Hamilton Algorithm and Overview of Novel Optimization Techniques

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1. Introduction

A single-port compact array antenna, i.e., electronically steerable parasitic array radiator (Espar) antenna [1], have shown the potential for application to wireless communications systems, and especially to mobile terminals. The (M+1)-element Espar antenna has only an active radiator connected to the receiver. The remaining M elements are parasitic. The antenna pattern is formed according to the values of the loaded reactance on these parasitic radiators.

Because of the configuration of the Espar antenna, we face the following three difficulties [1][2] in the development of optimum algorithms: a) The signals on all elements cannot be observed. Only the single-port output can be observed. b) The RF currents on the elements are not independent but mutually coupled with each other. c) The single-port output is a highly nonlinear function of the variable reactances that includes the admittance matrix inverse. In addition, unlike digital beamforming antennas, conventional criteria such as MMSE (Minimum Mean Square Error) are useless for the optimization of the Espar antenna, since the amplitude of the antenna output is difficult to be adjusted [2].

In this paper, we give an overview of criteria and optimization algorithms for beamforming and design of the Espar antenna. The criteria are maximum power, maximum cross-correlation coefficient and maximum m-th order moment [1][2][3]. We describe the optimization algorithms including random search algorithm [4], gradient-based algorithm [2][5], and Hamilton algorithm [6][7][8]. For the optimization of the Espar antenna, the gradient-based algorithm converge fast but sometimes unwillingly fall into a local minimum depending upon the initial value for one of their parameters. On the other hand, the random search algorithm tolerate local-minimum problems but rather slow to reach the final goal. Hamiltonian algorithm intends to meet the two conflicting requirements, i.e., to be deterministic and to be free from local problems. The algorithm is especially expected to work effectively in case that the number of parameters are very large.

2. Signal Model of Compact Array Antenna

In an (M+1)-element Espar antenna, the 0th element is an active radiator located at the center of a circular ground plane. It is a $\lambda/4$-length monopole (where λ is the wavelength) and is excited from the bottom in a coaxial fashion. The remaining elements of $\lambda/4$-length monopoles are parasitic radiators surrounding the active radiator symmetrically, with the circle's radius. Each of these elements is terminated by a variable reactance x_m, (m=1,2,…,M). The reactance vector is denoted by $\mathbf{x}=[x_1, x_2,\ldots, x_M]$. In [1][2], the RF current vector in elements is given by $\mathbf{i}=V_s(\mathbf{Z}+\mathbf{X})^{-1}\mathbf{u}_0$, where V_s is a constant, and $\mathbf{u}_0=[1,0,\ldots,0]^{\mathrm{T}}$. The diagonal matrix $\mathbf{X}=\mathrm{diag}[Z_0, jx_1,\ldots, jx_M]$ is called the reactance matrix, and $\mathbf{Z}=\left[z_{kl}\right]_{(M+1)\times(M+1)}$ is referred as to the impedance matrix, with z_{kl} expressing the mutual impedance between the elements k and l ($0\le k,l\le M$).

Suppose there are a total number of Q signals $u_q(t)$ with DOAs ϕ_q, ($q=1,2,\ldots,Q$**).** The output of the Espar antenna is $y(t)=\sum_{q=1}^{Q}\mathbf{i}^{\mathrm{T}}\mathbf{a}(\phi_q)u_q(t)+n(t)$, where $\mathbf{a}(\phi_q)=[a_0(\phi_q),\ldots,a_M(\phi_q)]^{\mathrm{T}}$ is the steering vector, and $n(t)$ is noise. Note that $y(t)$ is a function of the reactance vector $\mathbf{x}$.

3. Criteria for Optimization

This section gives three criteria for optimum control and design of the Espar antenna.

3.1 Maximum Power

The signal power of the antenna output is defined by $J_{\mathrm{P}}(\mathbf{x})=E[y(t_s)y^*(t_s)]$. When only a single signal from the direction ϕ is impinged, $J_{\mathrm{P}}(\mathbf{x})$ is seen as the power $J_{\mathrm{P}}(\mathbf{x},\phi)$ to be received in the direction ϕ of arrival of the signal. The criterion of the maximum power is used for sector beamforming [3] and antenna design to get a maximum gain [8].

3.2 Maximum Cross-Correlation Coefficient

Instead of MMSE, we propose a Maximum Cross-Correlation Coefficient (MCCC) as a trained criterion for antenna beamforming [1]. It is well known that the cross-correlation coefficient represents the similarity of two signals, while the error represents the difference. For a given radio environment, the normalized cross correlation coefficient $J\rho(\mathbf{x})$, between the output signal $y(t)$ and the reference signal $r(t)$, varies over the range [0, 1], as the reactance vector is controlled. The interference signals in the output signal $y(t)$ are suppressed when $y(t)$ becomes similar to the reference signal $r(t)$, regardless of their difference in amplitude. Employing the cross-correlation function avoids the need for an extra amplitude control (e.g., automatic gain control) on $y(t)$. For the Espar antenna, as shown below, this provides an effective solution to the difficulty of adjusting the amplitude of the output signal so that it equals the amplitude of the reference signal.

