Performance Improvement of a Reconstruction Algorithm in Electromagnetic Inverse Scattering Problem

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1. Introduction

Electromagnetic chracterization of internal properties, size, shape, and location of a scattering object has been one of the most important and challenging topics in electromagnetics community due to its practical applications in biomedical diagnostics, nondestructive testing of materials, and detection of buried objects. In recent years, many inversion methods have been proposed to solve the inverse scattering problems [1]-[8].

It is the purpose of this paper to consider the performance improvement of an iterative inversion algorithm of reconstructing the relative permittivity of a lossy dielectric cylinder based on the multigrid optimization method. The object located in a homogeneous background medium is illuminated with multifrequency cylindrical electromagnetic waves in microwave region. A cost functional is defined as the norm of a difference between the scattered electric fields measured and calculated. Then the electromagnetic inverse scattering problem can be treated as an optimization problem where the contrast function is determined by minimizing the cost functional. We employ the multigrid optimization method to solve the optimization problem. This method is composed of the frequency-hopping technique with a weighting factor, the multigrid method with a V-cycle [9], and the conjugate gradient method [1]. Computer simulations are performed for a lossy and homogeneous dielectric circular cylinder to show the effect of the weighting factor on the reconstruction accuracy.

2. Theory

Consider a lossy dielectric cylinder of relative permittivity $\varepsilon_s(\boldsymbol{\rho})$, which is situated in a homogeneous background medium of relative permittivity ε_b. The object with cross section Ω is assumed to be infinitely long along the z-direction. Now the object is illuminated by TM cylindrical waves with electric field $\mathbf{E}_p^i$ $(=\mathbf{u}_z E_p^i(\theta;\, \boldsymbol{\rho}))$ corresponding to the p-th frequency of f_p, where $\mathbf{u}_z$ is the unit vector in the z-direction, $p = 1, 2, \cdots, P$, and $f_1 < f_2 < \cdots < f_P$. Line sources generating $\mathbf{E}_p^i$ are located at positions with polar coordinates $(\rho,\, \theta + \pi)$. For each illumination, the scattered electric field $\mathbf{E}_p^s$ $(=\mathbf{u}_z E_p^s(\theta;\, \boldsymbol{\rho}))$ are measured at the observation points with polar coordinates $(\rho,\, \phi)$. The geometry of the problem is shown in Fig. 1. The material property of the

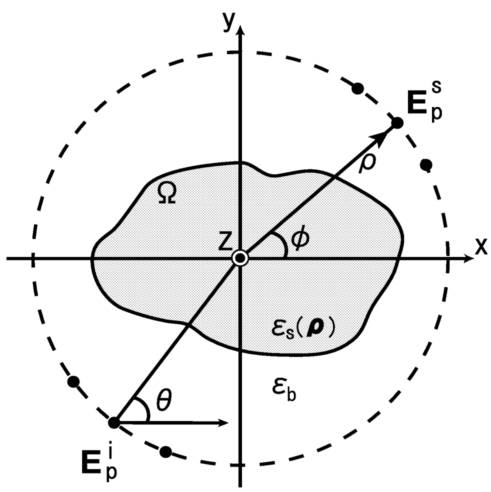

Figure 1: Geometry of the problem.

object is characterized by a contrast function,

$$c(\boldsymbol{\rho}) = \varepsilon_s(\boldsymbol{\rho}) - \varepsilon_b. \tag{1}$$

The z-component of the scattered electric field outside the object is expressed as

$$E_p^s(c;\,\theta;\,\boldsymbol{\rho}) = k_p^2 \iint_\Omega c(\boldsymbol{\rho}')\, E_p^t(c;\,\theta;\,\boldsymbol{\rho}') G_p(\boldsymbol{\rho};\,\boldsymbol{\rho}')\, d\boldsymbol{\rho}', \quad \boldsymbol{\rho} \in \overline{\Omega}, \tag{2}$$

where $\overline{\Omega}$ indicates a domain outside the object, k_p is a free-space wavenumber at the p-th frequency, and $G_p(\boldsymbol{\rho};\,\boldsymbol{\rho}')$ denotes the two-dimensional Green's function for the background medium. The total electric field $E_p^t(c;\,\theta;\,\boldsymbol{\rho})$ inside the object, which is expressed as the sum of $E_p^i(\theta;\,\boldsymbol{\rho})$ and $E_p^s(c;\,\theta;\,\boldsymbol{\rho})$, satisfies the integral equation,

$$E_p^t(c;\,\theta;\,\boldsymbol{\rho}) = E_p^i(\theta;\,\boldsymbol{\rho}) + k_p^2 \iint_\Omega c(\boldsymbol{\rho}') E_p^t(c;\,\theta;\,\boldsymbol{\rho}')\, G_p(\boldsymbol{\rho};\,\boldsymbol{\rho}')\, d\boldsymbol{\rho}', \quad \boldsymbol{\rho} \in \Omega. \tag{3}$$

