

# Study on Improvement of Convergence Characteristics of the Blind MMSE Adaptive Array in OFDM Transmission System

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## 1. Introduction

It is well known that the OFDM (Orthogonal Frequency Division Multiplexing) transmission technology is used in the terrestrial digital TV broadcasting and wireless LAN. The OFDM is featuring high efficiency of frequency utilization and also effective use of FFT in the modulation scheme. In addition, a guard interval (GI) is inserted into the head of each OFDM symbol to overcome the delay spread of the channel. Hence, the communication performance of OFDM is superior to that of a single carrier in a multipath environment where the delay times of multipath waves are within the GI length.

On the other hand, inter-carrier interference is caused by incidence of multipath waves with delay times exceeding the GI length or by Doppler shift at mobile reception, which results in serious degradation in the OFDM transmission performance. For suppressing the inter-carrier interference, beamforming and null-steering techniques by adaptive array antennas have been investigated [1]–[3]. As one of them, the MMSE (Minimum Mean Square Error) adaptive array utilizing the GI in OFDM was proposed which is a blind adaptive processor of pre-FFT type [1]. However, due to the blind processing, the improvement of the convergence characteristics of the adaptive system remains to be addressed [4].

In this paper, therefore, we utilize in the MMSE adaptive system the techniques of multiple weight generation in each OFDM symbol [5], smoothing of the correlation vector, and time average using a window function [1]. Through computer simulation, we demonstrate how significantly convergence characteristics of the system are improved by the additional techniques.

## 2. Signal Model

Suppose that a  $K$ -element array antenna receives  $L$  ( $L \leq K$ ) multipath waves whose directions of arrival are different from each other. Figure 1 shows the MMSE adaptive array system utilizing the GI in OFDM. Here, the vector forms of array element signals:  $x_1(t), \dots, x_K(t)$  and weights:  $w_1, \dots, w_K$ , and the combined array output  $y(t)$  are expressed respectively as follows:

$$\mathbf{X}(t) = [x_1(t), x_2(t), \dots, x_K(t)]^T, \quad \mathbf{W} = [w_1, w_2, \dots, w_K]^T, \quad y(t) = \mathbf{W}^H \mathbf{X}(t) \quad (1)$$

In Fig.1, CHOP-H and CHOP-T stand for signal extraction during the head guard interval (Head GI) and during the tail guard interval (Tail GI), respectively, of the desired OFDM signal that is a synchronized signal. This is depicted in Fig.2 which shows the OFDM signal (desired signal) along with the delayed signal (undesired signal) in time domain. We let  $x_{hk}(t)$  and  $x_{tk}(t)$  ( $k = 1, 2, \dots, K$ ) represent those extracted input signals and  $\mathbf{X}_h(t)$  and  $\mathbf{X}_t(t)$  represent their vector forms. Similarly, we have the extracted array output signals  $y_h(t)$  and  $y_t(t)$  through CHOP-H and CHOP-T, which are expressed as  $y_h(t) = \mathbf{W}^H \mathbf{X}_h(t)$  and  $y_t(t) = \mathbf{W}^H \mathbf{X}_t(t)$ , respectively.

## 3. Optimization Algorithms

### 3.1 MMSE adaptive array utilizing the GI in OFDM

Head GI and Tail GI are identical in each OFDM symbol of the desired signal as shown in Fig.2. Using this property, we determine the weight vector  $\mathbf{W}$  on the MMSE criterion [1]. Using  $y_t(t)$  as a

