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Numerical and Experimental Control in a Parametric Pendulum using Delayed Feedback Method

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Abstract– This paper addresses the application of continuous chaos control method for avoiding bifurcations in a parametrically excited pendulum. Specifically, a delayed feedback control method is employed to maintain stable period-one rotating solution of the pendulum. The motivation of this analysis is the energy harvesting from sea waves where the idea consists in converting the base oscillations of a structure into a rotational motion of the pendulum mass. In such case, the oscillations of the structure are caused by the sea waves, whereas the pendulum rotational motion provides a driving torque for an electrical generator. In this context, bifurcation diagram is investigated by considering forcing amplitude variation. Basically, it is investigated a situation where the desired rotational solution loose stability. Numerical and experimental results are presented showing that chaos control method can be successfully applied to perform bifurcation control.

1. Introduction

The idea of energy harvesting from various renewable sources has been gaining an increasing interest and importance in recent years. The currently available energy harvesting or extraction devices convert solar and wind energy effectively. The sea waves though possessing the largest renewable energy source are practically untapped. This is mainly due to the fact that most of the current devices have so-called end points resulting in inability to design them for extreme weather conditions. Wiercigroch [1] proposed an energy generation concept from the sea waves using a pendulum system by converting the base oscillations of the structure into a rotational motion of the pendulum mass. In such case, the oscillations of the structure are caused by the sea waves, whereas the pendulum rotational motion provides a driving torque for an electrical generator [2-4]. In order to explore potentials of this concept, the dynamics of the pendulum system has to be carefully considered and the means of maintaining the periodic rotational solutions have to be developed.

Although rotating solutions are found and studied by different authors, it should be pointed out that they only exist over limited parameters range and there are a lot of bifurcations of the system that destabilize this kind of motion. Thus, the bifurcation control can be very useful in maintaining the rotational solution of the system and crucial in potential energy extraction applications. In this same context, Yokoi & Hikihara [5] and De Paula *et al.* [6] present interesting results on the stabilization of rotational solutions in a parametrically excited pendulum.

In this work continuous chaos control method proposed by Socolar *et al.* [7] is employed in order to maintain the rotating solution of the pendulum system by stabilizing unstable periodic orbits of the system. The main goal is to avoid bifurcations that destabilize the rotating motion, being useful for energy harvesting purposes. The control is carried out numerically and experimentally.

2. Chaos Control Methods

Chaos control methods can be split into discrete and continuous methods. Continuous methods are based on continuous-time perturbations to perform stabilization. This approach was first proposed by Pyragas [8] and deals with a dynamical system modelled by a set of ordinary nonlinear differential equations as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{Q}(\mathbf{x}, t) + \mathbf{B}(t) \tag{1}$$

where t is time, $\mathbf{x}(t) \in \Re^n$ is the state variable vector, $\mathbf{Q}(\mathbf{x}, t) \in \Re^n$ defines the system dynamics, while $\mathbf{B}(t) \in \Re^n$ is associated with the control action.

Socolar *et al.* [7] proposed a control law named as the extended time-delayed feedback control (ETDF) considering the information of time-delayed states of the system represented by the following equations:

$$\mathbf{B}(t) = \mathbf{K}\left[(1-R)\mathbf{S}_\tau - \mathbf{x}\right] \\ \mathbf{S}_\tau = \sum_{m=1}^{\infty} R^{m-1}\mathbf{x}_{m\tau} \tag{2}$$

where $K \in \Re^{n\times n}$ is the feedback gain matrix, $0 \le R < 1$ is a control gain, $\mathbf{S}_\tau = \mathbf{S}(t-\tau)$ and $\mathbf{x}_{m\tau} = \mathbf{x}(t-m\tau)$ are related to delayed states of the system and τ is the time delay. The UPO stabilisation can be achieved by a proper choice of **K** and *R*. Note that for any gain defined by **K** and *R*, perturbation of Eq.(1) vanishes when the system is on the UPO since $\mathbf{x}(t-m\tau) = \mathbf{x}(t)$ for all m if $\tau = T_i$, where T_i is the periodicity of the *i*th UPO. It should be pointed out that when *R* = 0, the ETDF turns into the original time-delayed feedback control method (TDF) proposed by Pyragas [8].

