Simple Models for Multiplexing Throughputs in Open- and Closed-Loop MIMO Systems with Fixed Modulation and Coding for OTA Applications

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***Abstract*-In this paper, we study the multiplexing throughputs of open- and closed-loop multiple-input multiple-output (MIMO) systems with fixed modulation and coding (i.e., fixed maximum data rate). For the open-loop case, we assume the simplest linear decoder, i.e., zero forcing (ZF). For the closed-loop case, we assume singular value decomposition (SVD). Using the threshold receiver model (accounting for the advanced coding), the throughput for either case can be readily obtained via simulations. MIMO antenna degradations such as correlation and imbalanced antenna efficiencies can be readily included in the developed throughput model. In addition to the usual presentation of the average throughput in a fading channel, we introduce the user-distributed detection probability of the (data) streams.**

I. INTRODUCTION

The long term evolution (LTE) of the universal mobile telecommunications system (UMTS) is attracting more and more attentions due to its throughput enhancement capability. In order to characterize (and improve) the LTE system, currently there is a growing interest in developing efficient over-the-air (OTA) system for testing LTE devices [1]-[5]. To complement the OTA measurements, throughput models have been presented in [3], [5]. Specifically, a single-input multiple-output (SIMO) throughput model based on the concept of the threshold receiver was presented in [3], a useful approximation also being studied in [6]. The SIMO throughput model was extended to multiple-input multiple-output (MIMO) systems with full spatial multiplexing in [5]. Due to the unavailability of closed-loop MIMO testing instruments, all these previous studies are for open-loop MIMO systems, where the channel state information (CSI) is only available at the receive side.

In this paper, we extend the MIMO throughput model further to include the closed-loop MIMO system (where the CSI is known at both MIMO sides). For simplicity, we assume fixed modulation and coding (i.e., fixed maximum data rate) as in [3], [5], and beamforming based on singular-value-decomposition (SVD) at both MIMO sides. Using the developed throughput model, the effects on the MIMO system throughput of correlation and power imbalance due to different antenna efficiencies are studied. In addition to the usual perception of the average throughput in a fading channel, we introduce a user-distributed throughput presentation, which can be interpreted as the probability of the supported streams under certain signal to noise ratio (SNR) values.

II. MULTIPLEXING THROUGHPUT

A. *SISO Throughput Model*

The single-input single-output (SISO) throughput model serves as a building block for developing the MIMO throughput model later on. It has been presented in [3]. Nevertheless, for the sake of completeness, we present it here briefly.

In order to model the advance coding in the modern telecommunications system with lowest possible complexity, we resort to the threshold receiver, whose block-error-rate (BLER) in an additive white Gaussian noise (AWGN) channel can be expressed as

$$P_e(\gamma) = \begin{cases} 1, & \gamma < \gamma_{th} \\ 0, & \gamma > \gamma_{th} \end{cases} \tag{1}$$

where γ is the received SNR in the AWGN channel and γ_{th} is the threshold value. In a fading channel, the average BLER is

$$\overline{P_e}(\overline{\gamma}) = \int_0^{\gamma_{th}} f(\gamma/\overline{\gamma})d\gamma = F\left(\gamma_{th}/\overline{\gamma}\right) \tag{2}$$

where $\overline{\gamma}$ represents the average γ and F denotes the cumulative distribution function (CDF) of γ. Interestingly, (2) agrees with the outage theorems [7], which states that with powerful coding the average BLER can be well approximated by the outage probability of the fading channel.

The throughput of a SISO system can be easily modeled as

$$T_{\text{put}}\left(\overline{\gamma}\right) = T_{\text{put,max}}\left(1 - F(\gamma_{th}/\overline{\gamma})\right) \tag{3}$$

where $T_{\text{put,max}}$ denotes the maximum data rate. Note that 1-F in (3) is the complementary CDF (CCDF) of the fading channel.