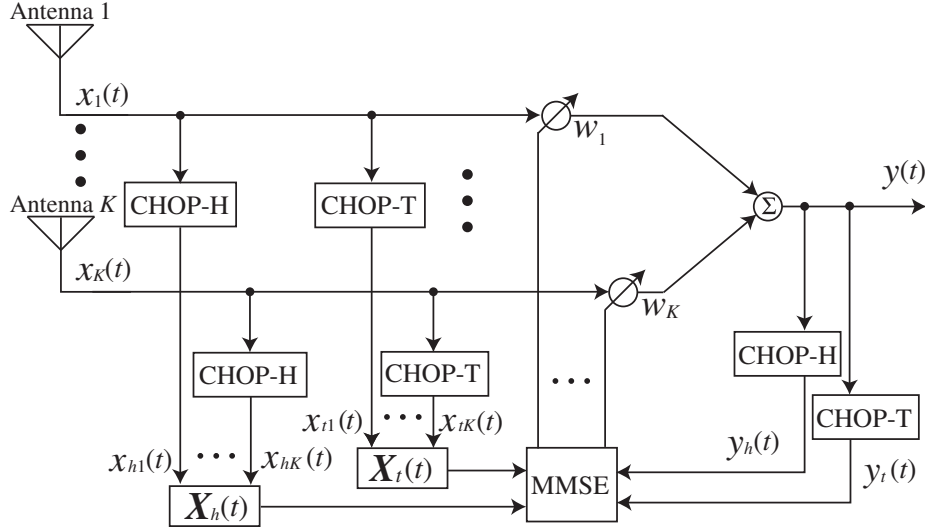


Figure 1: MMSE adaptive array system utilizing the GI in OFDM ( $K$ -element array).

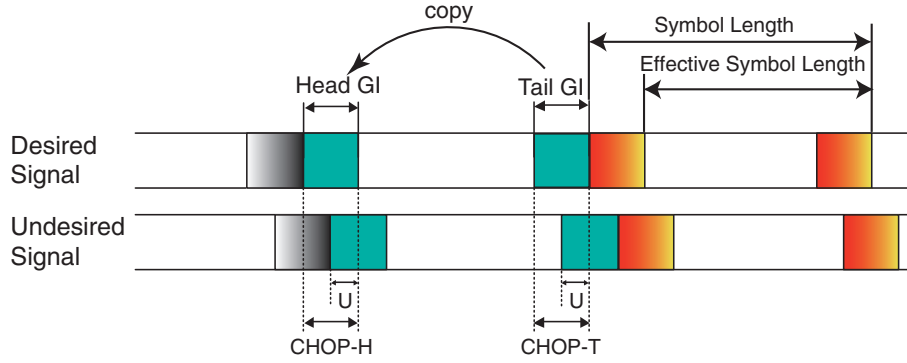


Figure 2: Relation between guard intervals of the desired and undesired signals.

reference signal for MMSE, the cost function which is minimized with respect to  $\mathbf{W}$  is expressed as

$$E[|e(t)|^2] = E\left[|\mathbf{W}^H \mathbf{X}_h(t) - y_i(t)|^2\right] \quad (2)$$

where  $E[\cdot]$  means the expected value. Thus, the SMI (Sample Matrix Inversion)-based algorithm to update recursively  $\mathbf{W}$  is given by

$$\mathbf{W}(m) = \left\{ave\left[\mathbf{X}_h(m, t)\mathbf{X}_h^H(m, t)\right]\right\}^{-1} ave\left[\mathbf{X}_h(m, t)\mathbf{X}_t^H(m, t)\right] \mathbf{W}(m-1) \quad (m = 1, 2, \dots) \quad (3)$$

where  $m$  is the iteration number which is equal to the symbol number, and  $ave[\cdot]$  means time average during the GI, which takes the place of  $E[\cdot]$  in Eq.(2). In Eq.(3),  $\mathbf{X}_h(m, t)$  and  $\mathbf{X}_t(m, t)$  express the samples of extracted input vectors at the  $m$ th symbol, and  $ave\left[\mathbf{X}_h(m, t)\mathbf{X}_t^H(m, t)\right] \mathbf{W}(m-1)$  represents the correlation vector  $\mathbf{r}_{xr}(m)$  in the MMSE criterion.

