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Comparison of random and deterministic characteristics of chaotic signals issued from a one-dimensional piecewise linear map

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Abstract— This paper presents the characteristics of the power spectral density (PSD) of chaotic signals generated by a one-dimensional piecewise linear map. The majority of previous research on chaos show that chaotic signals are rather broadband with impulsive auto-correlation sequence (ACS). However, recent studies of the skew tent map [4, 5] have shown that the PSD and ACS are modified according to certain values of the bifurcations. We propose to extend this work to other maps and to study relations between bandwidth and Lyapunov exponents.

1. Introduction

The chaotic signals are aperiodic signals and have a sensitivity to initial conditions (SIC) [1]. The sensitivity to initial conditions means that the signals obtained with similar initial conditions can become very different when the number of iterations tends to infinity. Usually in the existing publications the chaotic signals are characterized by an impulsive auto-correlation sequence (ACS) and they have a broadband frequency [6, 9, 12]. Using this property, different applications have been considered with chaotic signals. Anyhow, it is possible to construct chaotic signals with different kinds of bandwidth, this is one of our purposes in this paper. So, chaotic signals could be used for other applications than those implying broadband frequency.

The spectral analysis of signals is used in several domains to extract information and verify the distribution of energy or power in the frequency range [10]. Spectral models are used in many processes of modulation, voice processing, compression and voice recognition [7]. In medicine, the spectral analysis of electrocardiograms and electroencephalograms may provide useful information in diagnostics [10]. Spectral analysis of signals is also crucial in the field of telecommunications. The specters must be known and clearly defined, due to insufficient frequencies available for communication and limited frequency bands in guided media. In this paper, we propose to study the possibility to obtain chaotic signals with different types of bandwidth, using signals obtained from a discrete-time dynamic system. We have chosen a piecewise linear map depending on parameters for which previous studies have shown that chaotic signals can be observed [3].

2. One-dimensional piecewise linear map with two parameters

We will be more particularly interested in a one-dimensional map depending on two parameters, modeled under the following form:

$$x_{n+1} = f(x_n, k, \phi), \tag{1}$$

with $n \in \aleph$; $x_n \in I$ interval of $\Re$; k and $\phi \in \Re$.
We will try to find a relation between the different possible forms of chaotic attractors, the Lyapunov exponent and the power spectral density (PSD). We consider the specific map $f : [-1, 1] \to [-1, 1]$ proposed in [3]:

$$\begin{cases} z_n = mod((k.|x_n|) + \phi, 1) \\ x_{n+1} = sign(x_n).((2.z_n) - 1) \end{cases} \tag{2}$$

which depends on two parameters ϕ and k (cf. Fig.1). The parameters can be changed to produce different sequences, $(0 \leq k \leq 2)$ and $(0 \leq \phi \leq 2)$. Due to an obvious symmetry property [3], the initial condition x_0 will be chosen in the interval $0 < x_0 < \frac{1}{k}$.

2.1. Bifurcation diagram

A fundamental problem of nonlinear dynamics is the study of bifurcations in the parameter space. A bifurcation corresponds to a qualitative change in the behavior of the system when one parameter, for instance ϕ, crosses through a critical value ϕ_b. A bifurcation may correspond to the appearance or disappearance of new singularities, or also to the change in the shape of a chaotic attractor.

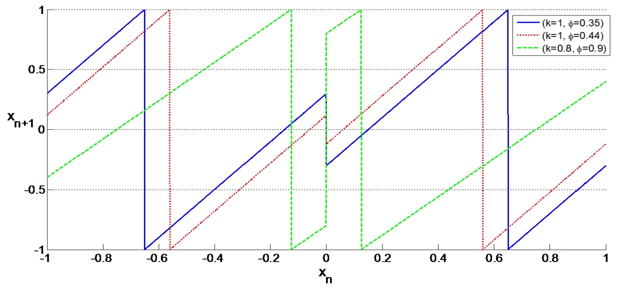

Figure 1: Chaotic map (2) with ($k = 1$, $\phi = 0.35$), ($k = 1$, $\phi = 0.44$) and ($k = 0.8$, $\phi = 0.9$).