B. *MIMO Channel*

In a flat fading channel, the MIMO system can be modeled as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{4}$$

where $\mathbf{H}$ is the MIMO channel matrix, $\mathbf{x}$ and $\mathbf{y}$ are the transmitted and received signal vectors, respectively, and $\mathbf{n}$ is the noise vector with independent identically distributed (i.i.d.) Gaussian variables. Note that, for an LTE MIMO system working in a frequency-selective and quasi-static fading channel [8], the model (4) can be regarded as one subcarrier of the orthogonal frequency division multiplexing (OFDM) system, which can be easily extended to model the frequency-selective fading channel [3], [5].

In MIMO OTA tests, the device under test (DUT) is usually measured as a receiver. In order to characterize the DUT alone, and the transmit antennas (whose efficiencies are calibrated out) are usually uncorrelated. The MIMO channel including the overall antenna effect (at the receive side) then can be expressed as [9]

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{H}_w \tag{5}$$

where $\mathbf{H}_w$ denotes the spatially white MIMO channel with i.i.d. complex Gaussian elements, $\mathbf{R}^{1/2}$ is the Hermitian square root of $\mathbf{R}$, which is

$$\mathbf{R} = \mathbf{\Xi} \circ \mathbf{\Phi} \tag{6}$$

where $\mathbf{\Xi} = \sqrt{\mathbf{e}}\sqrt{\mathbf{e}}^T$ with $\mathbf{e}$ denoting a vector consisting the embedded radiation efficiencies at each port of the MIMO antenna, the superscript ^{T} is the transpose operator, $\sqrt{\ }$ is the element-wise square root, $\circ$ denotes element-wise product, and the correlation matrix $\mathbf{\Phi}$ consists of the complex correlation coefficients of the MIMO antenna.

C. Open-Loop MIMO System

In an open-loop MIMO system with full spatial multiplexing (i.e., the number of independent data streams equals the number of transmit antennas N_T), we assume a ZF receiver [10] for simplicity.

Let $\mathbf{h}_i$ be the ith column of $\mathbf{H}$ and x_i be the ith element of $\mathbf{x}$, (4) can be rewritten as

$$\mathbf{y} = \mathbf{h}_i x_i + \sum_{j \neq i} \mathbf{h}_j x_j + \mathbf{n}. \tag{7}$$

The first term in the right side of (7) stands for the ith stream, and the second term represents the interferences from all the other streams with respect to (w.r.t.) the ith stream.

The ZF equalizer for the ith stream corresponds to the ith row of the pseudo-inverse of $\mathbf{H}$, $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$, where the superscript ^{H} stands for conjugate transpose. It projects the ith data stream into the subspace orthogonal to the one spanned by $\mathbf{h}_1,\ldots, \mathbf{h}_{i-1}, \mathbf{h}_{i+1},\ldots, \mathbf{h}_{N_T}$. Assuming equal allocation of transmit power and left multiplying the pseudo-inverse of $\mathbf{H}$ to (4), the SNR of the ith stream can be easily derived as

$$\gamma_i = \frac{1}{\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\right]_{i,i}} \tag{8}$$

where $[\mathbf{X}]_{i,i}$ denotes the ith diagonal element of the matrix $\mathbf{X}$. Note that, without loss of generality, (8) assumes $\mathrm{E}[|x_i|^2]/\mathrm{E}[|n_i|^2] = 1$. The MIMO throughput is the sum of the throughputs of all the streams:

$$T_{\text{put}}\left(\bar{\gamma}\right) = \sum_i T_{\text{put,max},i}\left(1 - F\left(\gamma_{th}/\bar{\gamma}_i\right)\right) \tag{9}$$

where $T_{\text{put,max},i}$ denotes the maximum data rate of the ith stream. Since the CSI is unknown at the transmitter, $T_{\text{put,max},i} = T_{\text{put,max}}/N_T$ and $\bar{\gamma}_i = \bar{\gamma}/N_T$.

