Polarization Inside Homogeneous Media Characterized by Higher-order Complex Susceptibility: One-dimensional Simulation using Characteristic-based Method

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1 Introduction

The main objectives of this paper are to apply the characteristic-based method to the numerical solutions of Maxwell's equations involved with higher-order complex susceptibility, and then to observe the effects of the nonlinear, complex-valued permittivity on the nonlinear behavior of the electromagnetic pulses inside media by plotting the numerical results side-by-side. The computed results of the time-domain Maxwell's curl equations involved with nonlinear susceptibility were obtained through the application of the recently developed characteristic-based method. The computed electric field intensities and effective permittivity, complex quantities and functions of time, were demonstrated alongside and compared. Hopefully, it is desirable to explore the possibility of such application to the nonlinear optical problems using the characteristic-based method.

Generally, the analytical solutions of nonlinear problems are not likely obtainable, accurate numerical procedures are consequently demanded providing accurate numerical approximations to problems of interest. In the area of computational Electromagnetics, for instance, numerical methods are employed to predict the reflection and transmission of the electromagnetic fields when dielectric material is illuminated by electromagnetic source. By presenting accurate computational results, numerical techniques always serve as important tools giving scientists a better understanding of physics. Thanks to the dramatic advances in computing power and the development of accurate numerical methods, the heavy demands on memory storage, speed, and scheme accuracy for computation are somewhat satisfactory for detailed numerical modeling.

When electromagnetic fields propagate through dielectric material, nonlinear optical phenomena are observed as a result of the interaction between electromagnetic energy with matter. Studies of such nonlinear behavior are quite complicated and difficult for the following reasons: the governing equations may be in a closed form yet the physics behind the problem may be very complicated and whose analytical solutions are usually not available. It is then desirable, if numerically possible, to investigate the nonlinear behavior of electromagnetic fields inside nonlinear dielectric materials, particularly those with higher-order complex susceptibilities. The electric polarization in medium describes material characteristics that can be arranged in certain relations to the externally applied electric fields. Assuming that the magnetic effects are negligible, the electric polarization can be expressed as the power expansion in the electric field where the linear and nonlinear susceptibility are respectively accountable for the linear and nonlinear properties of medium.

Since the early 60s various computational techniques have been applied to many fields of research. For electromagnetic related problems numerical methods approximate Maxwell's equations either in the frequency domain or time domain providing researchers valuable information. The method of moment and the finite-difference time-domain (FDTD) technique have been the two most widely used numerical methods since they were proposed in the 1960s. The numerical technique employed in the present paper was developed for direct approximations of the time-dependent Maxwell curl equations in the mid of the 1990s. About a decade earlier, the characteristic-based method was proposed by Whitfield and Janus for the solutions of fluid dynamic problems [1]. Few years later, Shang adapted this method to solve the time-domain Maxwell's equations [2] by explicitly applying the central-difference scheme for both temporal and spatial derivatives. Its implicit formulation was developed for the same purpose and soon found to have good agreement with FDTD [3] where the scattered electromagnetic fields and the induced currents on an infinite long perfect strip under the

illumination of a plane Gaussian electromagnetic pulse. It is also shown that the characteristic-based method can predict the reflection of electromagnetic fields from moving / vibrating perfect conductor in one dimension [4, 5], the effects of finite conductivity on the reflection / transmission of electromagnetic fields [6], and the reflection / transmission of electromagnetic field propagation onto moving dielectric half space [7]. The basic difference between the present method and traditional techniques, Moment Method and FDTD where all field variables are allocated at the grid nodes, is that the former defines all field quantities in the center of grid cell. It directly solves time-dependent Maxwell curl equations in the curvilinear system by evaluating the flux across every cell interface and then balancing them within each computational cell.