### 3.2 Multiple weight generation in each OFDM symbol

Multiple weight generation in each OFDM symbol is the way to update efficiently the weight vector for relatively long duration of one OFDM symbol [5]. Increasing the number of multiple updates in each symbol enables the weight vector to converge rapidly to the optimum one. The algorithm in this case is given by

$$\begin{cases} \mathbf{W}(m, i) = \left\{ave\left[\mathbf{X}_h(m, t)\mathbf{X}_h^H(m, t)\right]\right\}^{-1} \mathbf{r}_{xr}(m, i) & (i = 1, 2, \dots, J) \\ \mathbf{r}_{xr}(m, i) = ave\left[\mathbf{X}_h(m, t)\mathbf{X}_t^H(m, t)\right] \mathbf{W}(m, i-1) \end{cases} \quad (4)$$

where  $\mathbf{W}(m, i)$  and  $\mathbf{r}_{xr}(m, i)$  represent the weight vector and correlation vector, respectively, at the  $i$ th iteration of the  $m$ th symbol, and  $J$  is the total number of multiple updates in each OFDM symbol. The last weight of the  $m$ th symbol is used as the initial weight of the next  $(m+1)$ th symbol, i.e.,  $\mathbf{W}(m, J) = \mathbf{W}(m+1, 0)$ .

### 3.3 Smoothing of the correlation vector

To suppress the undesired signal effectively, the correlation vector has to include only the desired signal component. However, as observed in Fig.2, CHOP-H and CHOP-T signals include the correlated undesired signal component denoted by “U”, and hence the correlation vector comes to involve the undesired signal component, which results in slow or unstable convergence of the algorithm.

Therefore, we propose here the smoothing technique of correlation vector to try to overcome the instability due to the undesired signal component in the correlation vector. In this paper, the correlation vector is determined on the basis of the last weight of the previous symbol as follows:

$$\mathbf{r}_{xr}(m, i) = \text{ave} \left[ \mathbf{X}_h(m, t) \mathbf{X}_t^H(m, t) \right] \{ (1 - \beta) \mathbf{W}(m - 1, J) + \beta \mathbf{W}(m, i - 1) \} \quad (i = 1, 2, \dots, J) \quad (5)$$

where  $\beta$  is the real value called the forgetting factor which satisfies  $0 < \beta \leq 1$ .

### 3.4 Time average using a window function

Here, we introduce time average using a window function into the algorithm to reduce directly the effect of the undesired signal component on the correlation vector [4]. The algorithm using Eq.(5) is described as follows:

$$\begin{cases} \mathbf{W}(m, i) = \left\{ \text{ave} \left[ f(t) \mathbf{X}_h(m, t) \mathbf{X}_h^H(m, t) \right] \right\}^{-1} \mathbf{r}_{xr}(m, i) & (i = 1, 2, \dots, J) \\ \mathbf{r}_{xr}(m, i) = \text{ave} \left[ f(t) \mathbf{X}_h(m, t) \mathbf{X}_t^H(m, t) \right] \{ (1 - \beta) \mathbf{W}(m - 1, J) + \beta \mathbf{W}(m, i - 1) \} \end{cases} \quad (6)$$

where  $f(t)$  is the window function and it is obvious that the ideal window function is the one excluding the samples denoted by “U” from CHOP-H and CHOP-T samples in Fig.2.

## 4. Performance Analysis by Computer Simulation

Through computer simulation, we carry out the performance analysis of MMSE adaptive array utilizing GI with the techniques of multiple weight generation in each symbol, smoothing of the correlation vector and time average using the window function. Tables 1 and 2 show the simulation conditions and radio environment, respectively. It is assumed that two plane waves are incoming with no fading and also that symbol and frequency synchronization to the desired signal (wave 1) is completely performed. The initial weight vector is  $\mathbf{W}(0) = \mathbf{W}(1, 0) = [1, 0, 0, 0]^T$ , and the forgetting factor  $\beta$  is 0.5.

