

# A CFAR Circuit of Detecting Spatially Correlated Target for Automotive UWB Radars

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## 1 Introduction

Automotive radars for potential accident reduction and for autonomous cruise control were available in 1999, and 77 GHz long-range radars and 24 GHz short-range radars are developing [1]. 79-GHz short-range automotive radars have a bandwidth of 4 GHz, a maximum range of 30 m, and a range resolution of 5 cm.

Radar reception signals often contain unwanted echoes, called “radar clutter.” Moving target indication (MTI) is a possible way for detecting a target with a motionless radar and the clutter objects are motionless, too. Automatic target detection with a fixed threshold encounters an increase in a probability that the radar faulty detect a target when there is no target. Constant false alarm rate (CFAR) circuits measure local mean amplitude to suppress an increase in the false alarm rate. CFAR adjusts the threshold adaptively using the average samples [2].

Radar echoes tend to have high peak-to-average power ratio an increase in a resolution, then the the echoes are referred to as spiky. For sea clutter, there are many echoes from sea surface facets within the radar illumination area and the resulting signal obeys Rayleigh distribution because of the central limit theorem. With high resolution radars, however, a few facets contribute the resulting signal and their mutual interferences characterize spiky clutter [3]. The amplitude fluctuation has been modeled as Weibull distribution, log-normal distribution, and  $K$ -distribution. Various modifications of CFAR circuits, such as ordered-statistics CFAR, cell-average CFAR, and and-or CFAR, are examined for automotive radars [4].

In this paper, a CFAR circuit that measures local mean powers of both target and clutter is proposed. High resolution automotive radars would receive spiky signals not only for clutter but also for target. Spatial correlation is introduced to evaluate performance of the proposed method, because automotive radars have higher range resolution that target and clutter would occupy several range cells, and spatial correlation increases a required target signal power for detection.

## 2 Target Detection with Automotive UWB Radars

### 2.1 CFAR

Here, we define the false alarm probability  $P_F$  as the probability that a radar detects a target when a target does not exist, while the detection probability  $P_D$  is the probability that a radar detects a target when a target exist.

Figure 1 shows the conventional CFAR. We consider the case that video amplitude input is clutter only and the statistics follows the Rayleigh distribution  $p(x) = \frac{2x}{\bar{x}^2} \exp\left(-\frac{x^2}{\bar{x}^2}\right)$ . The probability that

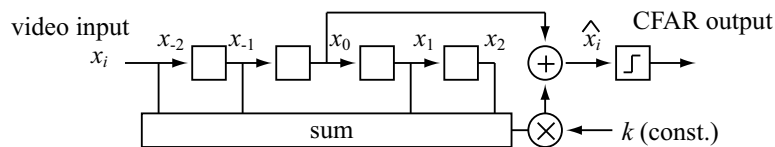


Fig. 1. A conventional CFAR circuit that uses local mean values to detect a target.

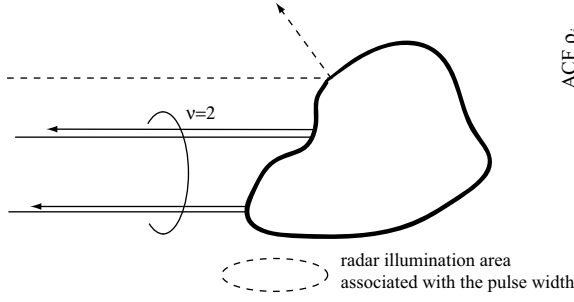


Fig. 2. Gamma-distributed target and clutter in an automotive radar.