2 Governing Equations

The governing equations for electromagnetic problems in free space are the Maxwell's equations:

$$\frac{\partial B}{\partial t} + \nabla \times E = 0 \tag{1}$$

$$\frac{\partial D}{\partial t} - \nabla \times H = 0 \tag{2}$$

with the constitutive relations being $D = \varepsilon_o \varepsilon_r E$ and $B = \mu_o \mu_r H$ where ε_o and μ_o are the permittivity and permeability of vacuum, ε_r and μ_r are the dielectric constant and relative permeability of medium, respectively. In the presence of isotropic dielectric materials characterized by nonlinear susceptibility the electric flux density D is written in terms of the linear and nonlinear components and is written as

$$D = \varepsilon_o (1 + \chi^{(1)} + \chi^{(2)} E + \chi^{(3)} E \cdot E + \dots) E \tag{3}$$

or in terms of the induced polarization P, the above expression becomes

$$D = \varepsilon_o E + P \tag{4}$$

In (3), the dielectric constant ε_r is expressed as a power series of the electric field intensity and parenthesized; for linear materials $\chi^{(1)}$ is real and all higher-order χ's in (3) are zero in magnitude. For non-magnetic materials the magnetization effects are negligible and μ_r is equal to unity.

To investigate the effective permittivity, we simply calculated

$$\frac{D}{\varepsilon_o E} = 1 + \chi^{(1)} + \chi^{(2)} E + \chi^{(3)} E \cdot E + \dots \tag{5}$$

and then illustrated various results alongside. As can be seen the effective permittivity is a function of the externally applied electric field. For simplicity and easy observation on how various susceptibility terms affect on the transmitted electromagnetic pulses, high-order χ's are assumed that their real and imaginary components are both constants.

3 Cases Studied

To minimize the number of parameters within a controllable range, the numerical model was simplified as below. The excitation source normally illuminate upon a half space made of dielectric medium characterized by only one term of complex nonlinear susceptibility. The incident EM pulse used in this simulation has the following properties. The maximum value of the electric field intensity of the incident pulse is assumed to be 1 V/m. The electromagnetic pulse is plane with a Gaussian distribution form in the time domain, truncated by a rectangular window with 100 dB with respect to the peak value, and has a pulse width of 1.042 fs corresponding to highest frequency content in the spectrum about 732.935 THz.

As stated earlier, for simplicity and clear investigations on the effect, each medium used in the model is specified by only one term of complex susceptibility. This means that for some media whose first order susceptibility $\chi^{(1)}$ is a constant value of 3 and one term of higher-order susceptibility term, $\chi^{(2)}$ or $\chi^{(3)}$ in one of the following forms: $\pm 0.4 \pm 0.4$ i. The exaggerated magnitude of these variables is for the purpose of magnifying their effects on the transmitted electromagnetic pulses inside various

media. The value 0.4 is chosen so that it is 10 % of the real valued dielectric constant, $1 + \chi^{(1)} = 4$. The field components as well as the ratio of the electric displacement to the electric field intensity are recorded at four different sampling points (A, B, C, and D) which are shown in Figure 1.

4 Results

Depicted in Figure 1 is the definition of the worked problem along with four different sampling points inside medium. Also given are some details of the incident Gaussian pulse: its electric field component, its pulse width, and its initial position measured from the air-medium interface. Listed are all possible specifications of the higher-order complex susceptibility. To observe the effects of complex susceptibility on the electromagnetic pulse inside medium, two series of electric fields recorded at four various depths inside two different media, along with those of linear medium, $\varepsilon_r = 1 + \chi^{(1)} = 4$, are illustrated in Figures 2 and 3 respectively corresponding to $\chi^{(2)} = -0.4 + 0.4$ i and $\chi^{(3)} = + 0.4 - 0.4$ i. From these two groups of plot the followings are observed. The positive real part of higher-order susceptibility accelerates the peak of electric fields with respect to that inside linear medium while the negative one lags. This is clearly shown by examining the location of the pulse peak with respect to that of linear medium. Yet, there is no regular relationship can be found. The deeper the electromagnetic pulse propagates into medium the more distortion are caused and the $\chi^{(2)}$ term deforms electromagnetic pulse more than $\chi^{(3)}$ does. The imaginary part of higher-order susceptibility answers for the imaginary part of the transmitted pulse.