The inverse scattering problem discussed here may be formulated as the solution to a nonlinear integral equation for the contrast function, which is obtained by replacing $E_p^s(c;\,\theta;\,\boldsymbol{\rho})$ with $\widetilde{E}_p^s(\theta;\,\boldsymbol{\rho})$ in the left-hand side of Eq. (2). Here $\widetilde{E}_p^s(\theta;\,\boldsymbol{\rho})$ is the scattered electric field measured. Now the measured data are simulated by solving the direct scattering problem for the true contrast function using the FFT-CG method [10].

The line sources are placed at the positions with polar angles $\theta = \theta_l$ for one frequency, where $l = 1, 2, \cdots, L$. For each illumination, the measurements of the scattered electric field are made at M observation points with polar angles $\phi = \phi_m$, where $m = 1, 2, \cdots, M$. The square investigation domain, which contains the object and the background medium, is subdivided into small square cells. Then the method of moments with pulse-basis functions and point matching [11] is employed to discretize Eqs. (2) and (3).

Let us define the cost functional at the p-th frequency of f_p as follows:

$$F_p(c) = \sum_{q=1}^{p} \sum_{l=1}^{L} \sum_{m=1}^{M} w_q^p \left| E_q^s(c;\,\theta_l;\,\phi_m) - \tilde{E}_q^s(\theta_l;\,\phi_m) \right|^2, \quad p = 1,\, 2, \cdots, P. \tag{4}$$

Here $\tilde{E}_p^s(\theta_l;\,\phi_m)$ and $E_p^s(c;\,\theta_l;\,\phi_m)$ are the scattered electric fields measured and calculated for an estimated contrast function, respectively. Furthermore $w_q^p = \gamma^{p-q}$ and γ indicates a positive number. It should be noted in Eq. (4) that the cost functional is defined as the weighted sum of p residual errors in the scattered electric fields at the p-th frequency. Introducing the cost functional, the electromagnetic inverse scattering problem is reduced to an optimization problem where the contrast function at the p-th frequency is determined by the minimization of $F_p(c)$. The multigrid optimization method, which is composed of the frequency-hopping technique with a weighting factor, the multigrid method with a V-cycle [9], and the conjugate gradient method [1], is employed to solve the optimization problem. Then one can derive an iterative scheme of reconstructing the contrast function. The reconstruction scheme terminates if the convergence criterion for δ_p is finally less than a value prescribed at the highest frequency of f_P. The parameter δ_p is defined by the right-hand side of Eq. (4) normalized by the norm of the scattered electric field measured.

3. Numerical Results

Numerical results are presented for a lossy and homogeneous dielectric circular cylinder to examine the performance improvement of the proposed algorithm using the multifrequency scattering data. The effect of the weighting factor on the reconstruction accuracy is explored. We employ six frequencies of $f_1 = 1$GHz, $f_2 = 2$GHz, $f_3 = 3$GHz, $f_4 = 4$GHz, $f_5 = 5$GHz, and $f_6 = 6$GHz. The current frequency hops to the next higher-frequency after two V-cycles are completed. However many V-cycles greater than two are exceptionally used at the highest frequency until δ_p becomes the prescribed value. Note that the V-cycle is constructed from two fine grids and one coarse grid. 24 positions of line sources and 24 measurement points for each illumination are uniformly distributed along a circle of radius 2λ, where λ is the wavelength at the highest frequency in the background free space. The $2\lambda \times 2\lambda$ square investigation domain containing the object and the background medium is uniformly subdivided into 48×48 or 24×24 small square cells corresponding to the fine grid or the coarse one. The initial guess of the contrast function is asuumed to be zero, and the number of relaxation calculation based on the conjugate gradient method at each grid level is set to 5. The reconstruction schemes with $\gamma = 0.5, 1.0$, and 2.0 are called the method I, the method Ⅱ, and the method Ⅲ, respectively.