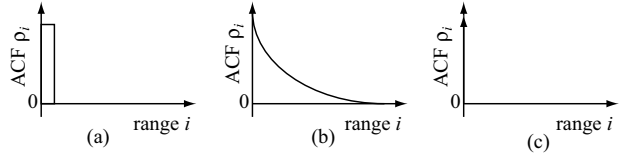


Fig. 3. Possible autocorrelation assumptions for target and clutter: (a) unit-step function (large structure), (b) negative exponential (randomly placed structure), and (c) delta function (receiver noise).

exceed a threshold  $x_{th}$  is  $P_F = \exp\left(-\frac{x_{th}^2}{\bar{x}^2}\right)$ . Therefore,  $P_F$  increases as an increase in the mean amplitude  $\bar{x}^2$ . If we obtain the (local) mean amplitude  $\langle x \rangle = \int_0^\infty x p(x) dx = \frac{\sqrt{\pi}}{2} \bar{x}$ , and the probability that CFAR output  $\hat{x} = x - k\langle x \rangle$  exceeds zero is  $P_F = \exp\left(-\frac{k^2 \pi}{4}\right)$ , which does not depend on  $\bar{x}$  and the false alarm rate becomes constant.

## 2.2 Reception Power Distribution

According to study on laser speckle patterns, the statistics of the spiky power variation can be modeled as gamma distribution [5]. A gamma distribution is characterized in terms of the shape parameter  $\nu$  and the scale parameter  $\beta$ , and  $\nu$  is interpreted as the number of reflection waves (Fig. 2). A lower  $\nu$  (say, less than 3) leads to a spiky signal. For automotive radars, a target power, as well as clutter power, would obey a gamma distribution because the size is several times as large as the resolution, and the power variation increases the required signal-to-clutter power ratio (SCR).

A probability density function (pdf) that obeys gamma distribution is expressed as  $p(x) = \frac{x^\nu}{\Gamma(\nu)\beta^\nu} \exp\left(-\frac{x}{\beta}\right) u(x)$  ( $0 < \nu, 0 < \beta$ ), where  $u(\cdot)$  is the unit step function. The statistics of gamma-distributed random variable (RV)  $x$  are  $\langle x \rangle = \nu\beta$  and  $\text{var}(x) = \nu\beta^2$ . Therefore, we obtain

$$\nu = \frac{\langle x \rangle^2}{\text{var}(x)}, \quad \beta = \frac{\text{var}(x)}{\langle x \rangle}. \quad (1)$$

## 2.3 Spatial Correlation

A possible correlation in target and clutter is shown in Fig. 3. For RVs  $x_1, x_2, \dots, x_n$ , the correlation coefficient  $\rho_i$  is expressed as  $\rho_{j-i} = \frac{\langle (x_j - \bar{x})(x_i - \bar{x}) \rangle}{\text{var}(x)}$ , where  $\bar{x} = \langle x \rangle$ . With the renewal characteristics of gamma distribution, the mean and variance of sum  $x_1 + x_2 + \dots + x_n$  are [6]:

$$\langle x_1 + x_2 + \dots + x_n \rangle = n\langle x \rangle, \quad \text{var}(x_1 + x_2 + \dots + x_n) = \text{var}(x) \left\{ n + 2 \sum_{i=1}^{n-1} (n-i)\rho_i \right\}. \quad (2)$$

## 3 Proposed CFAR with Multiple Target Detection Cells

Supposed that we average successive range cells whose number is associated with the target size, the number of radiowaves increases, and it results in reduction in required SCR. The proposed CFAR is shown in Fig.4.

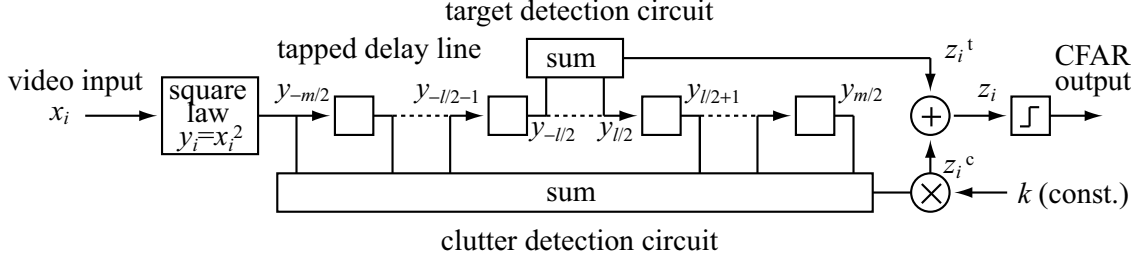


Fig. 4. Proposed CFAR.

