Array Configuration Conversion Method Using CSS Processing for DOA Estimation

Takuya Ohara Nobuyoshi Kikuma Hiroshi Hirayama Kunio Sakakibara
Department of Computer Science and Engineering,
Nagoya Institute of Technology, Nagoya, 466-8555, Japan
E-mail: kikuma@m.ieice.org

1. Introduction

Recent development of wireless communication and radar systems is remarkable. On the other hand, miscellaneous radio waves make our radio environments much complicated. Therefore, it is important to understand appropriately the radio wave propagation structures so as to attain high performance of the wireless systems. For the purpose, it is effective to estimate DOAs (directions of arrival) of individual incoming waves with array antennas [1]. As the DOA estimation algorithms, MUSIC and ESPRIT [1] receives much attention because of their high resolution and high estimation accuracy. In addition, non-searching algorithms such as Root-MUSIC [1] and ESPRIT are computationally efficient although they require a uniform linear or rectangular array. Therefore, when we use the estimation approach of non-searching type for an arbitrary or irregular array, we need the conversion processing of the received signal from the real array to a virtual uniform linear array as an example [2].

In this paper, we propose the array configuration conversion method using CSS (Coherent Signal Subspace) processing [3], and we show through computer simulation that this conversion method performs well in the DOA estimation using the circular array.

2. Signal Model and Array Configuration Conversion Method

2.1 Signal Model

Consider that the array used for DOA estimation is a K-element array shown in Fig.1, and also that it receives L ($L < K$) narrow-band waves whose respective DOAs are $\theta_1, \theta_2, \cdots, \theta_L$ and complex amplitudes are $s_1(t), s_2(t), \cdots, s_L(t)$. When the array response vector $\boldsymbol{x}(t)$ of the lth incoming wave is given by $\boldsymbol{a}(\theta_l)$ ($l = 1, 2, \cdots, L$), the array input vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l)\, s_l(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_L)]\ ,\ \ \boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \tag{2}$$

where $\boldsymbol{A}$ is the array response matrix, $\boldsymbol{s}(t)$ is the signal vector, and $\boldsymbol{n}(t)$ is the internal additive noise vector in receiving all incoming waves.

2.2 Array Configuration Conversion Method Using CSS Processing

To carry out the configuration conversion for the K-element array, we receive individually N narrow-band reference signals with DOAs: $\hat{\theta}_1, \cdots, \hat{\theta}_N$ being known. Then, each of the array input vectors is expressed as

$$\boldsymbol{x}_i(t) = \boldsymbol{a}(\hat{\theta}_i)\hat{s}_i(t) + \boldsymbol{n}_i(t) \qquad (i = 1, 2, \cdots, N) \tag{3}$$

where $\boldsymbol{a}(\hat{\theta}_i)$ and $\hat{s}_i(t)$ are the array response vector and the complex amplitude of the ith reference signal, respectively. Also, $\boldsymbol{n}_i(t)$ is the internal additive noise vector in receiving the ith reference signal.

Now, we define $\overline{\boldsymbol{a}}(\hat{\theta}_i)$ as the array response vector of virtual uniform linear array (broadside array) and $\boldsymbol{a}_e(\hat{\theta}_i)$ as the normalized eigenvector corresponding to the dominant eigenvalue of the covariance matrix of $\boldsymbol{x}_i(t)$. Between $\overline{\boldsymbol{a}}(\hat{\theta}_i)$ and $\boldsymbol{a}_e(\hat{\theta}_i)$, there is the relation given by

$$\overline{\boldsymbol{a}}(\hat{\theta}_i) = \boldsymbol{T}c_i\boldsymbol{a}_e(\hat{\theta}_i) \qquad (c_i : \text{complex constant}) \tag{4}$$

where $\boldsymbol{T}$ is the conversion matrix from the real array to the virtual uniform linear array. Estimating the conversion matrix $\boldsymbol{T}$ is the purpose of this configuration conversion method.