D. Closed-Loop MIMO System

When the CSI is known at the transmitter, it is natural to use SVD-based MIMO configuration [10]. However, due to limited feedback in frequency-division duplex (FDD) systems, codebook-based precoding [11] is used instead. The codebook-based precoding uses, in essence, quantized CSI (i.e., partial CSI), which incurs performance degradation. In this paper, for simplicity, we assume full CSI at the transmitter (e.g., SVD-based MIMO). The corresponding results can be regarded as upper bounds for the codebook-based precoding counterparts.

Let the SVD of $\mathbf{H}$ be $\mathbf{H} = \mathbf{U\Lambda V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are the unitary matrices, and $\mathbf{\Lambda}$ is a diagonal matrix consisting the singular values of $\mathbf{H}$. The precoding and power allocation is done by multiplying the signal vector $\mathbf{s}$ by $\mathbf{VP}$, $\mathbf{x} = \mathbf{VPs}$, where $\mathbf{P}$ is a diagonal matrix whose elements correspond to the allocated power for each stream. The decoding is done by multiplying $\mathbf{y}$ by $\mathbf{U}^H$, r = $\mathbf{U}^H$ $\mathbf{y}$. The resulting (interference-free) parallel MIMO channel is

$$\mathbf{r} = \mathbf{\Lambda P s} + \mathbf{z} \tag{10}$$

where $\mathbf{z} = \mathbf{U}^H\ \mathbf{n}$. Note that $\mathbf{s}$ and $\mathbf{z}$ have the same statistics as $\mathbf{x}$ and $\mathbf{n}$, respectively, in that $\mathbf{V}$ and $\mathbf{U}$ are unitary.

Note that the well known water-filling algorithm for optimally allocating power [10] (i.e., determining optimal $\mathbf{P}$) is derived from Shannon's extended capacity formula, not for the practical throughput case. Thus it is not suitable for practical LTE system [12]. Actually, equal power allocation is implemented in current LTE systems [11]. As a result, this paper assumes equal power allocation (at the transmitter), i.e., $\mathbf{P} = \mathbf{I}$. (Quantized precoding with limited feedback will be studied in future work.) With the assumption of $\mathrm{E}[|x_i|^2]/\mathrm{E}[|n_i|^2] = 1$, the SNR of the ith stream equals the square of the ith element of $\mathbf{\Lambda}$,

$$\gamma_i = \lambda_i^2\,. \tag{11}$$

The SVD-based MIMO throughput is the sum throughputs of all the parallel streams as expressed in (9).

Next, we show that combining precoding $\mathbf{W} = \mathbf{V}$ (that is also a unitary matrix) with ZF can achieve the same performance of a SVD-based MIMO system.

Proof: A 2×2 MIMO system with a precoder $\mathbf{W}$ (in a flat-fading channel) can be modeled as

$$\mathbf{y} = \mathbf{HWs} + \mathbf{n} = \mathbf{U\Lambda V}^H\mathbf{Ws} + \mathbf{n}\,. \tag{12}$$

Since $\mathbf{V}$ and $\mathbf{W}$ are unitary matrices, one can choose $\mathbf{W}$ such that

$$\mathbf{V}_{eq} = \mathbf{V}^H\mathbf{W} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{13}$$

where $\theta \in [0, \pi/2]$. The SNR of the 2×2 MIMO system with precoding $\mathbf{W}$ and ZF receiver is

