DOA Estimation Using Adaptive Beamspace EM Algorithm

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1. Introduction

To understand radio propagation structures and consider signal recovering techniques in mobile communications, it is most effective to estimate DOAs (directions of arrival) of individual incoming waves with array antennas. Also, in radar systems, it is required to discriminate the desired signal from interference [1]. Recently, DOA estimation using EM (expectation-maximization) algorithm [2] based on the maximum-likelihood (ML) approach receives much attention. It is because the EM algorithm remains stable in scenarios involving small numbers of snapshots, coherent signals and low SNR. However, the ML approach generally has the high computational complexity caused by optimization of the likelihood function. In addition, the convergence rate is slow, if the incoming waves are coherent and their DOAs are closely spaced.

In this paper, we apply the beamspace processing using DCMP (directionally constrained minimization of power) criterion [3] to the EM algorithm for improving the convergence rate. Furthermore, we investigate the effect of the error of initial values in the beamspace EM algorithm on the DOA estimation. Through computer simulation, we show that the beamspace processing provides improved performance for DOA estimation.

2. Signal Model and DOA Estimation

2.1 Signal Model

Consider that the array antenna used for DOA estimation is a K-element linear array shown in Fig.1, and also that it receives L ($L < K$) narrow-band waves whose respective DOAs are $\theta_1,\ \theta_2,\ \cdots,\ \theta_L$ and complex amplitudes are $s_1(t),\ s_2(t),\ \cdots,\ s_L(t)$. When the array response vector (mode vector) of the lth incoming wave is given by $\boldsymbol{a}(\theta_l)$ $(l = 1,\ 2,\ \cdots,\ L)$, the array input vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l) s_l(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1),\ \boldsymbol{a}(\theta_2),\ \cdots, \boldsymbol{a}(\theta_L)],\ \boldsymbol{s}(t) = [s_1(t),\ s_2(t),\ \cdots,\ s_L(t)]^T \tag{2}$$

where $\boldsymbol{A}$ and $\boldsymbol{s}(t)$ are called the array response matrix (mode matrix) and the signal vector, respectively, and $\boldsymbol{n}(t)$ is the internal additive noise vector.

2.2 EM Algorithm

The EM algorithm is the method based on maximum-likelihood estimation [2]. In the EM algorithm, iterative calculation is carried out for getting DOAs from unobservable complete data rather than observed incomplete data $\boldsymbol{x}(t)$. Each iteration consists of two steps: E-step (expectation) which approximates the complete data by conditional expectation and M-step (maximization) which maximizes the likelihood of the complete data. The mth iteration of the EM algorithm proceeds as follows.

E-step : The maximum likelihood estimate of complete data $\boldsymbol{x}_l^{(m)}(t)$ is calculated by using the DOA estimate $\theta_l^{(m)}$ and the complex amplitude estimate $s_l^{(m)}(t)$ of the lth wave, which is given by

$$\boldsymbol{x}_l^{(m)}(t) = s_l^{(m)}(t)\boldsymbol{a}(\theta_l^{(m)}) + \beta\left[\boldsymbol{x}(t) - \boldsymbol{A}^{(m)}\boldsymbol{s}^{(m)}(t)\right] \quad (l = 1,\ 2,\ \cdots,\ L) \tag{3}$$

where $\boldsymbol{a}(\theta_l^{(m)})$ is the response vector of the lth wave at the mth iteration, and $\boldsymbol{A}^{(m)}$ is the corresponding array response matrix. Also, β is a non-negative coefficient of noise term, and it affects the convergence characteristics.

M-step : The updated values $\theta_l^{(m+1)}$ and $s_l^{(m+1)}(t)$ of the lth wave are obtained by using the covariance matrix of complete data: $\boldsymbol{C}_l^{(m)} = E\left[\boldsymbol{x}_l^{(m)}(t)\boldsymbol{x}_l^{(m)}(t)^H\right]$, as shown below.

$$\theta_l^{(m+1)} = \arg\max_{\theta} \frac{\boldsymbol{a}(\theta)^H \boldsymbol{C}_l^{(m)} \boldsymbol{a}(\theta)}{\boldsymbol{a}(\theta)^H \boldsymbol{a}(\theta)},\ s_l^{(m+1)}(t) = \frac{\boldsymbol{a}(\theta_l^{(m+1)})^H \boldsymbol{x}_l^{(m)}(t)}{\boldsymbol{a}(\theta_l^{(m+1)})^H \boldsymbol{a}(\theta_l^{(m+1)})} \quad (l = 1,\ 2,\ \cdots,\ L) \tag{4}$$

Both E-step and M-step above-mentioned are repeated until estimated parameters converge.