CSS processing requires a preliminary estimation of the DOAs. Here, we let θ_n represent the preliminary estimated angle of a certain incoming wave, $\hat{\theta}_i$ and $\hat{\theta}_{i+1}$ the angles of two adjacent reference signals which are in the directions closest to θ_n, and $a_{ec}(\hat{\theta}_i)$ the component that is in the phase center of $\boldsymbol{a}_e(\hat{\theta}_i)$. Using the following matrices:

$$\overline{\boldsymbol{A}} = [\,\overline{\boldsymbol{a}}(\hat{\theta}_i), \overline{\boldsymbol{a}}(\hat{\theta}_{i+1})]\ ,\ \boldsymbol{A}_e = [\boldsymbol{a}_e(\hat{\theta}_i), \boldsymbol{a}_e(\hat{\theta}_{i+1})]\ ,\ \boldsymbol{\Lambda} = \mathrm{diag}\{1/a_{ec}(\hat{\theta}_i), 1/a_{ec}(\hat{\theta}_{i+1})\} \quad (5)$$

we make the cost function J based on Eq.(4) and CSS, which is given by

$$J = \|\overline{\boldsymbol{A}} - \boldsymbol{T}\boldsymbol{A}_e\boldsymbol{\Lambda}\|_F \quad \text{subject to} \quad \boldsymbol{T}^H\boldsymbol{T} = \boldsymbol{I} \quad (6)$$

By minimizing J with respect to $\boldsymbol{T}$, the conversion matrix $\boldsymbol{T}$ can be obtained in the following equation

$$\boldsymbol{T} = \boldsymbol{V}\boldsymbol{U}^H \quad (7)$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are the $K \times K$ matrices, and their columns are composed of left and right singular vectors of $\boldsymbol{A}_e\boldsymbol{\Lambda}\overline{\boldsymbol{A}}^H$, respectively [3].

When the number of waves is two or more, both the estimation of $\boldsymbol{T}$ and the DOA estimation are repeated for individual of all incoming waves.

2.3 Array Configuration Conversion Method Using CSS-sp Processing

The normalized eigenvectors $\boldsymbol{a}_e(\theta)$ for θ other than $\hat{\theta}_i$ $(i = 1, 2, \cdots, N)$ can be obtained by applying spline interpolation to $\boldsymbol{a}(\hat{\theta}_i)$. Using the interpolated angles and the following matrices:

$$\overline{\boldsymbol{A}} = [\,\overline{\boldsymbol{a}}(\theta_n - \Delta), \overline{\boldsymbol{a}}(\theta_n + \Delta)]\ ,\ \boldsymbol{A}_e = [\boldsymbol{a}_e(\theta_n - \Delta), \boldsymbol{a}_e(\theta_n + \Delta)]\ ,\ \boldsymbol{\Lambda} = \mathrm{diag}\{1/a_{ec}(\theta_n - \Delta), 1/a_{ec}(\theta_n + \Delta)\} \quad (8)$$

we make the cost function J in the same manner as Eq.(6) where Δ is an angular interval. Then, the conversion matrix $\boldsymbol{T}$ is obtained by minimizing the cost function J. We call the method CSS-sp.

3. Performance Analysis by Computer Simulation

Under conditions shown in Fig.2 and Tables 1–4, the computer simulation is carried out to clarify the performance of the proposed method. The angular interval Δ for CSS-sp processing is 1deg. In the simulation, the initial DOAs are estimated at first, and after the array conversion processing of received data, the final DOA estimates are obtained by using Root-MUSIC method. The performance is evaluated in terms of RMSE (Root Mean Square Error) computed from independent trials of 100.