3.3 Maximum *m*-th Order Moment

The blind criteria used in digital beamforming antennas (such as the well-known CMA criterion) are again unsuitable for adaptive control of analog smart antennas. We instead propose the Maximum *m*-th order Moment Criterion (MMMC) [1]. This is defined similarly to the MCCC, but the objective function is $J_M(\mathbf{x}) = |E[y^m(t)]|^2 / |E[y^m(t)]^2|$, where m is the modulation index of the desired signal. This operation is effective for any signal modulated in m-ary phase shift keying. Say $m = 2$ for BPSK and $m = 4$ for QPSK. It works without *a priori* training code. The only information the receiver has to know beforehand is the index m. The statistical behavior of the objective function is somewhat more complex than that of MCCC but can be derived as a monotonous function of SINR.

4 Optimization Algorithms

This section describes the three optimum algorithms for optimizing the Espar antenna. In general, the three criteria we stated in Section 3 can be used to the three optimum algorithms.

4.1 Random Search Algorithm

A simple approach to obtain an optimum reactance vector is by random search algorithm [4], also called Monte Carlo method. The algorithm is algebraically describe as

$$\mathbf{x}_{opt} = \max_{n=1,2,\dots N} J(\mathbf{x}(n))$$

where $J(\mathbf{x}(n))$ is an estimate of an objective function based on samples of y with $\mathbf{x}=\mathbf{x}(n)$. For example, the objective function can be the cross-correlation coefficient $J\rho(\mathbf{x}(n))$.

4.2 Gradient-Based Algorithm

The random search algorithm above has the drawback that nothing is learned when a trial at step n is completed. The next trial at step n+1 is independent from the previous step. It does not take any local continuity properties of the objective function surface. The gradient-based adaptive algorithm gives a good solution of this problem. The goal of the gradient-based adaptive algorithm of the Espar antenna is to find a reactance vector $\mathbf{x}$ such that an objective function, one of the three functions in Section 3 (e.g., cross-correlation coefficient), is as large as possible. In the steepest gradient algorithm [2], the update value of the reactance vector at time n+1 is computed by using the simple recursive relation

$$\mathbf{x}(n+1) = \mathbf{x}(n) + \mu \nabla J(\mathbf{x}(n))$$

where μ is a positive real-value scalar constant that controls the convergence speed, and $\nabla J(\mathbf{x}(n))$ is the gradient vector of function $J(\mathbf{x}(n))$.

There may be some difficulty when we compute the gradient vector. As we have stated above, this arises from the facts that a) it may not be easy to analytically represent the gradient vector as a function of $\mathbf{x}$ because of the presence of an intractable matrix inverse in the representation of $y(t)$, and b) the signal vector impinging on each element of the antenna cannot be observed. An estimate of the gradient vector may be derived by the use of a finite-difference approximation of derivatives [2]. In this approximation, only one component of the vector $\nabla J(\mathbf{x}(n))$ is calculated at a time n from the output of the antenna. All of the components of reactance vector **x** are sequentially perturbed in order to get one gradient vector for each iteration. This sequential perturbation of the reactance requires K+1 times transmission of the signal (with length L bit) for one iteration. Thus, a total of $L(K+1)N$ symbols are required for N iterations. In addition, details for a simultaneous perturbation to calculate the gradient vector can be found in [5].

4.3 Hamiltonian Algorithm for Multiple-Parameter Optimization

There have been developed several sorts of algorithms to find optimum set of multiple parameters for a given criterion or objective function of the parameters. They are generally classified into two categories: 1) deterministic; and 2) randomized. Deterministic ones, such as the gradient-based algorithm, converge fast but sometimes unwillingly fall into a local minimum depending upon the initial value for one of their parameters. On the other hand, randomized ones, such as the random search algorithm tolerate local-minimum problems but rather slow to reach the final goal. Ones based on the genetic concept seem to be deterministic at first glance, but they actually involve mutation in a random fashion to escape from local traps.

Hamiltonian algorithm [6][7] intends to meet the two conflicting requirements, i.e., to be deterministic and to be free from local problems. The algorithm is especially expected to work effectively in case that the number of parameters are very large.

Hamiltonian algorithm was applied to antenna parameter optimization first for Espar antenna [8]. Hamilton algorithm originally stems from a heuristic idea with autonomous motion of lumped mass in a friction-free potential space. The point of mass moves according to the law of energy conservation, i.e., kinetic energy plus potential energy keeps constant during the motion. The unknown reactance vector **x** and the scalar objective function J, we stated in Sec. 3, are compared to the location and the potential energy of the moving mass, respectively. Table 1 shows the analogy from dynamics, mathematics, to antenna engineering. One can find that the key feature of Hamiltonian algorithm lies in the momentum, which does not appear either in mathematics or in antenna engineering.