3. Adaptive Beamspace Processing

3. 1 Beamforming Based on DCMP Criterion

Utilizing the estimated values $\theta_l^{(m)}$ $(l = 1,\ 2,\ \cdots,\ L)$ in the DCMP criterion of the adaptive antenna, the optimum weight vector for receiving only the lth wave is computed as follows.

$$\boldsymbol{w}_l(u_l) = \frac{\boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(u_l)}{\boldsymbol{a}(u_l)^H \boldsymbol{R}_{xx}^{-1}\boldsymbol{a}(u_l)} \qquad \left(\boldsymbol{w}(u_l)^H \boldsymbol{a}(u_l) = 1\right) \tag{5}$$

$$\boldsymbol{a}(u_l) = \left[1,\ \exp\left(-j\frac{2\pi}{\lambda}du_l\right),\ \cdots,\ \exp\left(-j\frac{2\pi}{\lambda}(K-1)du_l\right)\right]^T \tag{6}$$

Here, $u_l = \sin\theta_l^{(m)}$, and $\boldsymbol{a}(u_l)$ is the array response vector with the phase reference at the 1st antenna element. Also, $\boldsymbol{R}_{xx}$ is the covariance matrix and in this case it is made up as

$$\boldsymbol{R}_{xx} = \sum_{l=1}^{L} \boldsymbol{a}(\theta_l^{(m)})\boldsymbol{a}(\theta_l^{(m)})^H + \alpha\boldsymbol{I} \tag{7}$$

where α is a small positive number (pseudo noise power) for $\boldsymbol{R}_{xx}$ to be non-singular. Using $\boldsymbol{w}_l$ yields an array pattern with the mainlobe in the direction of the lth wave and nulls in the other waves. In addition, it is possible to control the whole ability of creating nulls by adjusting α in Eq.(7). For example, small value of α gives deep nulls, while large value of α contributes to making almost only mainlobe.

When the number of beams formed for the lth incoming wave is three, the beamforming matrix $\boldsymbol{W}_l$ is constructed as follows by using the weight vector $\boldsymbol{w}_l$.

$$\boldsymbol{W}_l = \left[\boldsymbol{w}_{ln}\left(u_l - \frac{2}{K}\right),\ \boldsymbol{w}_{ln}(u_l),\ \boldsymbol{w}_{ln}\left(u_l + \frac{2}{K}\right)\right] \tag{8}$$

In Eq.(8), $\boldsymbol{w}_{ln}$ is the normalized $\boldsymbol{w}_l$ with $||\boldsymbol{w}_l||$. $\boldsymbol{W}_l$ is applied to $\boldsymbol{x}_l^{(m)}(t)$ as $\boldsymbol{W}_l^H\boldsymbol{x}_l^{(m)}(t)$, leading to beamspace EM.

3. 2 Broad Null in Beam Patterns

In this paper, we introduce the broad null into the DCMP beam patterns. This is a countermeasure to the estimation error, and we add nulls in the directions shifted by ±2° from each estimated angle of incoming waves which are suppressed. We apply this broad null to the DCMP when the number of iterations of EM is within five times.

4. Performance Analysis by Computer Simulation

Under conditions shown in Tables 1–3, the computer simulation is carried out to clarify the performance of the proposed algorithm. For DOA estimation, the EM algorithms using elementspace beamformer, DFT-beamspace beamformer [4] and DCMP-beamspace beamformer are compared. As the evaluation measure of estimated results, RMSE (root mean square error) is used, which is calculated through 200 independent trials. The number of incoming waves is assumed to be estimated exactly in any simulation of DOA estimation.

