Finite-Difference Frequency-Domain Method Analysis of Periodic Nano-Plasmonic Waveguides

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1. Introduction

Recently, guiding electromagnetic (EM) waves with a mode at subwavelength scale has attracted great attention, in particular, at optical wavelengths. The related phenomenon is that of surface plasmon polaritons (SPPs) that are bound non-radiative surface waves propagating at metal-dielectric interfaces with the fields decaying exponentially away from the interface [1]. Structures supporting such subwavelength waveguiding modes, which can be called plasmonic waveguides, are essential components in future miniaturized light circuitry. Longitudinally uniform nano-plasmonic waveguides with various transverse structures have been proposed and studied [2-5], such as metallic nanowires [4] and V-shaped grooves [5]. In this paper, we consider longitudinally periodic waveguiding structures [6].

Real metals are lossy materials. Computation of band diagrams for periodic metallic structures including such real metal effects as an eigenvalue problem is known to be difficult, in particular for the transverse-electric (TE) mode, although a few methods have been developed leading to nonlinear eigenvalue problems, such as that based on multiple multipole expansions [7]. On the other hand, the finite-difference time-domain (FDTD) method based on the Yee mesh has been a popular numerical technique for the band-diagram calculation [8] because of its flexibility and easiness in dealing with dispersive materials. However, the conventional FDTD method employs orthogonal and staggered grids, and modifications at material interfaces are needed in order to improve numerical accuracy and mode-finding resolution in the calculation of band diagrams [9-10], which requires more complicated mathematical algorithms in treating material interfaces. In this paper, based on the finite-difference frequency-domain (FDFD) method using the Yee mesh, we formulate a standard eigenvalue problem for computing band diagrams or mode dispersion curves of two-dimensional (2-D) waveguiding structures with 1-D periodicity and involving lossless dispersive metallic materials such as silver nanorods. High enough numerical accuracy is obtained without using complicated algorithms to treat material interfaces, and the mode solutions are found to agree with the FDTD analysis results [6].

2. Formulation

Maxwell's equations for *z*-propagating TE wave mode in a linear, isotropic, and nonmagnetic material are written as

$$j\omega\varepsilon_0\varepsilon_x E_x = \frac{\partial H_z}{\partial y} \quad (1)$$

$$j\omega\varepsilon_0\varepsilon_y E_y = -\frac{\partial H_z}{\partial x} \quad (2)$$

$$j\omega\mu_0 H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \quad (3)$$

where ε_0 and μ_0 are free-space permittivity and permeability, respectively, and the relative permittivities ε_x and ε_y account for the possible material anisotropy. In the present study, we employ the free-electron model

$$\varepsilon_x(\omega)=\varepsilon_y(\omega)=\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2} \tag{4}$$

for real metals, where ω_p is the bulk plasma frequency. The metal is considered to be lossless in the free-electron model. After some mathematical work, the following equations for an eigenvalue problem are obtained

$$-\omega^2\mu_0\varepsilon_0\left(1-\frac{\omega_{px}^2}{\omega^2}\right)E_x=\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right) \tag{5}$$

$$-\omega^2\mu_0\varepsilon_0\left(1-\frac{\omega_{py}^2}{\omega^2}\right)E_y=\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}\right) \tag{6}$$

where we write ω_{px} and ω_{py} to account for the possible material anisotropy. By utilizing the central difference scheme and 2-D Yee's mesh, (5) and (6) can be derived into the matrix equation

$$k_0^2\begin{bmatrix}\mathrm{E}_x\\ \mathrm{E}_y\end{bmatrix}=\begin{bmatrix}-\mathrm{U}_y\mathrm{V}_y+\omega_{px}^2 & \mathrm{V}_y\mathrm{U}_x\\ \mathrm{V}_x\mathrm{U}_y & -\mathrm{V}_x\mathrm{U}_x+\omega_{py}^2\end{bmatrix}\begin{bmatrix}\mathrm{E}_x\\ \mathrm{E}_y\end{bmatrix} \tag{7}$$

where $k_0^2=\omega^2\mu_0\varepsilon_0$ is the eigenvalue, E_x and E_y are column vectors with elements being corresponding field components at grid points, ω_{px}^2 and ω_{py}^2 are both diagonal matrices, and the matrices U_x, U_y, V_x, and V_y are given below