Table 1: Simulation conditions.

real array configuration	5-element uniform circular array
real array radius	one wavelength
virtual array configuration	5-element uniform linear array
virtual element spacing	half wavelength
antenna element	isotropic
number of snapshots for DOA estimation	50

Table 2: Radio environment 1.

number of waves	1
DOA	$\theta = -180$ deg to 180 deg
SNR	20dB

Table 3: Radio environment 2.

number of waves	2 (uncorrelated, equal power)
DOA of the 1st wave	$\theta_1 = -180$ deg to 150 deg
DOA of the 2nd wave	$\theta_2 = \theta_1 + 30$ deg
SNR	20dB

Table 4: Setup of reference signals.

number of reference signals	37
DOA	$-180, -170, \cdots, 170, 180$ [deg]
SNR	30dB
number of snapshots	100

First, the performances of the proposed conversion methods are examined when the number of waves is one (Table 2). Beamformer method [1] is used for the the preliminary (initial) estimation. The results of estimation are shown in Fig.3 along with Cramer-Rao bound (CRB) [4]. In this figure, the result of using the Rectangular Weighting See method [5] with rectangle width of 7 and lower level α of 10^{-5} is also shown for comparison. From the figure, it is found that three conversion methods perform well for all DOAs and also that the method using CSS-sp is the most effective.

Secondly, the aperture length and element spacing of real circular array to which CSS processing is applied are examined. In this simulation, the aperture length and element spacing of the circular array are optimized to give the minimum average of RMSE over all DOAs. The results are shown in Fig.4. In addition, the resultant averages of RMSE are depicted in Fig.5. From the figures, it follows that as the number of elements increases the difference between apertures of the real circular array and the virtual linear array becomes large, resulting in large conversion error. Also, even if the number of elements increases, the optimum element spacing of the real circular array are approximately one wavelength.

Finally, the performances of the proposed conversion methods are examined when the number of waves is two (Table 3). MUSIC method [1] is used for the preliminary estimation in this case. The results of estimation are shown in Fig.6, which demonstrates that CSS-sp provides the best performance.

4. Conclusion

In this paper, we have proposed and discussed the array configuration conversion method using CSS processing for DOA estimation. Since this method transforms the arbitrary array to the uniform linear array, we can use the non-searching DOA estimators such as Root-MUSIC. Through computer simulation, we have shown the effectiveness of proposed method in employing a circular array as the real array. Especially, it is demonstrated that the proposed method using spline interpolation together with CSS processing (CSS-sp) performs much effectively. Furthermore, it is confirmed that the difference between the apertures of the optimum real circular array and the virtual linear array grows and so the conversion error becomes large as the number of elements of the circular array increases. As the future work, we will make a more detailed guideline for the optimum conversion matrix and evaluate the performance of the proposed conversion method by experiments.

References

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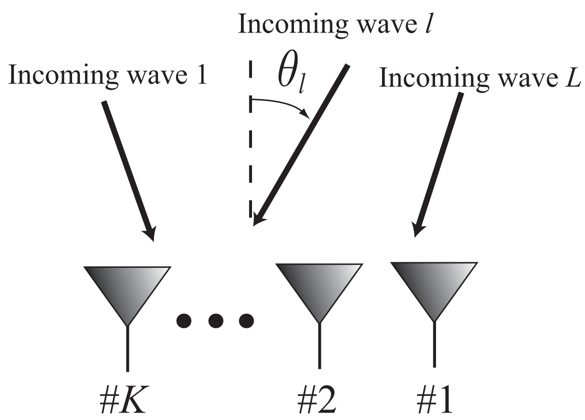

Figure 1: K-element array.

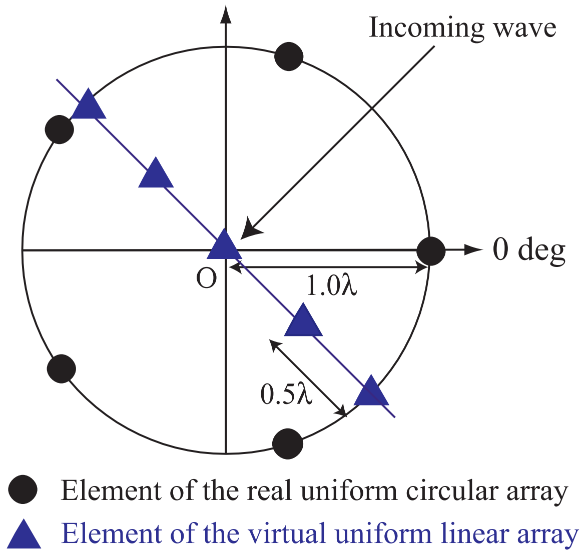

Figure 2: Array configuration.