First, the convergence characteristics of various EM algorithms are examined when the incoming waves are coherent and their DOAs are closely spaced. The radio environment is described in Table 2. The incoming waves are perfectly out of phase and completely correlated with each other. Figure 2 shows an example of the DCMP beam patterns which receive only the wave of 0° direction. The results of estimation are shown in Fig.3 along with Cramer-Rao bound (CRB) [5]. From the figure, it is found that the convergence rate becomes rapid by employing the adaptive beamspace processing using DCMP with $\alpha = 10^{-6}$.

Next, the performance to the initial value error of various EM algorithms is examined. The radio environment is described in Table 3. In this simulation, the initial value error θ_e is an independent uniform random number $[0, \Delta\theta]$ for each incoming wave, and θ_e is set as shown in Table 3. In addition, the broad null is used in the beam patterns. Figure 4 illustrates an example of the DCMP beam patterns which have the broad null in the direction of 30°. As a result, Figure 5 shows the convergence of RMSE of estimates in the case of $\Delta\theta = 10°$, and Figure 6 shows the RMSE of estimates versus the initial value error when the number of iterations of EM is three. From the figures, it is confirmed that DCMP with $\alpha = 10^6$ and DCMP with $\alpha = 10^{-6}$ and the broad null provide good performance.

Table 1: Simulation conditions.

Array configuration	Uniform linear array of isotropic elements
Element spacing	0.5λ
Number of elements	8
Number of beams	3
Number of waves	2
SNR	20dB

Table 2: Radio environment 1.

DOA from array broadside	(0°, 10°)
Initial value of EM	(−5°, 15°)

Table 3: Radio environment 2.

DOA from array broadside	(0°, 30°)
Initial value of EM	$(0° + \theta_e, 30° - \theta_e)$

5. Conclusion

Via computer simulation of DOA estimation, we have investigated the performance of the EM algorithm with the beamspace processing using DCMP criterion. In the convergence characteristics, DCMP-beamspace EM has shown high speed of convergence. In the characteristics to the initial value error, even if the error is large, it is possible to estimate DOAs successfully by adjusting α (pseudo noise power) and introducing the broad null into the beam patterns. As the future work, we will examine how to set the initial values of EM involving the estimation of the number of waves.

References

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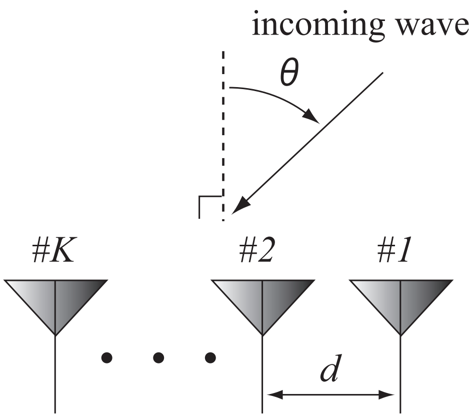

Figure 1: K-element uniform linear array (element spacing: d)

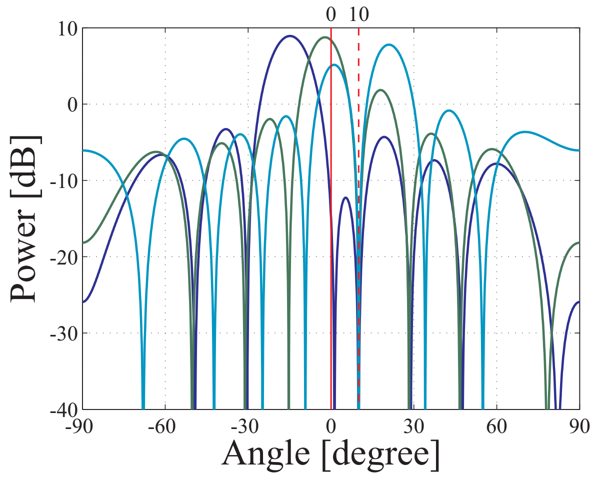

Figure 2: Multibeam patterns using DCMP for receiving the wave from 0° and suppressing the one from 10°.