$$\mathrm{U}_x=\frac{1}{\Delta x}\begin{bmatrix}-1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & -1 & 1\\ & & & & & -1\end{bmatrix},\quad \mathrm{U}_y=\frac{1}{\Delta y}\begin{bmatrix}-1 & & & 1 & & \\ & -1 & & & 1 & \\ & & \ddots & & & 1\\ & & & \ddots & & \\ & & & & -1 & \\ & & & & & -1\end{bmatrix} \tag{8}$$

$$\mathrm{V}_x=\frac{1}{\Delta x}\begin{bmatrix}1 & & & & & \\ -1 & 1 & & & & \\ & -1 & \ddots & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \\ & & & & -1 & 1\end{bmatrix},\quad \mathrm{V}_y=\frac{1}{\Delta y}\begin{bmatrix}1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ -1 & & & \ddots & & \\ & \ddots & & & 1 & \\ & & -1 & & & 1\end{bmatrix}. \tag{9}$$

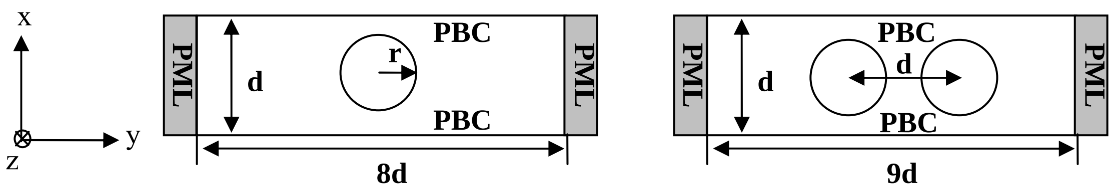
Considering the periodicity in the *x*-direction, we have the periodic boundary condition (PBC) along with Bloch's condition over one unit cell as

$$E(x+d)=\exp(ik_x d)E(x) \tag{10}$$

where k_x is the *x*-component of the wave vector, *d* is the period, and E represents E_x or E_y field component for the TE mode. In the *y*-direction, we use perfectly matched layer (PML) absorbing boundary conditions at appropriate locations. Consequently, U_x, U_y, V_x, and V_y will be slightly modified.

3. Numerical Results

In order to demonstrate our method, we consider a 1-D silver circular-nanorod periodic array along the *x*-direction. Figure 1(a) shows the one-period unit cell as the computing domain for calculating the dispersion diagram. The radius of the silver nanorod is $r = 2.5\times10^{-8}$ m and the period is $d = 7.5\times10^{-8}$ m. The plasma frequency is $\omega_p = 9.39\times10^{15}$ rad/s. The Bloch's PBCs and PMLs in *x*- and *y*-directions, respectively, are as shown in the figure. The FDFD calculated dispersion diagram is shown in Fig. 2 (a) for the TE mode.

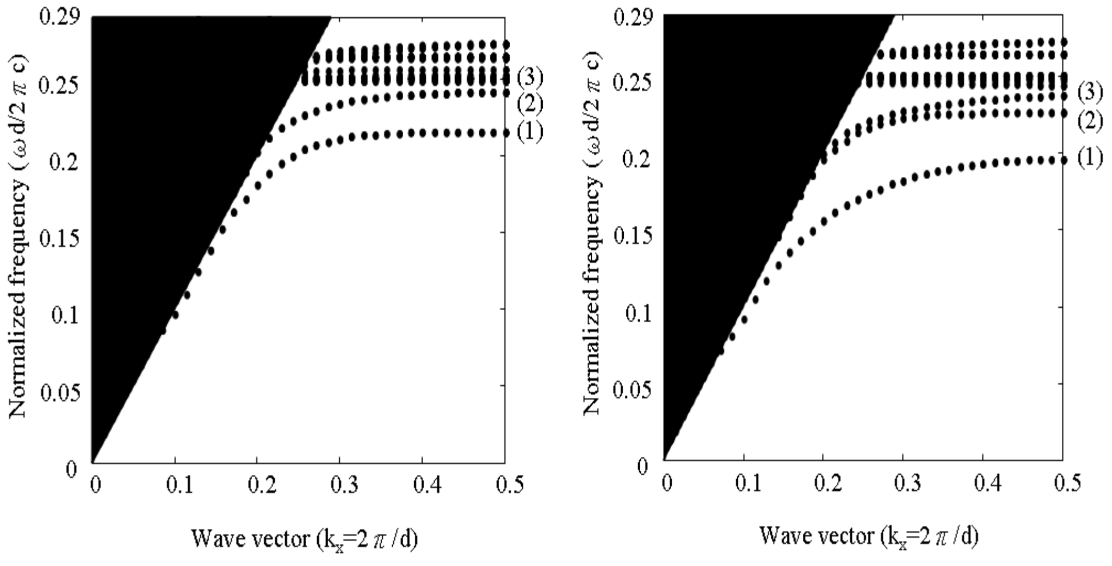


(a) (b)