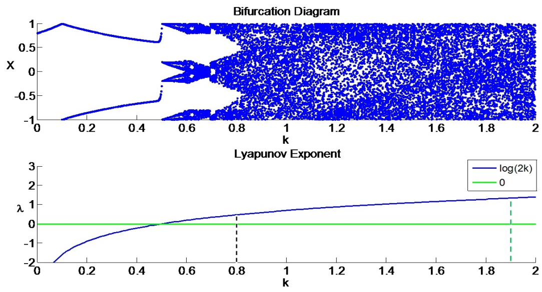

Figure 3: $\phi = 0.9$, $k \in [0, 2]$

2.2. Lyapunov exponent

The Lyapunov exponent measures the sensitivity to initial conditions. When the Lyapunov exponent of f (f differentiable) exists, it is defined by:

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \left(\sum_{n=0}^{N-1} \ln \left| f'(x(n, x_0)) \right| \right) \quad (3)$$

The sequence generated by (1) is chaotic if the Lyapunov exponent is positive. When f is piecewise linear or piecewise affine, it is also possible to calculate the Lyapunov exponent. The value of the Lyapunov exponent is given in [3], it only depends on the parameter k as indicated by the following equation:

$$\lambda = ln(2k) \quad (4)$$

When k is fixed and ϕ varies in the interval $[0, 2]$, we can see that the chaotic attractor changes its shape (cf. Fig.1), but the Lyapunov exponent is positive and remains constant (cf. Fig.2).

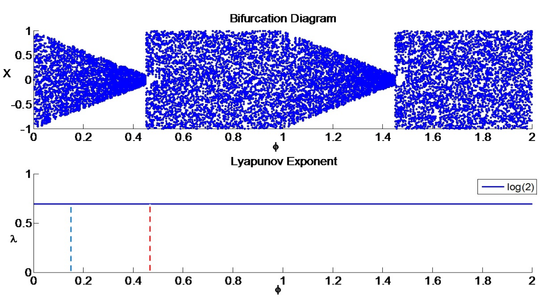

Figure 2: $k = 1$, $\phi \in [0, 2]$

In Fig. 3, ϕ is constant and k varies in the interval $[0, 2]$, then we obtain a chaotic attractor with a different appearance and the Lyapunov exponent increases with k.

3. Spectral analysis

Some calculations of the spectrum of a chaotic signal are given in [11, 5, 8, 2]. The chaotic signals generated by a particular map can be considered as deterministic individual signals or as sampling functions of an ergodic stochastic process. These two types of presentation give rise to different steps when calculating the PSD. In this paper, we consider a discrete time signal, both as deterministic and random. Using the function $f(.)$ of equation (2) and an initial condition $x(0) = x_0$, the sequence $x(n, x_0)$ is defined when $n > 0$ and the autocorrelation can be calculated by:

$$R(m, x_0) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x(n, x_0).x^*(n - m, x_0) \quad (5)$$

such as x^* is the conjugate of x. In this formula, m is an integer and $x(n - m, x_0) = 0$ when $(n - m)$ is a negative number. The power spectral density $X(f, x_0)$ can be obtained by applying the Discrete-Time Fourier Transform (DTFT) to $R(m, x_0)$ considering m as the time variable:

$$X(f, x_0) = \sum_{m=-\infty}^{\infty} R(m, x_0) e^{-jwm} \quad (6)$$

To analyze the signal by computer simulations, we use a Welch's method to determinate an estimation of the spectral density. This method starts by dividing the time series data into segments, then computing a modified periodogram of each segment and finally averaging the PSD estimates.

In Fig.4 and Fig.5, we have simulated the map (2) with different initial conditions and different values of parameters k and ϕ. These figures contain the orbits, the histogram (the vertical axis concerns the number of x values), the autocorrelation sequence and the power spectral density with $N = 20000$.