The controlled dynamical system consists of a set of delay differential equations (DDEs). The solution of this system is done by establishing an initial function $\mathbf{x}_0 = \mathbf{x}_0(t)$ over the interval $(-m\tau, 0)$. This function can be estimated by a Taylor series expansion as proposed by Cunningham [9]:

$$\mathbf{x}_{m\tau} = \mathbf{x} - m\tau\,\dot{\mathbf{x}} \tag{3}$$

Under this assumption, the following system is obtained:

$$\dot{\mathbf{x}} = \mathbf{Q}(\mathbf{x},t) + \mathbf{K}\left[(1-R)\mathbf{S}_\tau - \mathbf{x}\right]$$

$$\text{where} \begin{cases} \mathbf{S}_\tau = \sum_{m=1}^{\infty} R^{m-1}\left[\mathbf{x} - m\tau\,\dot{\mathbf{x}}\right], \text{ for } (t-m\tau) < 0 \\ \mathbf{S}_\tau = \sum_{m=1}^{\infty} R^{m-1}\mathbf{x}_{m\tau}, \text{ for } (t-m\tau) \ge 0 \end{cases} \tag{4}$$

3. Pendulum-Shaker System

Motivated by the cited idea of harvesting energy from sea waves, Xu *et al.* [10] and Horton *et al.* [11] analysed the behaviour of a parametric pendulum excited by electro-dynamical shaker, which is chosen to be studied in this work. Figure 1 shows a picture of the experimental apparatus. The setup consists of a pendulum rig fixed to an electromechanical shaker. The harmonic excitation of the system is provided by a shaker and two independent pendulums, with adjustable length and bob masses at the ends, are threaded to pendulum rods on the common supporting base. The rod is attached to a shaft, supported by needle bearing, in order to minimize friction. The shaft has a gear with a belt that provides the coupling to a low inertia DC servo motor of 1600rpm, which actuates over the system and provides the control torque. A 3 channel hollow shaft encoder with 500ppr monitors the angular position of the pendulum. An accelerometer is fixed on the base of whole rig in order to measure the actual excitation. In the study presented in this paper, only one pendulum is used, the other one is kept immovable.

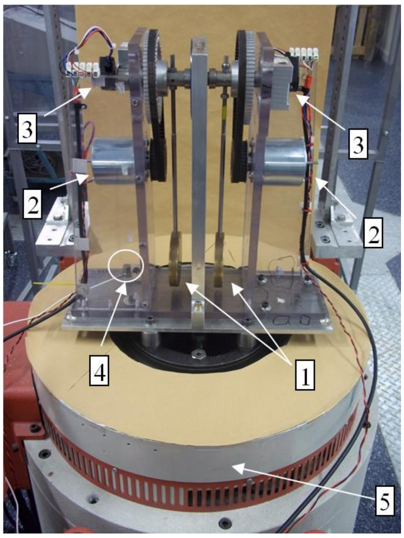

Figure 1 - Experimental rig: (1) Independent pendula; (2) Servo motors; (3) Encoders; (4) Accelerometer; (5) Shaker.

Figure 2 presents a schematic picture of this system identifying mechanical and electrical parts. The mechanical system (Figure 2a) is comprised of three masses: the pendulum mass, *M*, the armature assembly mass, M_a, and the body mass, M_b, that represents the mass of the magnetic structure containing the field coil. The excitation is provided by an axial electromagnetic force, F_{em}, which is generated by the alternating current in the constant magnetic field represented by the electrical system.

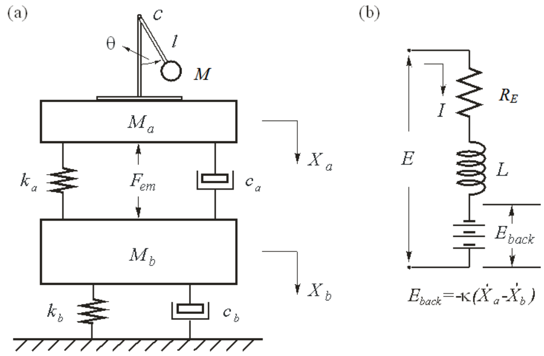

Figure 2 - Physical model of the pendulum-shaker system with mechanical and electrical components. Adopted from [10].