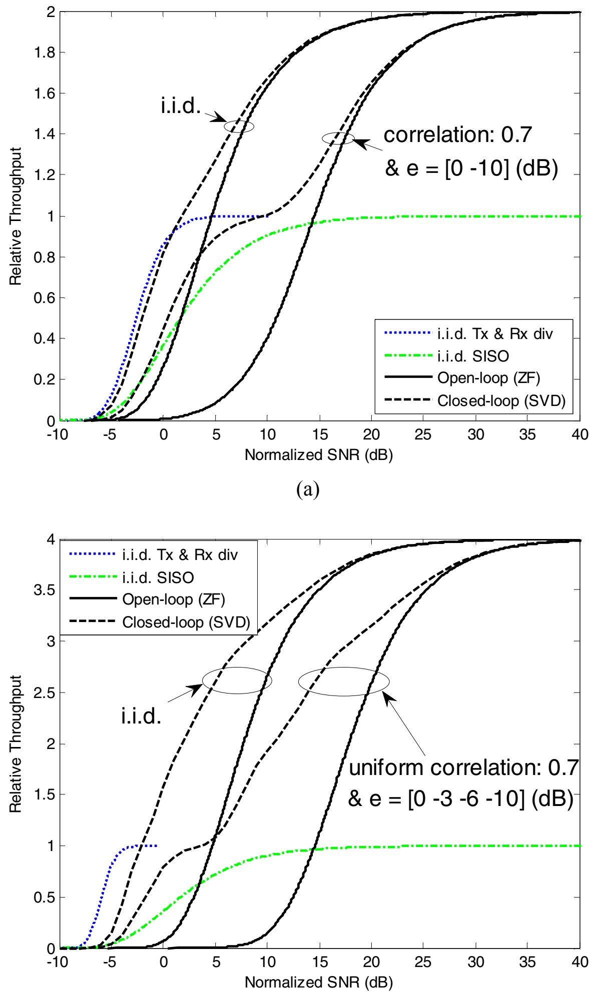

Figure 1. MIMO Throughput: (a) 2×2 MIMO systems; (b) 4×4 MIMO systems. As references, the throughputs of SISO and transmit (Tx) and receive (Rx) diversity systems in i.i.d. channels are plotted in the same figure. Note that the maximum throughput of a single stream is normalized to one in both graphs. Thus, the maximum throughput 2×2 and 4×4 MIMO systems are 2 and 4, respectively.

$$\gamma_i = \frac{1}{\left[\left(\left(\mathbf{HV}_{eq}\right)^H \mathbf{HV}_{eq}\right)^{-1}\right]_{i,i}} = \frac{1}{\frac{\cos^2\theta}{\lambda_i} + \frac{\sin^2\theta}{\lambda_l}} \qquad (14)$$

where $i,l \in \{1,2\}$ and $i \neq l$. Let $\theta = 0$ (i.e., $\mathbf{W} = \mathbf{V}$), (14) reduces to (11). □

This observation is rather intuitive. When $\theta = 0$, $\mathbf{V}^H\mathbf{W}$ becomes identity matrix, i.e., the precoder **W** diagonalizes the transmit singular vector matrix; and the receive singular vector matrix can be diagonalized similarly by a linear ZF decoder. Thus, parallel channels identical to the SVD case are obtained.

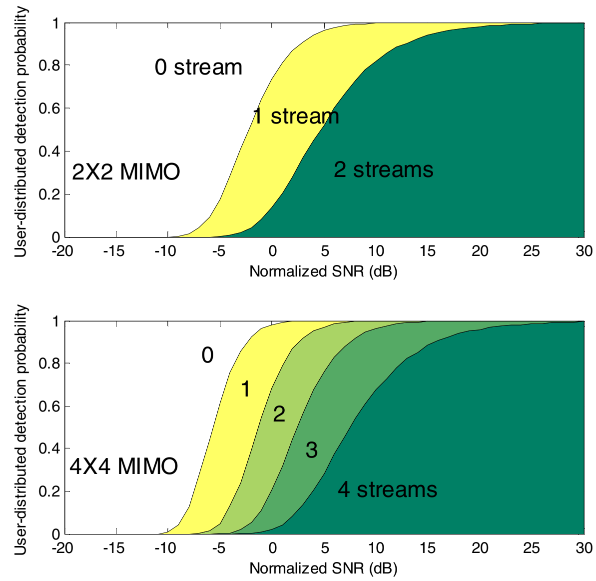

Figure 2. User-distributed detection probability of 2×2 (upper) and 4×4 (lower) open-loop MIMO systems. The different color regions represent the probability of the maximum number of streams supported.