Fig.4 is obtained with the fixed parameter values $\phi = 0.35$ and $k = 1$ but with the change of the initial

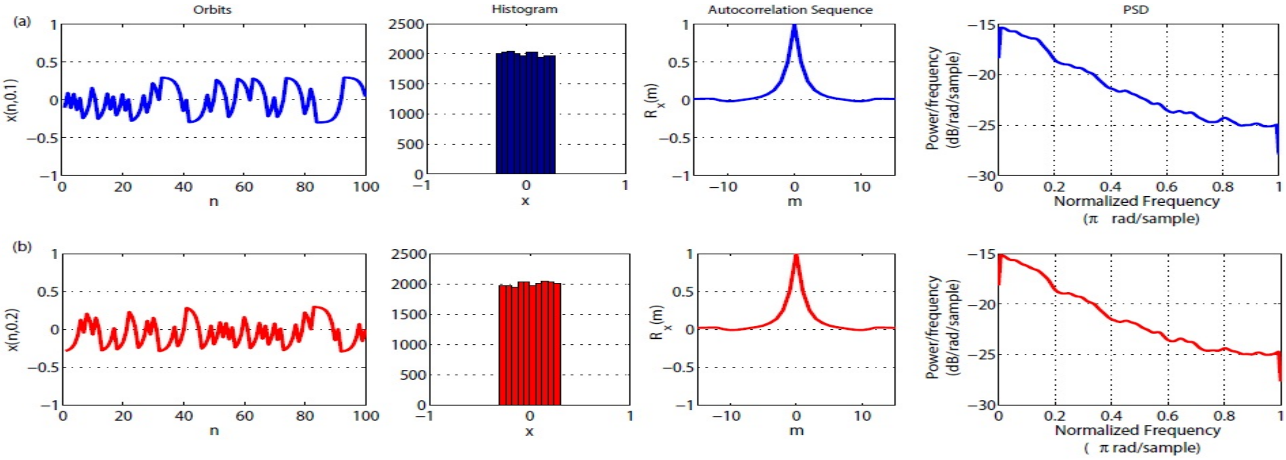

Figure 4: ϕ= 0.35, k= 1

condition. We can remark that when we change the initial condition, there is no effect on the histogram, the autocorrelation and the power spectral density. This result is normal because the chaotic attractor remains the same whatever the initial condition.

In Fig.5 we have used one color for each line. In the first line with a blue color ($k = 1, \phi = 0.15$), we can notice that the values of x vary inside an interval smaller than [-1,1], the autocorrelation sequence has a positive peak at 0 and the spectrum is low pass. In Fig.2, the bifurcation diagram shows that for the considered parameter values, the chaotic attractor occupies $[-0.7, 0.7]$ a smaller range in $[-1, 1]$.

In the second line of Fig.5, we have simulated the red curves for $k = 1$ and $\phi = 0.44$, the distribution of points is given in two intervals and the number of x values is very low when x_n is close to zero. In Fig.2, we can see a hole around the value $x = 0$. The ACS has a positive impulse and two negative impulses, the spectrum is high-pass.

In the third line of Fig.5, we have chosen $k = 1.9$, $\phi = 0.9$ and the curves are drawn in green. The points are distributed in all the interval $[-1, 1]$ (cf. Fig.3), the ACS has a positive peak which tends to zero very quickly. This chaotic signal is broadband.

In the fourth line we have found a chaotic sequence with a bandpass spectrum. The ACS of this sequence has two negative impulses as $R_x(m, x_0) \simeq -0.4$ and the x values is distributed on three intervals (cf. Fig.3).

4. Relation between Bandwidth and the Lyapunov exponent

The bandwidth B of a signal can be determined by the essential bandwidth defined as the frequency range where 95% of the total signal power is concentrated [5]. We use here a normalized version of frequency such as $0 \leq B \leq 1$.
After several simulations, we have found that the bandwidth is independent of the Lyapunov exponent for the map we have studied. When k is fixed and ϕ varies, we obtain different curves of the DSP with different bandwidths. For example, when we consider ($k = 1, \phi = 0.15$), the PSD is low-pass with a very low bandwidth ($B \simeq 0.25$) and for ($k = 1, \phi = 0.44$), the PSD shows a high pass bandwidth ($B \simeq 0.75$).

5. Conclusion

In the work of M.Eisencraft et al. [4], the authors identified two types of bandwidth (low-pass and high-pass) for the skew tent map. By using the map (1) [3], we have found several types of bandwidth. So, we have shown that it is possible to obtain different possible bandwidths using chaotic signals. Such signals can thus be used, depending upon the application we consider.
We intend to continue our work first by trying to calculate the PSD in an analytical way to facilitate the selection of the bandwidth, secondly by considering other kinds of maps, not only piecewise linear.

