FPGA Implementation of Complex Valued fastICA Algorithm for On-line Array Signal Processing

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1. Introduction

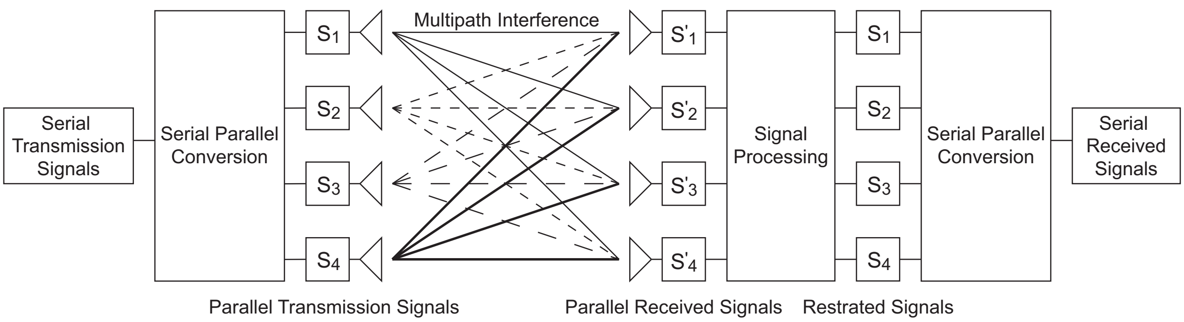

Figure 1 : MIMO Communication System

The blind estimation technique is useful for an adaptive antenna, and a MIMO communication system. These wireless technologies perform feedback control that the optimal system performance is obtained by adjusting a system parameter according to the radio wave situation observed with a certain means. In this system parameter determining method, Independent component analysis (ICA) is very useful.

ICA is the blind estimation technology which developed remarkably in recent years. Based on the statistical independency of a signal, it is the method of finding out a desired signal out of the crowded noise. The method has developed focusing on the field of image processing or speech signal processing. The application to various fields is progressing by the appearance of the robust and high-speed algorithm fastICA by Hyvarinen.

Since calculation of ICA is iteration of matrix calculation. Matrix calculation is suitable for FPGA.

2. Blind Estimation by ICA

When there are N sources of a signals and signals $s_i(t)$, i=1,…, n is transmitted from them, these are called Original signals. Suppose that these signals are observed as $x_i(t)$, i=1,…, n with M receivers, these are called Observation signals. Supposing it is alignment combination, the observation signals will become like the following formula using original signals.

$$x_i(t) = \sum_{j=1}^{n} a_{i,j} s_j(t),\ i = 1,\dots,m \qquad (1)$$

Here, $a_{i,j}$, i=1,…,n,j=1,…,m are the coefficients showing mixed strength. Signal **x**=**As** is observed when arbitrary signals s are transmitted. This A is called channel matrix.

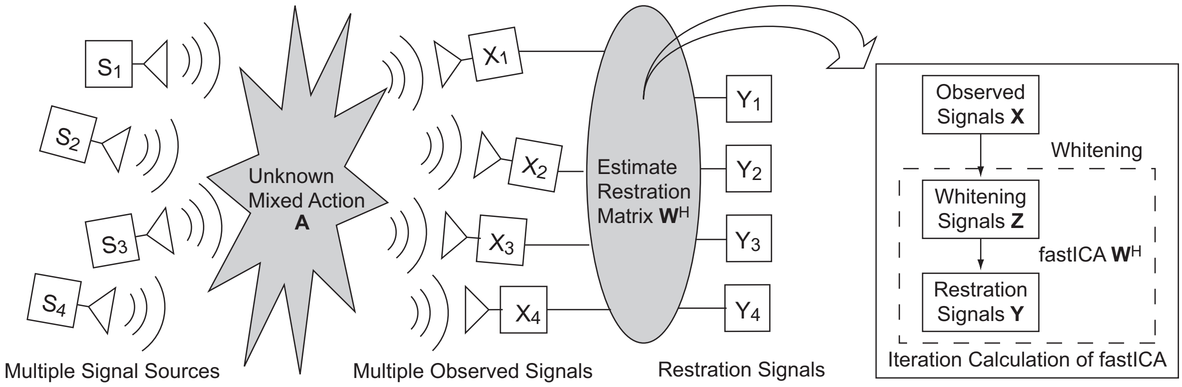

Figure 2: fastICA processing flow

Generally, since the channel matrix is unknown, it needs to estimate a separation vectors $\mathbf{b}_j^H$ by any way.

ICA determines a separation vectors $\mathbf{b}_j^H$, assuming the statistical independency between Original signals. The statistical independency of a microwave signal is defined by the following formula. Blind signal separation is realized using this.

$$E\{g_1(s_1)\cdots g_N(s_N)\} = E\{g_1(s_1)\}\cdots E\{g_N(s_N)\} \tag{2}$$

Here, $E\{g(x)\}$ shows the expected value of $g(x)$, and $g(x)$ shows arbitrary functions.