First, Fig.3 shows the difference signal  $|e(t)|^{\frac{1}{2}}$  between CHOP-H and CHOP-T signals as a function of time. It is seen that the difference signal  $|e(t)|^{\frac{1}{2}}$  and the ideal window function are quite alike, and so the difference signal can be used as the window function  $f(t)$ . In this paper,  $f(t) = |e(t)|^{\frac{1}{2}}$  is adopted for demonstration.

Next, Fig.4 shows the SINR convergence characteristics of the four algorithms which are described in Table 3. Algorithm 1 is the conventional one. Algorithm 2 using the multiple weight generation in

Table 1: Simulation conditions.

modulation scheme	64QAM
number of carriers	1405
effective symbol length	2048 samples
GI	128 samples
array configuration	4-element uniform linear array
element spacing	half wavelength of center frequency
number of waves	2
CNR	25dB

Table 2: Radio environment.

	wave 1 (Desired)	wave 2 (Undesired)
DOA from array broadside	40 deg	-20 deg
delay time	—	10 samples
DUR	—	0dB

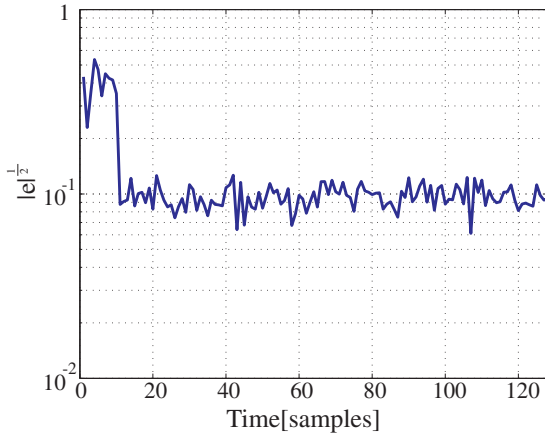


Figure 3: Difference signal  $|e(t)|^2$  between CHOP-H and CHOP-T signals.

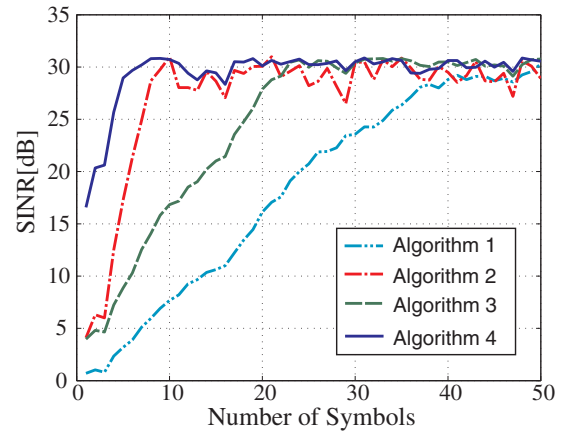


Figure 4: SINR convergence characteristics of MMSE adaptive arrays.

Table 3: Algorithms for MMSE adaptive array utilizing GI.

	number of updates in each symbol	smoothing of the correlation vector	window function
Algorithm 1	1	w/o	w/o
Algorithm 2	5	w/o	w/o
Algorithm 3	5	w/	w/o
Algorithm 4	5	w/	w/

each symbol gives faster convergence of SINR than the conventional one. However, SINR performance of Algorithm 2 is unstable in the stationary state after the 10th symbol. Although Algorithm 3 using additionally the smoothing of the correlation vector can improve the stability of SINR in the stationary state, the convergence is slow compared with Algorithm 2. On the other hand, it is found that Algorithm 4 using further time average with the window function keeps fast convergence and stable SINR performance compared with other three algorithms.

## 5. Conclusion

Performance of the blind MMSE adaptive array utilizing the OFDM guard interval has been examined in the case of using the techniques of multiple weight generation in each symbol, smoothing of the correlation vector and time average using a window function. As a result of computer simulation, it is shown that the combination of those techniques can provide the algorithm with fast and stable convergence.

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