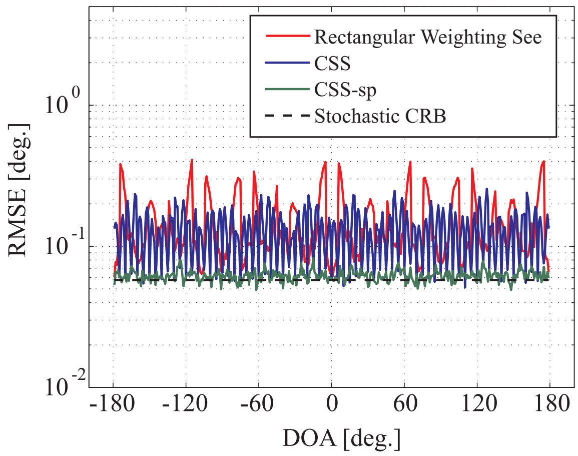

Figure 3: DOA estimation results when the number of waves is one.

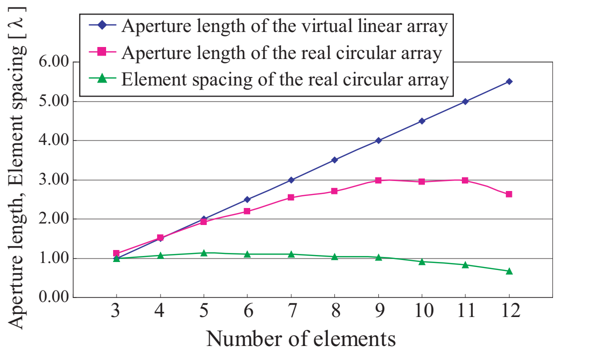

Figure 4: Aperture length and element spacing of the circular array as a function of number of elements in the case of using CSS processing.

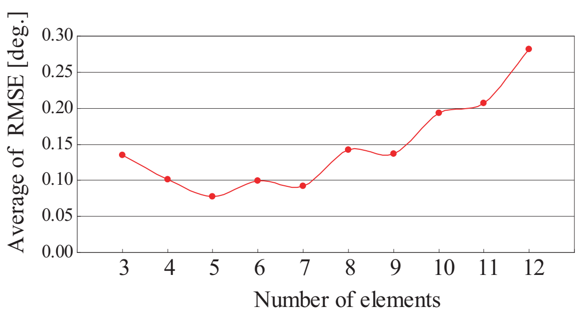

Figure 5: Averages of RMSE as a function of number of elements in the case of using CSS processing.

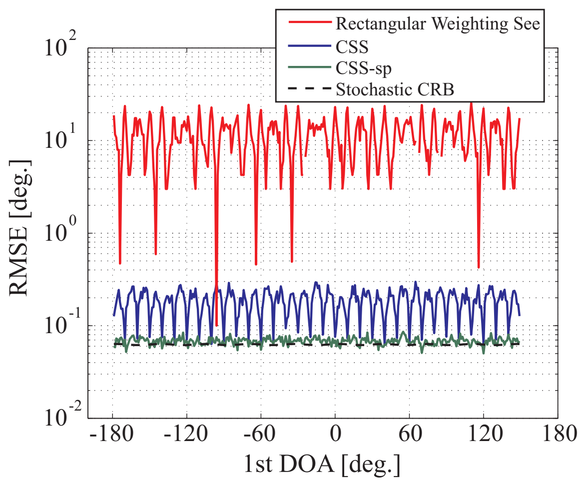

Figure 6: DOA estimation results when the number of waves is two.