As shown is a statistical central limit theorem, the probability distributions are deviated from a normal distribution, the original independent microwave signals are acquired. Therefore, an independent component $\mathbf{y}_j$ will be obtained, if the probability distributions of the separation signal $\mathbf{y}_j=\mathbf{b}_j^H\mathbf{x}$ which are alignment combination of an observation signal $\mathbf{x}=\mathbf{A}\mathbf{s}$ choose $\mathbf{b}_j^H$ so that it may separate from a normal distribution.

The negentropy $J(x)$ to the environment variable x which some measures of the distance of a certain probability distributions and normal distribution are proposed, and is especially defined by the following formula is often used.

$$J(x) = H(x_{gauss}) - H(x) \tag{3}$$

$$H(x) = -\int p_x(\eta)\log p_x(\eta)d\eta \tag{4}$$

X_{gauss} is a random variable which shows Gaussian distribution, and $H(x)$ shows entropy. The approximation formula $J(x) \propto [E\{G(x)\} - E\{G(x_{gauss})\}]^2, G(x) = \text{logcosh}(x)$ is proposed in consideration of calculation cost.

In ICA, it asks for bjH which makes the measure of such non-normality the maximum by the optimizing method. These are the steepest descent method, Newton method and fixed-point approach as typical optimizing method.

With many ICA algorithms, the preparation called whitening for efficient optimization is needed. The whitening signal $\mathbf{z}$ is acquired by the linear transformation $\mathbf{Vx}$. Whitening matrix $\mathbf{V}$ is expressed with $\mathbf{D}^{-1/2}\mathbf{E}^H$. Here, $\mathbf{D}$ is a diagonal matrix which has a characteristic value of $\mathbf{C} = \{\mathbf{x}\mathbf{x}^H\}/N_s$ the autocorrelation matrix of $\mathbf{x}$. As the elements, N_s are the number of samples on a time-axis. $\mathbf{E}$ is the matrix which put the eigenvectors corresponding to $\mathbf{D}$. By conducting ICA to this whitening signal, a separation matrix $\mathbf{W}^H$ is estimated and a separation signal $\mathbf{y}$ are acquired by $\mathbf{y} = \mathbf{W}^H\mathbf{z}$.

fastICA is the algorithm of ICA that robust at high-speed which was devised by Hyvarinen and extended for the Complex-valued. In fastICA, when whitening signal **z** is acquired, the separation vector **w** is calculated by following formula.

$$w \leftarrow E\{z(w^H z)^* g(|w^H z|^2)\} - E\{g(|w^H z|^2) + |w^H z|^2 g'(|w^H z|^2)\} w \quad (5)$$

Here, $g(x) = \tanh(x)$, $g'(x) = 1 - \tanh^2(x)$.

Next, orthogonalize to obtained **W**. Orthogonalization is given by formula $\mathbf{W} \leftarrow \mathbf{E}\mathbf{D}^{-1/2}\mathbf{E}^H\mathbf{W}$. Here, **E** is the matrix which put the eigenvector of **W** and **D** is the matrix which put the eigen value of **W**.
Finally, it iterates until it judges convergence of obtained **W** and fully converges. When fully converged by **W**, It is a separation matrix for the **W** to separate an observation signal into a transmitted signal.

3. Experimental Result

At this time, It performs Signal separation Experiment of wireless-communication system that 4 transmitting 4 reception (Fig. 3). The experimental parameter is as follows. Calculation accuracy is 18bit binary fixed point value and number of samples of observation signal is 512 point. The target device is Xilinx XC4VFX12. The resource of Table 1 was used this time.

Table 1: The used resource

Logic Utilization	Used
Number of Slices	4325 (79%)
Number of Slice Flip Flops	2418 (22%)

When, The transmission signal **S** is uniform distribution (Fig. 4), the observation signal **X** becomes Fig. 5. Fig. 6 is distribution of the signal **Z** what is performed whitening processing. Fig.7 is distribution of the separation signal **Y** when updating separation matrix **W** 14 times.
For evaluation of separation signal **Y**, it calculates the correlation of transmission signal **S** and separation signal $\mathbf{Y}^H$. Correlation matrix is calculated by $E\{\mathbf{S}(t_i)\, \mathbf{Y}^H(t_i)\}$.If **S** equal to **Y**, correlation matrix becomes to unit matrix. Consequently, the separation matrix **Y** approaches transmission signal **S**, the values of correlation matrix approaches 1. Fig.8 is the correlation matrix when updating separation matrix 14 times, it shows that signal separation is possible by using FPGA.
Table 2 is the calculation time in PC (Intel Celeron 1.73 GHz, RAM 2.5GB) and FPGA.