For analysis of  $P_F$  and  $P_D$  of the proposed CFAR, the characteristic function (chf) method and its Fourier expansion form are used for expressing the reception power distribution here. The bandwidth-limited chf of the original pdf  $p(x)$  is  $\Phi(\omega) = \int_{-\infty}^{\infty} p(x) e^{j\omega x} dx \approx \int_{-R}^R p(x) e^{j\omega x} dx$ , where  $\omega$  is an auxiliary variable and  $R$  is the maximum value of RV, and its Fourier series expansion can efficiently obtain the pdf form chf [7]. By termwise integration of the pdf, we obtain the probability  $P$  that the output is positive [8]:

$$P = \int_0^{\infty} p(x) dx = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \text{Im} \left\{ \Phi \left( \frac{n\pi}{R} \right) \right\} \quad (3)$$

First of all,  $P_F$  under the given threshold coefficient  $k$  is evaluated on the condition that all cells are filled with clutter with parameters of  $v_c$  and  $\beta_c$ . The statistics for target-detection circuit output  $z^t$  is expressed using Eqs.(1) and (2) as  $\langle z_i^t \rangle = (l+1) v_c \beta_c$  and  $\text{var}(z_i^t) = \left\{ (l+1) + 2 \sum_{i=1}^l (l+1-i) \rho_i^c \right\} v_c \beta_c^2$ , where  $\rho_i^c$  is the clutter autocorrelation function (ACF). Then, the scale parameter  $\beta_1$  and the shape parameter  $v_1$  are

$$\beta_1 = \frac{(l+1) + 2 \sum_{i=1}^l (l+1-i) \rho_i^c}{l+1} \cdot \beta_c, \text{ and } v_1 = \frac{(l+1) v_c \beta_c}{\beta_1}, \quad (4)$$

and the chf is  $\Phi_1(\omega) = (1 - j\omega\beta_1)^{-v_1}$ . On the other hand, the statistics of clutter-detection circuit output  $z^c$  is obtained considering the deficient cells. The scale parameter  $\beta_2$  and the shape parameter  $v_2$  are

$$\beta_2 = \frac{(m-l) + \sum_{i=1}^{m/2} (m-2i) \rho_i^c + \sum_{i=l+2}^{m/2+l+1} (i-l-1) \rho_i^c + \sum_{i=m/2+l+2}^{m+l} (m+l+1-i) \rho_i^c}{m-l} \cdot k \beta_c$$

$$v_2 = \frac{(m-l) k v_c \beta_c}{\beta_2}, \quad (5)$$

and the chf is  $\Phi_2(\omega) = (1 - j\omega\beta_2)^{-v_2}$ . Since  $z = z^t - z^c$ , the chf of the CFAR output  $\Phi_F(\omega)$  is

$$\Phi_F(\omega) = \langle e^{j\omega(z^t - z^c)} \rangle = \Phi_1(\omega) \Phi_2(-\omega) = (1 - j\omega\beta_1)^{-v_1} (1 + j\omega\beta_2)^{-v_2} \quad (6)$$

Substituting Eqs. (4)–(6) into Eq.(3), we obtain  $P_F$  under the coefficient  $k$ .