To convert antenna issues into dynamics, the space for M-dimensional reactance vector **x** is expanded to a 2M-dimensional space

$$[\,\mathbf{x}, \mathbf{p}\,] = [\,x_1, x_2, \ldots, x_M, p_1, p_2, \ldots, p_M\,]$$

where the appended M components imply the momentum vector of the mass. Hamiltonian function defined as the sum of potential energy and kinetic energy

$$H(\mathbf{x}, \mathbf{p}) = \psi(\mathbf{x}) + K(\mathbf{p})$$

plays the key role in dynamics. Function $H(\mathbf{x}, \mathbf{p})$ is kept at constant value, say E, provided that it does not explicitly contain time, where E is called *total energy*. The potential energy can be the negative one of the functions we stated in Section 3. For example, $\psi(\mathbf{x}) = -J_{\mathrm{P}}(\mathbf{x},\phi)$, where $J_{\mathrm{P}}(\mathbf{x},\phi)$ is the power of the antenna to be radiated in the direction ϕ. Kinetic energy K(**p**) in this particular system is considered as

$$K(\mathbf{p}) = \frac{1}{2m}\|\mathbf{p}\|^2 = \frac{1}{2m}\sum_{i=1}^{M} p_i^2$$

where the mass can be assumed to be unity for simplicity. This is valid since the movement is invariant against the mass. Thanks to the inertia of the mass i.e. momentum, it is not trapped by local minimums but effectively passes over them. Since the kinetic term itself is not concerned from the antenna engineering view point, we can introduce arbitrary positive constant index γ to modify the above formula into

$$K(\mathbf{p}) = \frac{1}{2m}\left(\sum_{i=1}^{M} p_i^2\right)^{\gamma}.$$

This modification effectively improves the efficiency of parameter optimization process. If and only if

Table 1 Analogy from mass dynamics to antenna engineering

dynamics	mathematics	ESPAR antenna
dimension	number of unknowns	number of varactors
location of mass	unknown vector	reactance vector
potential energy	objective function	directional gain
momentum	not concerned	not concerned

we choose $0 < \gamma < 1$, probability density is higher for the mass to stay at locations with lower potential while moving in the $2M$-dimensional space. This corresponds to higher probability for the antenna to reach a higher directional gain in the algorithm. Such a phenomenon indeed looks diametrically opposite from experiences in the actual space, but it is wonderfully true in multiple-dimensional spaces. See Refs. [6][7][8] for mathematical proof.

Function $H(\mathbf{x}, \mathbf{p})$ leads us to a system of canonical equations for x_i and p_i, formulated as

$$\begin{cases} \dfrac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \dfrac{\partial H(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}} \\ \dfrac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\dfrac{\partial H(\mathbf{x},\mathbf{p})}{\partial \mathbf{x}} \end{cases}$$

where partial derivative in terms of **p** or **x** indicates the gradient in each direction. We can trace the mass trajectory by alternately iterating the coupled equations. Simulations in [8] show that the moving mass hits the vicinity of optimum spot, i.e., the region of high directional gain of antenna, with high probability. Figure 1 shows histograms of the directional gain obtained by the Hamiltonian algorithm (HA) during 10,000 time steps. For the sake of comparison, also depicted on the same chart are gain histograms obtained from random search, i.e., Monte Carlo (MC), simulation in 10,000 random trials for the same condition as HA. It is found that HA exhibits significantly higher probability to hit high gains than MC does. This advantage is supposed to be even more enhanced in case the number of unknown parameters should increase [8].

5. Conclusion

We have given an overview of the optimum criteria and algorithms for Espar antenna. Among the three optimum algorithms, Hamiltonian algorithm intends to be deterministic and to be free from local problems. The algorithm is especially expected to work effectively in case that the number of parameters are very large.

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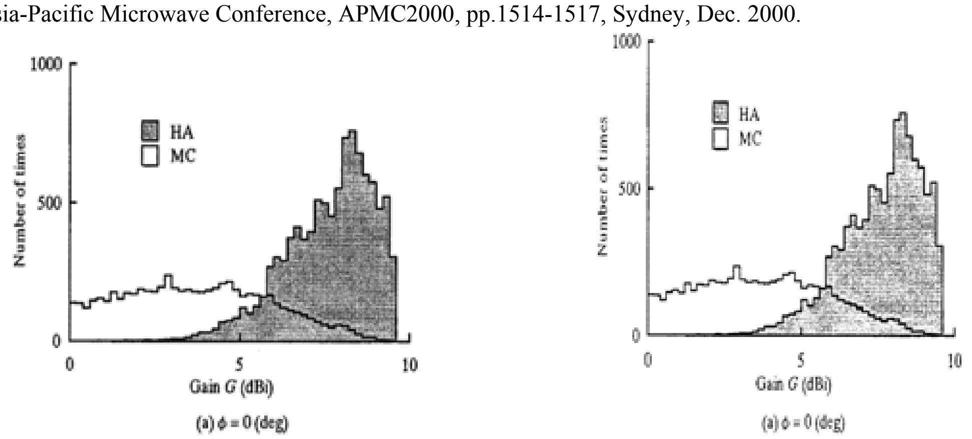

Fig. 1 Directional gain histograms counted by Hamiltonian algorithm (HA) and Montecarlo method (MC) [8].