as follows:

$$\mathbf{Y}^H\mathbf{Y} = \mathbf{U}\Lambda\mathbf{U} \tag{21}$$

Where

$$\mathbf{U}_1 = [eye(g-1), zeros((g-1)\times 1)] * \mathbf{U}_s \tag{22}$$

$$\mathbf{U}_2 = [zeros(g-1)\times 1, eye(g-1))] * \mathbf{U}_s \tag{23}$$

$\mathbf{U}_s$ is the P largest eigenvector of $\mathbf{U}$. we can get the DOA (θ_k) of the signals by solving following equation

$$\mathbf{\Phi} = \left(\mathbf{U}_1^\dagger\right)\mathbf{U}_2 = \mathbf{H}^{-1}\mathbf{\Psi}\mathbf{H} \tag{24}$$

$$\mathbf{\Psi} = diag(e^{j\varphi_1}, e^{j\varphi_2} \cdots, e^{j\varphi_P}) \tag{25}$$

$$\varphi_k = (2\pi d / \lambda)\sin\theta_k \tag{26}$$

Where $(\bullet)^\dagger$ denotes the pseudo-inverse,and $\mathbf{H}$ is a full rank matrix . In this way, when we do the de-coherent of the signals we don't need to do the spatial processes, which is more computation costly.

IV. SIMULATION RESULTS

In this section, the performance of the proposed method is evaluated by computer simulation. For comparison purpose , we also do the simulation of the spatial method ,which is based on the spatial smooth technology, firstly ,we calculate the covariance matrix ,then do its EVD decomposition ,after this, we apply ESPRIT algorithm to estimate the DOA. Another method ,which is called SCM-EVD method[4] is also performed there. For simplicity, the array herein is assumed to be an ULA with nine isotropic sensors whose spacing equals half-wavelength. Suppose that there are three completely coherent signals with equal power impinging upon the ULA and the true DOAs are $\{-20^\circ, 0^\circ, 40^\circ\}$. We assumption the background noise is a zero mean, white Gauss stationary process. In Fig.2 the snapshot is 200, and the number of Monte Carlo trials is 500, RMSE is calculated an follows:

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(\hat{\theta}_i - \theta\right)^2} \tag{27}$$

Where T is the number of trials, and $\hat{\theta}_i$ is the estimated angle, θ_i is the real angle

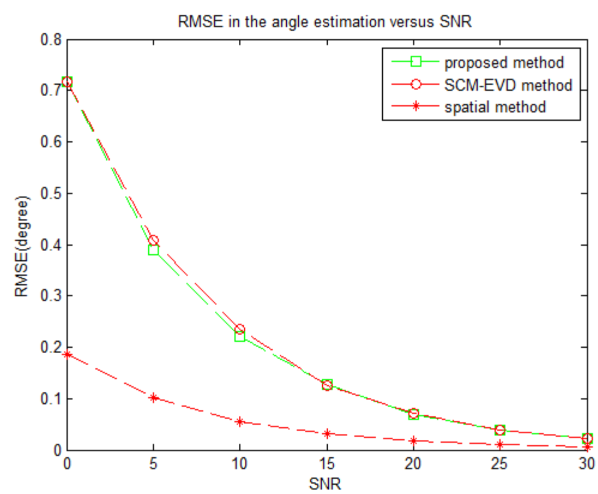

Fig. 2 RMSE in the angle estimation versus SNR of the three methods

we can observe that the RMSE of the proposed method is very closed to the SCM-EVD method, the curves of the two are almost coincide to each other, but both of them are a little biger than the spatial method. What's more, they all have the same trend, as the SNR increase the RMSE of the three methods are both decreased. To some extend, we can see that the proposed method achieved the computation simplicity with the sacrifice of its performance, compared with the SCM-EVD method, we have the same performace but with much less computation burden.

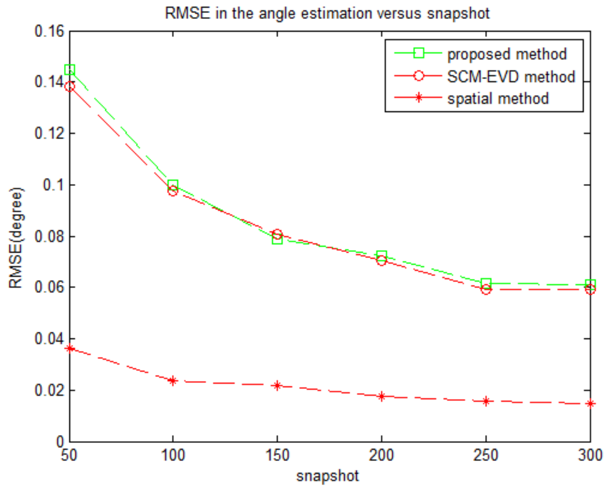

Fig. 3 RMSE in the angle estimation versus snapshot of the three methods

In Fig.3 the SNR is 20dB and the number of Monte Carlo trials is 500. From Fig.3 we can obtain the same conclusion as from the Fig.2. The reason why the proposed method have poor performance than the spatial method is that the Nyström-Based Esprit Algorithm noly use the $\mathbf{R}_{11}$ and $\mathbf{R}_{12}$ to calculate the signal subspace, while the spatial method have used all the sample data to calculate the signal subspace, and in this way its more closer to the real situation.

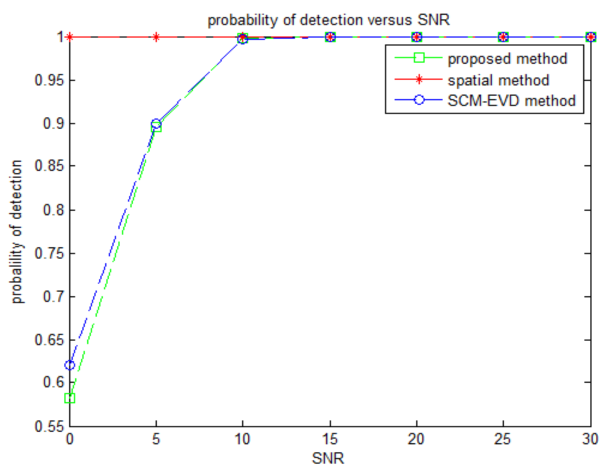

Fig. 4 probability of detection the three methods versus SNR

In Fig.4 the snapshot is 200 and the number of Monte Carlo trials is 500. From Fig.4 we can obtain the same conclusion as from Fig.2.and Fig.3. the probability of detection of the proposed method and the SCM-EVD method almost coincide. The detection probability of the three increase with the SNR, When the SNR is exceed 10 dB, the detection pabability is almost one.

V. CONCLUSION

In this paper, we use the Nyström-Based ESPRIT Algorithm for DOA Estimation of completely coherent signals. The total computation complexity of the proposed method is $o\left((M-P)NP+NP^2+MP^2\right)+o\left(m^2g+mg^2+g^3\right)+o\left(P^3\right)$. While the total computation complexity of the SCM-EVD method is $o\left(MN^2\right)+o\left(M^3\right)+o\left(m^2g+mg^2+g^3\right)+o\left(P^3\right)$, while the spatial method is $o(P-1)N(M-P+1)^2)+o\left((M-P+1)^3\right)+o\left(P^3\right)$. Generally, the snapshot is very large, but the sensor number and the number of signals are not large. Then we can see that the proposed method can greatly reduce the complexity. From the simulation results, we can obtain that the proposed method has strong de-correlation ability and high estimation accuracy. In addition, the performance of the proposed method almost achieves the same performance with the SCM-EVD method and the classical spatial smooth method ,but wih less computation burden.

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