Next, we evaluate  $P_D$ . The target signal and the clutter are independent, therefore the statistics of the superposition of target signal RV  $X^t$  on clutter RV  $X^c$  can be written as  $\langle X^t + X^c \rangle = \langle X^t \rangle + \langle X^c \rangle$  and  $\text{var}(X^t + X^c) = \text{var}(X^t) + \text{var}(X^c)$ . For obtaining  $P_D$ , we assume that the target signal fills with the target detection cells and does not spill over the clutter detection cells. The scale parameter  $\beta_3$  and the shape parameter  $v_3$  of the target detection circuit output due to the target signal is obtained using simple substitutions of variables in Eq.(4):

$$\beta_3 = \frac{(l+1) + 2 \sum_{i=1}^l (l+1-i) \rho_i^t}{l+1} \cdot \beta_t, \text{ and } v_3 = \frac{(l+1) v_t \beta_t}{\beta_3}, \quad (7)$$

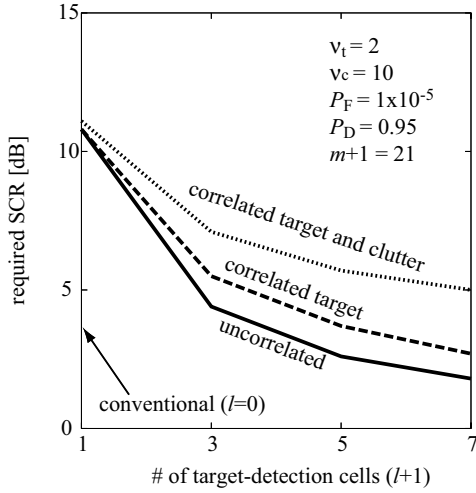


Fig. 5. SCR of the proposed CFAR for spiky target.

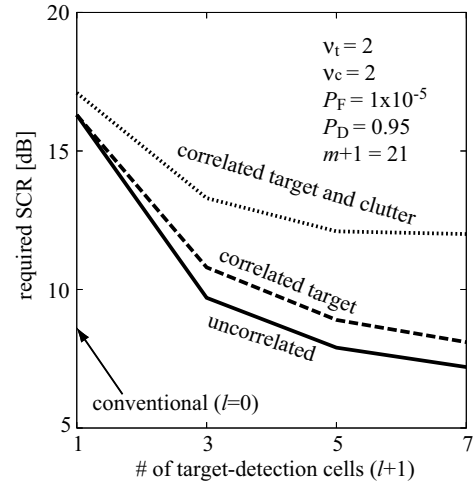


Fig. 6. SCR of the proposed CFAR for spiky target and clutter.

where  $\rho_i^t$  is the ACF of the target signal, and the chf is  $\Phi_3(\omega) = (1 - j\omega\beta_3)^{-v_3}$ . Then, the chf of the CFAR output  $\Phi_D(\omega)$  is

$$\Phi_D(\omega) = \Phi_1(\omega) \Phi_3(\omega) \Phi_2(-\omega) = (1 - j\omega\beta_1)^{-v_1} (1 - j\omega\beta_3)^{-v_3} (1 + j\omega\beta_2)^{-v_2} \quad (8)$$

We obtain  $P_D$  with Eqs.(3)–(8).

#### 4 Numerical Result and Conclusion

Required SCR under  $P_D = 0.95$  and  $P_F = 10^{-5}$  are calculated for various  $l$  and is shown in Fig.5. The SCR is obtained by  $v_t\beta_t/(v_c\beta_c)$ . We assume the number of entire cells of 21 ( $m = 20$ ),  $v_c = 10$  and  $v_t = 2$ . In the evaluation, ACF of  $1, e^{-1}, e^{-2}, \dots$  were also assumed for both  $\rho_i^t$  and  $\rho_i^c$ . The figure also shows uncorrelated cases. The proposed CFAR that uses the adjacent cells for target detection ( $l = 2$ ) reduced the required SCR by 6.4 dB for the uncorrelated case and by 4.0 dB for the correlated target and correlated clutter case. Also, required SCR for spiky target ( $v_t = 2$ ) and spiky clutter ( $v_c = 2$ ) was obtained and is shown in Fig. 6. Spiky clutter increased the required SCR, but the proposed CFAR reduced it. The proposed CFAR with multiple target detection cells reduced the required SCR.

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