Illustrated in Figures 4 and 5 are the real and imaginary parts of the effective permittivity inside several different media. It is clearly shown that the real components of the effective permittivity are independent of the imaginary component of susceptibility and that the second-order susceptibility shows to have greater effective permittivity in magnitude than the third-order one. The imaginary components of the effective permittivity are dependent of both real and imaginary portions of the third-order susceptibility while they depend only on the imaginary portions of the second-order susceptibility.

5 Conclusion

This paper has shown the computational results of the propagation of electromagnetic pulse through dielectric materials having one term of higher-order complex susceptibility. The characteristic-based method was used to numerically solve Maxwell's curl equations associated with the permittivity term is expressed in the power expansion of the external electric field. The computed data reveal reasonable general trends of the effects of the second- and third-order complex susceptibility on the transmitted electromagnetic pulses. Also shown is the effective permittivity in complex form under the excitation of electromagnetic fields as functions of complex susceptibility. It is planned in the future to develop the existing numerical procedure for three-dimensional problems.

References:

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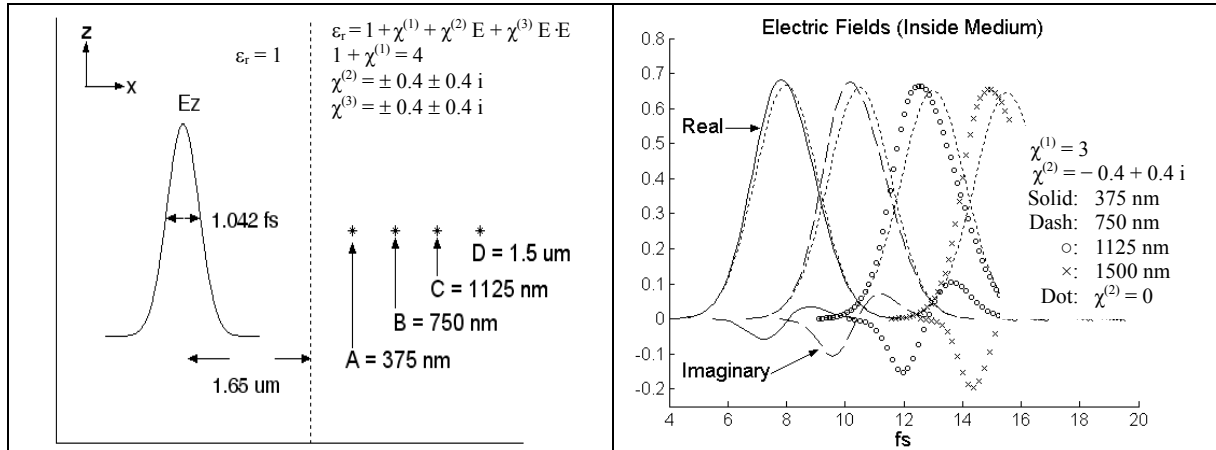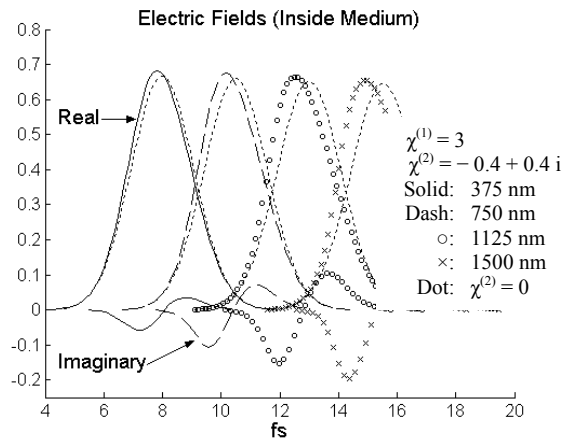
Figure 1: Definition of the worked problem with four sampling points.