The mechanical part of the pendulum–shaker system is described by three generalized coordinates: angular displacement of the pendulum, θ, and the vertical displacements of the body and the armature, X_b and X_a, respectively. The electrical system is described by the electric charge q, that is related to the current I by its derivative: $I = dq/dt$. Equations of motion for each degree-of-freedom of the parametric pendulum-shaker system are given by:

$$
\begin{aligned}
&\theta : Ml\ddot{\theta} + cl\dot{\theta} + \frac{2}{\pi}F_{\mu}\tan^{-1}(q\dot{\theta}) + Mg\sin\theta = M\ddot{X}_a\sin\theta + T_C \\
&X_a : (M_a + M)\ddot{X}_a + c_a(\dot{X}_a - \dot{X}_b) + k_a(X_a - X_b) \\
&\qquad = (M_a + M)g + Ml\ddot{\theta}\sin\theta + Ml\dot{\theta}^2\cos\theta - \kappa I \\
&X_b : M_b\ddot{X}_b + c_b\dot{X}_b - c_a(\dot{X}_a - \dot{X}_b) + k_bX_b - k_a(X_a - X_b) \\
&\qquad = M_bg + \kappa I \\
&I : R_EI + L\frac{dI}{dt} - \kappa(\dot{X}_a - \dot{X}_b) = E_0\cos(\Omega t)
\end{aligned}
\tag{5}
$$

where T_C corresponds to the control parameter actuation, which consists of a torque applied to the pendulum; and F_μ is the torque due to dry friction. Equations of motion are based on the formulation proposed by Xu *et al.* [10] adding terms related to dry friction and to control torque. For the numerical simulations, Eq. (5) is converted in a system of first order ODEs by considering the state variables $\boldsymbol{x} = \{\theta, \dot{\theta}, X_a, \dot{X}_a, X_b, \dot{X}_b, I\}$. By using the formalism presented for the ETDF control law, the control actuation T_C, with m=3, may be expressed as follows:

$$
T_C = \frac{Ml(M_a + M - M\sin^2 x_1)}{(M_a + M)} \times K[(1-R)(x_\tau + Rx_{2\tau} + R^2x_{3\tau}) - x], \tag{6}
$$

where $x_\tau = x_1(t-\tau)$, $x_{2\tau} = x_1(t-2\tau)$ and $x_{3\tau} = x_1(t-3\tau)$. Moreover, matrix gain $\mathbf{K}$ becomes a scalar K once the control action is only applied to one differential equation, the one related to time evolution of x_2, and its control law is only associated with delayed values of one state variable, x_1. In others words, only component K_{21} is different from zero and notation K is used for it.

Table 1: Experimentally determined parameters of the pendulum-shaker system.

M	0.709 kg	l	0.2605 m	c	0.0828 kg/s
M_a	27.58 kg	K_a	86175.9 kg/s^2	C_a	534.05 kg/s
M_b	820 kg	K_b	244284 kg/s^2	C_b	679.35 kg/s
R_E	0.3 Ω	L	2.626 × 10^{-3} H	κ	130 N/A
F_μ	0.0625 Nm				

Xu *et al.* [10] discussed experimental aspects of the pendulum-shaker dynamics. The parameters related to the shaker used in this work were experimentally determined in [10]. Parameters related to pendulum rig were also experimentally determined. All parameters are presented in Table 1.At first the uncontrolled behaviour of the system was studied. From an initial forcing amplitude with ω=1.51Hz, where the pendulum presents a period-1 rotation behaviour, the forcing amplitude was decreased to construct bifurcation diagrams shown in Figure 3 and obtained numerically (left) and experimentally (right). From the phase spaces and Poincaré sections presented in Figures 4 and 5 it is possible to observe that the initial period-1 rotation orbit leads to a quasi-periodic motion and then to a period-2 oscillatory motion.

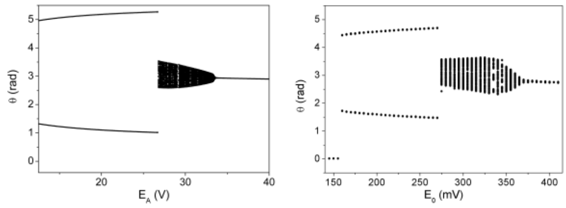

Figure 3 – Bifurcation diagram by decreasing forcing amplitude with ω=1.51Hz. Left: Numerical; Right: Experimental.