III. Simulations and Results

Using the MIMO channel and throughput models developed in the previous section, we can study the effects of correlations and power imbalance between antenna ports on the MIMO throughput in a fading channel. We generate 2×2 and 4×4 i.i.d. Gaussian matrices with 20000 realizations and introduce power imbalance and correlation to the MIMO channel via (5) and (6).

Fig. 1 shows the throughputs of the 2×2 and 4×4 MIMO systems with both open- and closed-loop configurations. The throughputs of SISO and transmit (Tx) and receive (Rx) diversity systems in i.i.d. channels are plotted in the same figure as references. Note that the relative throughput is defined as the throughput normalized to its maximum value, and that the normalized SNR is the average received SNR divided by the threshold value. Also note that the maximum ratio combining (MRC) is assumed in simulations. The uniform correlation of 0.7 (for the 4×4 MIMO case) means that the correlation coefficient between any two antenna ports is 0.7. As expected, the closed-loop MIMO configuration offers better throughput performance compared with its open-loop counterpart, and that both correlation and power imbalances (due to different antenna efficiencies) affect the throughput performance adversely. Note that the maximum throughput of the 4×4 MIMO system is double of that of the 2×2 MIMO system (at the expense of more SNR required to achieve the full spatial multiplexing). Another interesting observation is that the throughput of the 2×2 closed-loop MIMO system at low SNR is slightly smaller than that of the transmit and receive diversity system. This can be explained by the fact that the received SNR of the (MRC) diversity system corresponds to the summation of all the eigenvalues (11), whereas throughput performance at low SNR is given by the largest eigenvalue for the closed-loop (or SVD-based) MIMO system. The inferior throughput of the closed-loop MIMO with respect to the diversity system becomes more noticeable in that the diversity system uses all the four eigenvalues, whereas the closed-loop MIMO system still relies on the largest eigenvalue in the low SNR regime.

Fig. 1 corresponds to the average throughput of a single user in a fading environment. Another useful presentation of the simulated throughput is to plot the throughput as the user-distributed detection probability. For illustration purpose, we focus on the open-loop MIMO systems with ZF receivers in an i.i.d. flat-fading channel. Fig. 2 shows the user-distributed detection probability for the 2×2 and 4×4 open-loop MIMO system, where different color regions represent the probability of the maximum number of streams supported. For instance, when the average SNR equals the threshold value (i.e., at 0-dB normalized SNR), the probability of detecting one, two, three, and four streams by using the 4×4 open-loop MIMO system are 92%, 65%, 20%, and 2%, respectively.

IV. CONCLUSION

In this paper, based on the threshold receiver [3], [6], we have developed throughput models for open- and closed-loop MIMO systems. Specifically for the closed-loop MIMO system, we show that by diagonalizing the transmit eigenvector matrix with a unitary precoder matrix, for which CSI must be known on Tx side, a ZF decoder offers the same system performance as the SVD-based MIMO. We compared the open- and closed-loop MIMO configurations (corresponding to CSI not know and known, respectively, on Tx side) under different correlations and power imbalances. As expected, the closed-loop MIMO configuration has better performance than the open-loop MMIO configuration, especially at low SNR. Nevertheless, it is clearly seen in Fig. 1 that, at low SNR, the diversity scheme offers the highest throughput; spatial multiplexing can only be achieved at high SNR. In addition to the usual presentation of the average throughput, we present the user-distributed detection probability, based on which, the probability of the number of supported streams is readily shown. The detection probability of two bit streams can be easily obtained from the ZF algorithm, and it is then almost the same as obtained by SVD for large SNR. For low SNR the detection probability of one bit stream is better for ZF than for SVD. Thus, when characterizing quality of wireless devices we can use the detection probability of bit streams as a quality metric, with the multiple bit stream cases having the i.i.d. result obtained by a ZF algorithm as the maximum reference cases, and we can determine quality in terms of degradation in dBiid relative to this reference at a certain probability level such as 95%.

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