

# On the Effect of Spatial Smoothing and Conjugate Processing to DOA Estimation Accuracy of Coherent Signals for UCA

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## 1. Introduction

In the scheme of mobile telecommunications or wireless communication system, the real-world environment has been suffered from multipath caused by delay and reflection waves. Direction-of-arrival (DOA) is a significant parameter to estimate all incident signals. The approach of algorithms with high accuracy such as Root-MUSIC have been developed for DOA estimation. On the other hand, this techniques can be applied only for uniform linear array (ULA) where the array response vector can be given as a Vandermonde structure. However, ULA cannot provide 360° of coverage in the azimuthal plane which is necessary in many applications such as radar, sonar and wireless communications. The estimation in all azimuth angles with identical accuracy is accomplished by using a uniform circular array (UCA). Unfortunately, in the scenario of coherent signals estimation, the UCA cannot apply the forward/backward spatial smoothing preprocessing (FB-SSP) directly. To make this scheme accomplish, the array interpolation processing is necessary.

The idea of array interpolation have been studied to interpolate between available calibration point, and one application is to transform arbitrary configure data to particular array geometry. Although array interpolation process could approximately map arbitrary array signal into the ULA's, this process yields mapping errors that finally affect DOA estimation accuracy. In the issue of mapping the real array data onto the virtual array such as array interpolation [1], naturally, the larger the number of virtual elements is, the more the mapping error margin grows. Especially in the case of coherent signal, not only mapping error, but also signal correlativity suppression become significant point needed to be considered. As for the signal decorrelation approach, besides the FB-SSP, the Modified Conjugate ESPRIT (mod C-ESPRIT) [3] has been proposed recently. This technique achieved by modify and rearrange the input signals which is different from the FB-SSP that deals with shifting of covariance matrix.

In this paper, we focus the study on coherent signals estimation using Root-MUSIC algorithm for UCA and correlativity suppression method can be done by using an alternative conjugate-based approach [3], and confirm that their accuracy becomes completely the same even though the approach is very different.

## 2. Signal and Array Model

Consider  $L$  narrowband far-field signals  $s_\ell(t)$ , ( $\ell = 1, 2, \dots, L$ ) received by the uniform circular array (UCA) of radius  $r$  which composed of  $N$  sensors, assumed to be identical and omnidirectional. The signals arrive at azimuth angle  $\phi_\ell$  and elevation angle  $\theta_\ell$ . The complex input baseband signal vector  $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_K(t)]^T$ , where  $x_k$  is the input signal at  $n$ -th array element, can then be written as

$$\mathbf{x}(t) = \sum_{\ell=1}^L s_\ell(t) \mathbf{a}(\phi_\ell, \theta_\ell) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where  $\mathbf{a}(\phi_\ell, \theta_\ell)$  is a array response vector for  $\ell$ -th signal. In (1), array response matrix  $\mathbf{A}$ , incident signal vector  $\mathbf{s}(t)$ , and noise signal vector  $\mathbf{n}(t)$  are respectively given by

$$\begin{aligned}\mathbf{A} &= [\mathbf{a}(\phi_1, \theta_1), \mathbf{a}(\phi_2, \theta_2), \dots, \mathbf{a}(\phi_L, \theta_L)], \\ \mathbf{a}(\phi_\ell, \theta_\ell) &= [e^{j(2\pi/\lambda)r \sin \theta_\ell \cos \phi_\ell}, e^{j(2\pi/\lambda)r \sin \theta_\ell \cos(\phi_\ell - \varphi)}, \dots, e^{j(2\pi/\lambda)r \sin \theta_\ell \cos \phi_\ell - (N-1)\varphi}]^T, \\ \mathbf{s}(t) &= [s_1(t), s_2(t), \dots, s_L(t)]^T, \\ \mathbf{n}(t) &= [n_1(t), n_2(t), \dots, n_N(t)]^T\end{aligned}$$

where  $\lambda$  denotes wavelength, and  $\varphi = 2\pi/N$ .

### 3. Mapping Techniques

In this section we briefly review the high-accuracy performance Element-Space Root-MUSIC(ES-Root-MUSIC) [2] and conventional well known Array Interpolation [1].