Figure 1: The 2-D FDFD computing domain for a 1-D silver circular-nanorod periodic array structure. (a) The single-row array. (b) The two-row array.

Our calculated results agree with the FDTD method results given in [6]. We plot the normalized magnetic-field distributions of different modes at $k_x = \pi/d$ in Fig. 3(a). The field distributions can be distinguished into even and odd modes. The modes marked by (1) and (3) in Figs. 2(a) and 3(a) are even modes corresponding to the first and third bands, and that marked by (2) is an odd mode corresponding to the second band. The results clearly demonstrate that the formulated FDFD method can accurately predict the propagating modes and their dispersion curves on 1-D nano-structured arrays or periodic plasmonic waveguides, and band diagrams can be easily obtained.

We have discussed the one-row array so far. We further consider the two-row array, arranged in square lattice, as shown in Fig. 1(b). The calculated results are presented in Figs. 2(b) and 3(b). By comparing Fig. 3(b) with Fig. 3(a), we conclude that the first (1) and second (2) modes for the two-row array are even and odd ones related to the first mode of the one-row array and that the third (3) mode for the two-row array is an even mode related to the second mode of the one-row array.



(a) (b)

Figure 2: The dispersion diagrams for the 1-D silver circular-nanorod periodic array structure of Fig. 1. (a) The single-row array. (b) The two-row array

4. Conclusion

In conclusion, we have developed a simple and efficient FDFD method for analyzing plasmonic waveguides with 1-D periodicity in the propagation direction. The standard eigenvalue problem is established in the formulation such that the calculation of band diagrams becomes numerically efficient compared with the conventional FDTD method. One-row and two-row silver nanorod arrays are analyzed and compared.

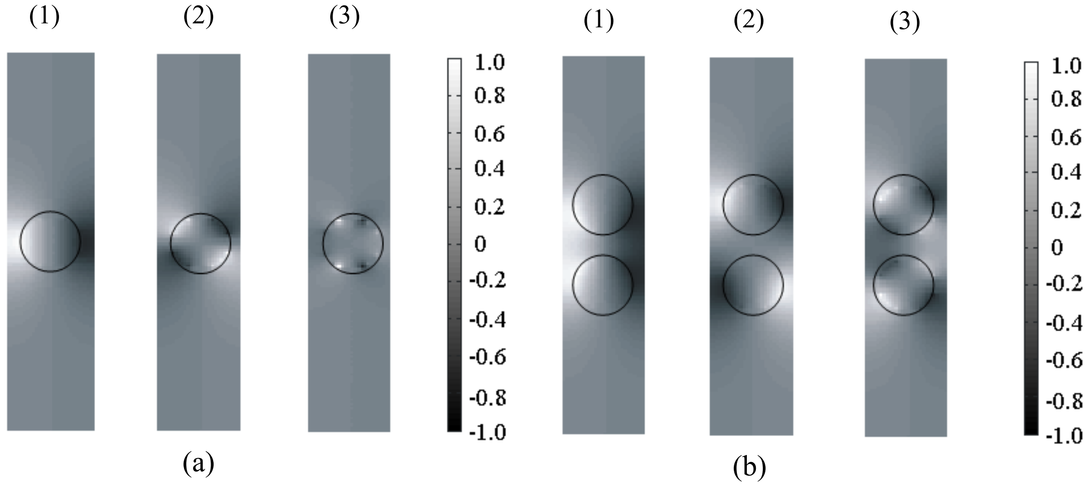


Figure 3: Normalized magnetic-field distributions of different modes at $k_x = \pi / d$ as marked in Fig. 2(a) and (b). Modes (1) and (3) are even ones and modes (2) are odd ones. From left to right, the corresponding normalized frequencies for the six mode fields are $\omega = 0.2155$, 0.2413, 0.2491, 0.1962, 0.2262, and 0.2374, respectively.

Acknowledgments

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