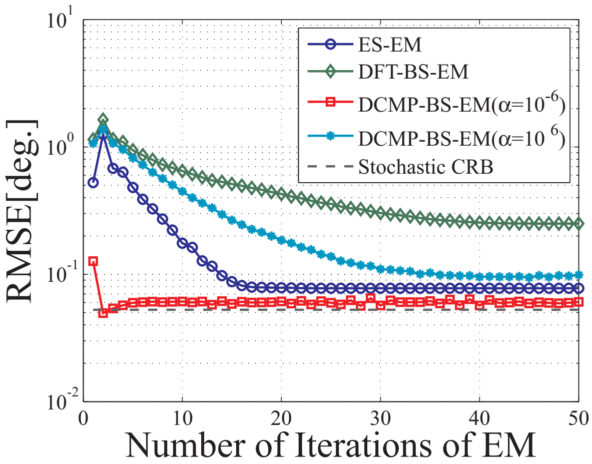

Figure 3: Convergence of RMSE of estimates. ($\beta = 1/\sqrt{L}$)

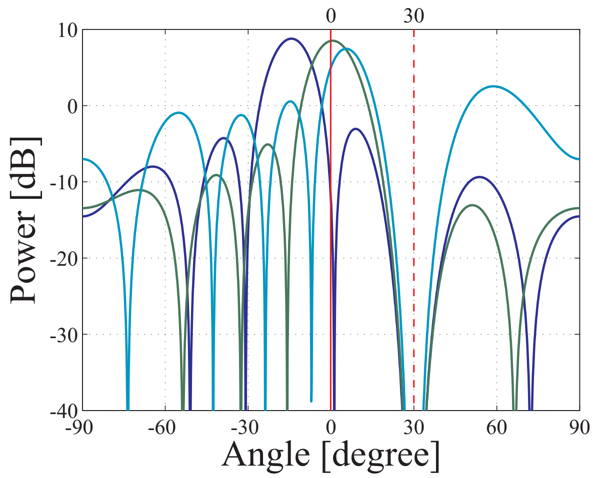

Figure 4: Multibeam patterns using DCMP which have the broad null in the direction of 30°.

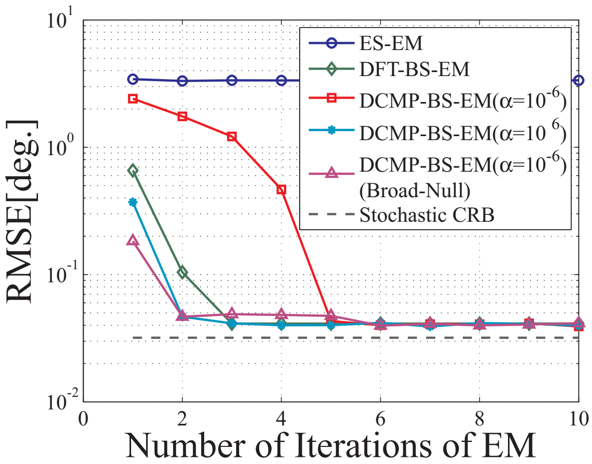

Figure 5: Convergence of RMSE of estimates. ($\beta = 1$, $\Delta\theta = 10°$, and broad null is used.)

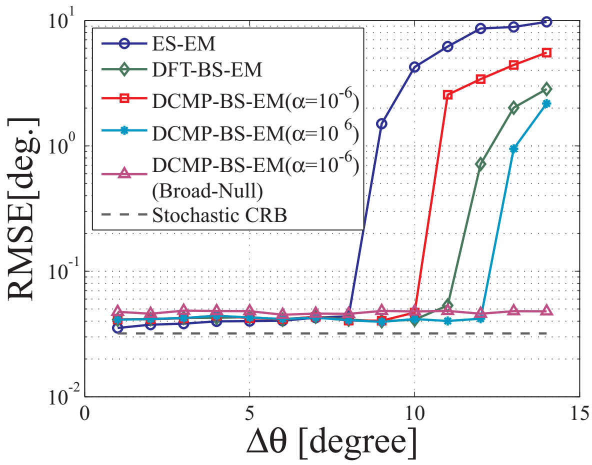

Figure 6: RMSE of estimates vs. initial value error. ($\beta = 1$, the number of iterations of EM is three.)