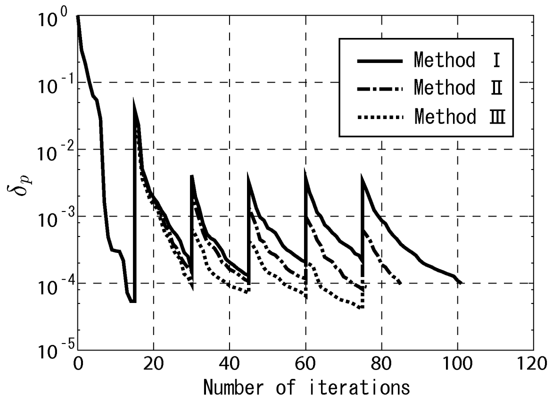

Figure 2: Relative residual errors in the scattered electric field.

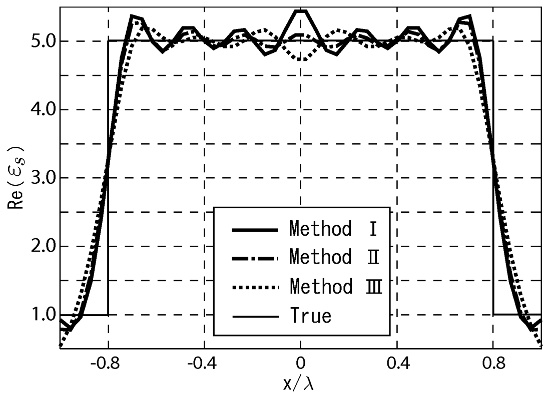

Figure 3: Reconstructions of the real part of relative permittivity.

Let us consider the reconstruction of a circular cylinder with the relative permittivity of $5.0 - j0.5$ and the radius of 0.8λ. Figure 2 illustrates the value of δ_p versus the number of iterations. The solid, dotted-dashed, and dotted lines present the results based on the method I, the method Ⅱ, and the method Ⅲ. Figures 3 and 4 show the reconstructions of the real part and the imaginary part of the relative permittivity for the three methods. For reference, the true profiles of the real and the imaginary parts of the relative permittivity are also depicted by the thin solid lines in these figures. The reconstructed results are obtained after 101, 85, and 76 iterations corresponding to the method I, the method Ⅱ, and the method Ⅲ, respectively. The convergence criterion is $\delta_6 < 10^{-4}$. Table 1 shows the relative mean squared errors in the real

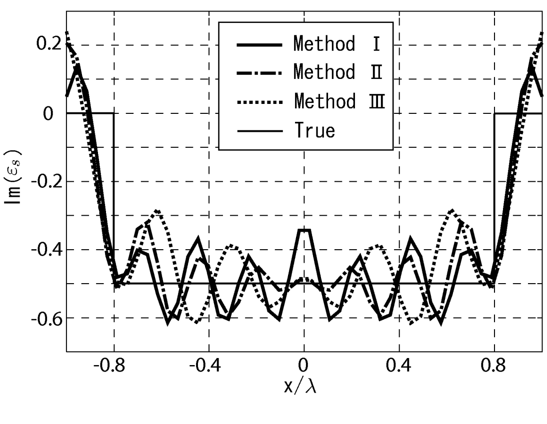

Figure 4: Reconstructions of the imaginary part of relative permittivity.

Table 1: Relative mean squared erros in the real and imaginary parts of relative permittivity based on the method I, the method Ⅱ, and the method Ⅲ.

Method	$\delta_\varepsilon^{(R)}$	$\delta_\varepsilon^{(I)}$
Ⅰ	3.37×10^{-2}	0.13
Ⅱ	3.56×10^{-2}	0.15
Ⅲ	4.42×10^{-2}	0.20

and imaginary parts of the relative permittivity. The parameters $\delta_\varepsilon^{(R)}$ and $\delta_\varepsilon^{(I)}$ correspond to the real and imaginary parts, respectively. It is seen from Figs. 3 and 4 and Table 1 that the reconstruction scheme with $\gamma = 0.5$ gives the best reconstruction accuracy.

4. Conclusion

The performance improvement of an iterative inversion algorithm of reconstructing the relative permittivity of a lossy dielectric cylinder based on the multigrid optimization method has been investigated. The electromagnetic inverse scattering problem is reduced to an optimization problem. Numerical results for a lossy and homogeneous dielectric circular cylinder are given to examine the effect of the weighting factor on the reconstruction accuracy. It is confirmed from the results that the reconstruction scheme with the weighting factor of 0.5 provides the best reconstruction accuracy. Research on the determination of the optimum weighting factor and the use of a regularization method remains topics for future work.

Acknowledgments

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