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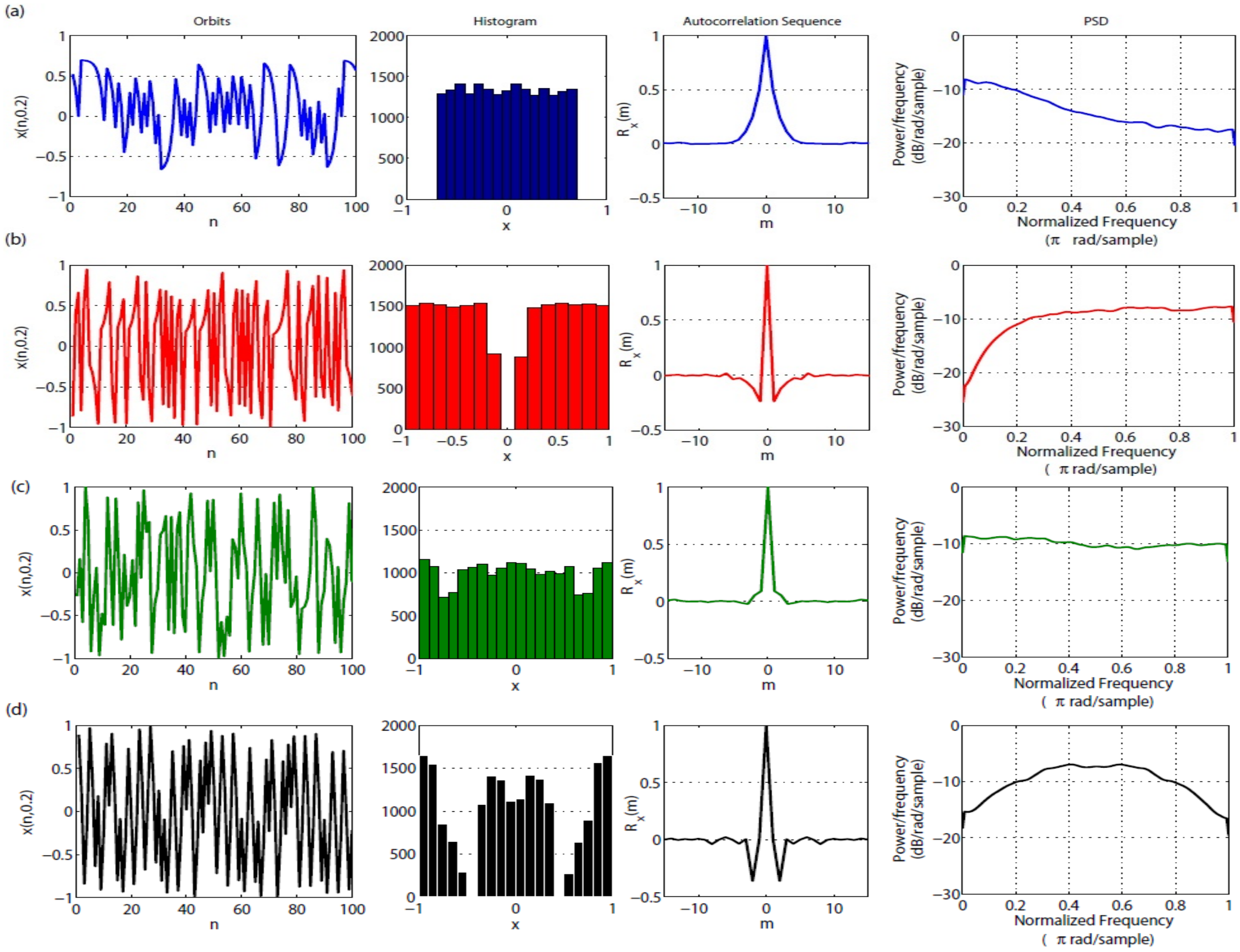

Figure 5: $(a)\phi = 0.15; k = 1; (b)\phi = 0.44; k = 1; (c)\phi = 0.9; k = 1.9; (d)\phi = 0.9; k = 0.8;$

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