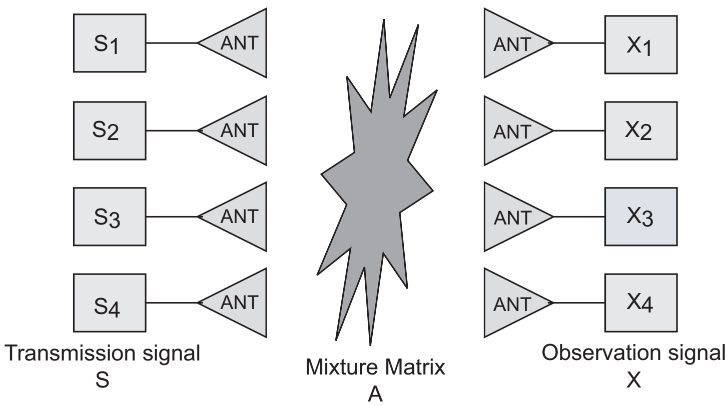

Figure 3: Experimental System

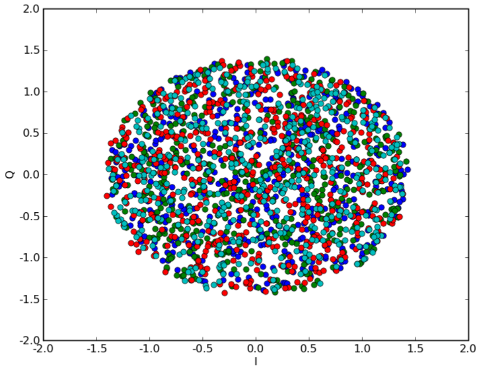

Figure 4: Distribution of transmission signal **S**

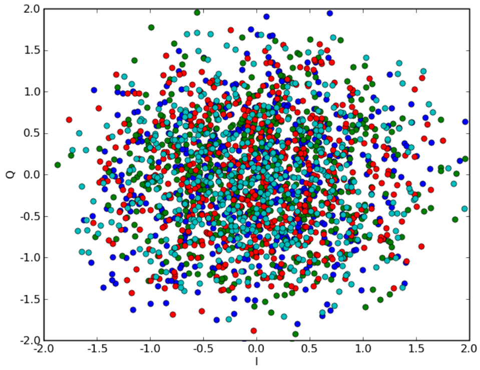

Figure 5: Distribution of observation signal **X**

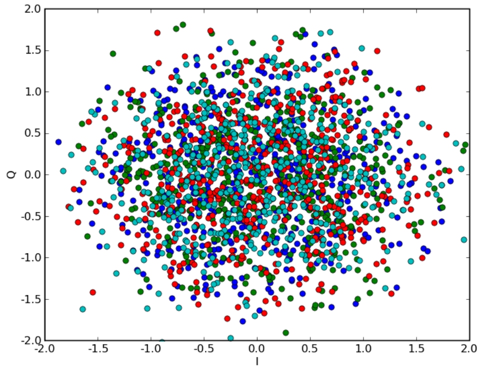

Figure 6: Distribution of whitening signal **Z**

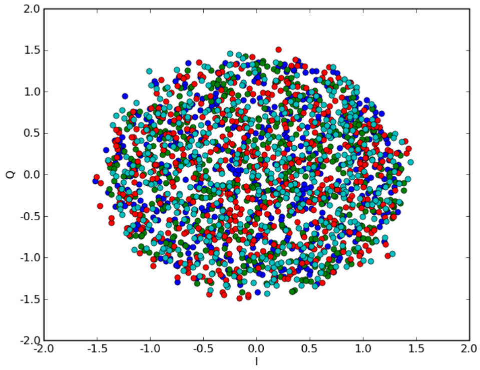

Figure 7: Distribution of separation signal **Y**

$$E\{S Y^H\} = \begin{bmatrix} 0.07 & 0.05 & \textcircled{0.99} & 0.08 \\ 0.06 & 0.10 & 0.06 & \textcircled{1.00} \\ \textcircled{1.00} & 0.06 & 0.05 & 0.09 \\ 0.03 & \textcircled{1.00} & 0.04 & 0.05 \end{bmatrix}$$

Figure 8: Correlation about transmission signal and separation signal

Table 2: Calculation Time

	Calculation Time
PC	4.84 [s]
FPGA	98.7 [ms]

4. Conclusion

This paper describes that implementation of the fastICA algorithm on FPGA, and it can check that high-speed processing is possible compare with PC. From now on, optimizing circuit composition and signal processing algorithm, accelerate calculation is possible. It is thought by implement processing by FPGA, that on-time processing is possible.

References

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