Figure 2: Electric fields inside medium with $\chi^{(1)} = 3$, $\chi^{(2)} = -0.4 + 0.4$ i.
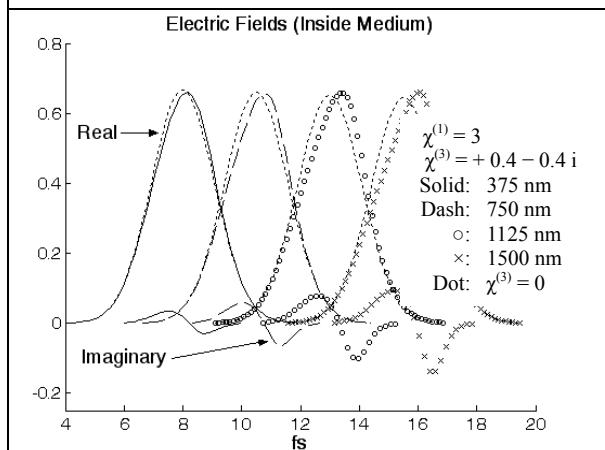

Figure 3: Electric fields inside medium with $\chi^{(1)} = 3$, $\chi^{(3)} = +0.4 - 0.4$ i.
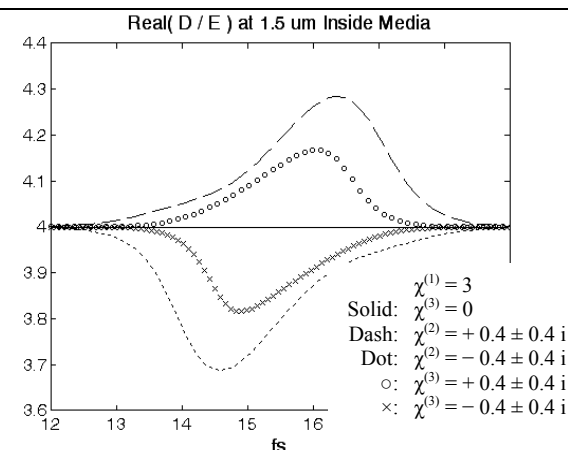

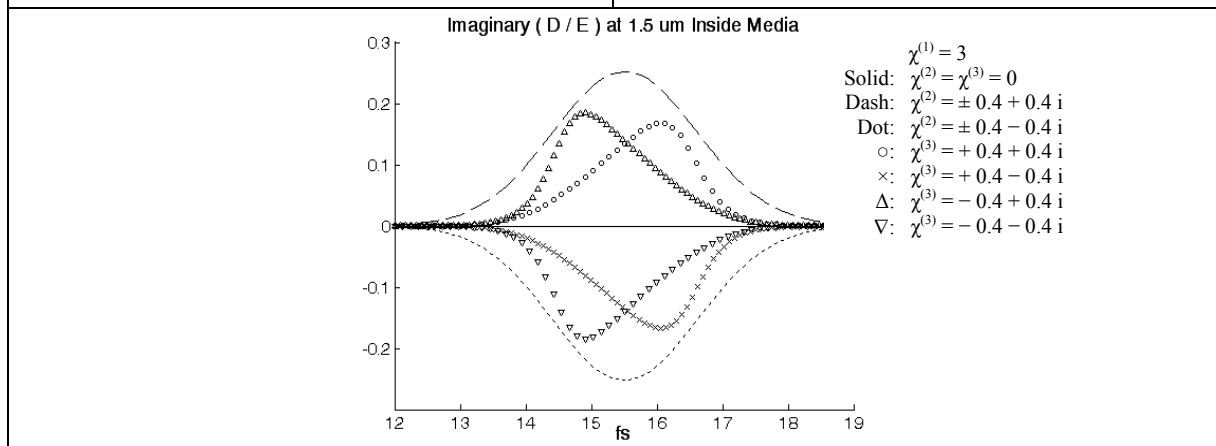Figure 4: Real part of the effective permittivity inside various media.

Figure 5: Imaginary part of the effective permittivity inside various media.