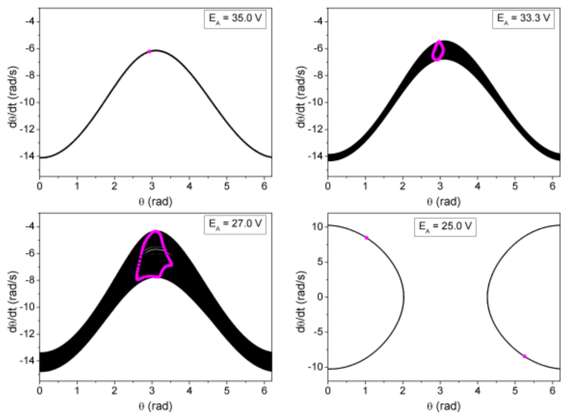

Figure 4 – Phase space together with Poincaré section for different forcing amplitude and ω=1.51Hz obtained numerically.

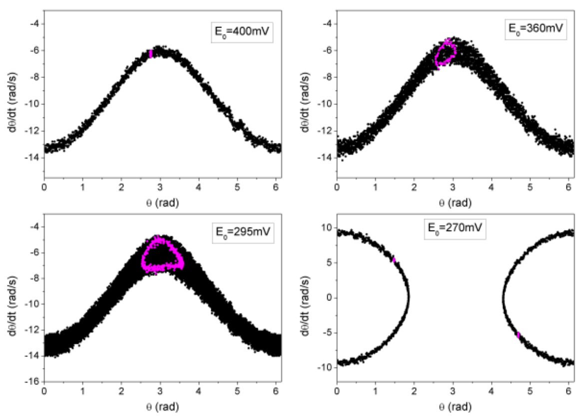

Figure 5 - Phase space together with Poincaré section for different forcing amplitude and ω=1.51Hz obtained experimentally.

The undertaken analysis exploits the idea of using the chaos control method in order to avoid bifurcations depicted in Figure 3 that destabilize periodic rotational solution of the pendulum. Essentially, bifurcation control is of concern.

Figure 6 presents controlled and uncontrolled bifurcation diagram obtained numerically (a) and experimentally (b) with controller parameters R=0 and K=1. The time delay, τ, is considered equal to the forcing period, $2\pi/\omega$, in both numerical and experimental approaches since the desired rotational orbit has periodicity one. It is important to mention that all points of Poincaré section are plotted in the diagram, including transient response. Note that in both cases the controller eliminates the bifurcation, preserving the periodic rotational behaviour.

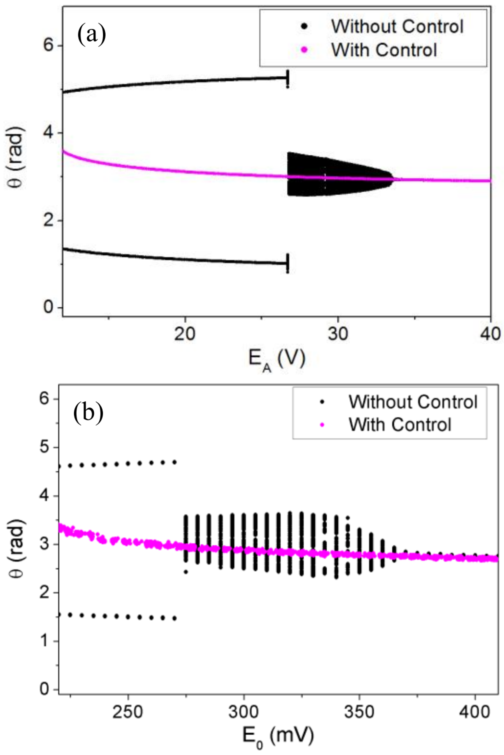

Figure 6 - Bifurcation diagram with and without control action decreasing the forcing amplitude with ω=1.51Hz. a) Numerical; b) Experimental.

4. Conclusions

This contribution deals with the application of chaos control methods to perform bifurcation control using a parametrically excited pendulum. This system is related to energy harvesting from sea waves and therefore, periodic rotational behaviour is the desired one. Bifurcation diagrams are constructed by considering forcing amplitude variation showing bifurcations from the desired rotational solution to non-rotational periodic response. Delayed feedback control is successfully employed numerically and experimentally to avoid these bifurcations, maintaining the rotational behaviour stable. In both controlled and uncontrolled system, numerical and experimental approaches present good agreement.

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