#### 3.1 ES-Root-Music

ES-Root-MUSIC [2] algorithm exploits the manifold separation technique to remodel the mode vector of an arbitrary array. By this way, the array mode vector can be represented by the product of characteristic matrix  $\mathbf{G}$  and a vector  $\mathbf{D}$  with Vandermonde structure, and the system model can be rewritten as  $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{G}\mathbf{D}\mathbf{s}(t) + \mathbf{n}(t)$ . The manifold separation can be written as a least squares problem as follow

$$\arg \min_G \left\{ \sum_{i=1}^T \left\| \mathbf{a}(\theta_i) - \mathbf{G}\mathbf{d}(\theta_i) \right\|_F^2 \right\} \quad (2)$$

By this concept, it becomes clearly that mapping recieved signals is unnecessary. From another viewpoint, this technique backwardly transforms the virtual array mode vector with Vandermonde structure onto the real array's, that means, it is free to decide the number of virtual array elements(up to #360) to reduce mapping error. By this way, this techique achieves very high accuracy in incoherent-sources estimation. Anyway, this approach need no mapping process, naturally FB-SSP is not applicable.

#### 3.2 Array Interpolation

Array interpolation [1] is a mapping technique that linearly transform real array manifold into specified virtual array geometry, i.e.,  $\mathbf{B}\mathbf{a}(\phi) \approx \tilde{\mathbf{a}}(\phi)$  where  $\mathbf{B}$  denotes interpolation matrix and  $\tilde{\mathbf{a}}(\phi)$  is a virtual array steering vector. To reduce the mapping error within the allowable levels, generally, the  $360^\circ$  coverage area has to be sectorized into  $L$  angular sectors and the interpolation matrix is designed by least squares solution as

$$\min_B \left\| \tilde{\mathbf{a}}(\theta_i) - \mathbf{B}\mathbf{a}(\theta_i) \right\|_F^2 \quad (3)$$

After calculate interpolation matrix  $\mathbf{B}$ , the real received signals is transformed onto ULA geometry. By this mapping process, FB-SSP and Root-MUSIC algorithm becomes applicable. However, in order to achieve a sufficiently low mapping error, this technique always require mapping from a real array to a virtual array with equal or smaller number of elements.

### 4. Array Interpolation via Conjugate Preprocessing

Here we focus attention on coherent signals estimation, and array interpolation based method will be considered. Firstly, the real input signal received by UCA is transformed onto ULA's by using conventional array interpolation. After that, mostly FB-SSP is applied to suppress correlativity of coherent signals, in this paper we proposed an alternative approach by using a Modified Conjugate ESPRIT(mod C-ESPRIT) [3] that breifly reviewed in section 4.1.

## 4.1 Modified Conjugate ESPRIT

The mod C-ESPRIT is a way to suppress signal correlativity which consider in a ULA with  $\tilde{N}$  elements ( $\tilde{N} = 2M + 1$ , where  $M = 1, 2, \dots$ ) and the central element is designated as reference point. Here, the input signals is defined as

$$\mathbf{X}(t) = \begin{bmatrix} x_0(t) & x_{-1}(t) & \dots & x_{-M}(t) \\ x_1(t) & x_0(t) & \dots & x_{-M-1}(t) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(t) & x_{M-1}(t) & \dots & x_0(t) \end{bmatrix} \quad (4)$$

the number of snapshots is assumed to be  $Q$  where  $t = 1, \dots, Q$ . The input signal is modified by using every  $Q$  to generate a  $(M + 1) \times Q$  matrix  $\mathbf{Y}_0$  as

$$\mathbf{Y}_0 = \begin{bmatrix} x_0(1) & x_0(2) & \dots & x_0(Q) \\ x_1(1) & x_1(2) & \dots & x_1(Q) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(1) & x_M(2) & \dots & x_M(Q) \end{bmatrix} \quad (5)$$

This operation is repeated in all the sequences of Eq.(4). In that way, a matrix  $\mathbf{Z}$  is generated by arrange the obtained  $\mathbf{Y}_0, \mathbf{Y}_1, \dots, \mathbf{Y}_M$  and presumed to be a new input signal.

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Y}_0 & \mathbf{Y}_1 & \dots & \mathbf{Y}_{M-1} & \mathbf{Y}_M \end{bmatrix} \quad (6)$$

When the conjugate process is executed, the input signal  $\mathbf{X}$  is mapped onto  $\mathbf{Z}$  with  $(\tilde{N} + 1)/2 \times Q$  dimension. After that, we apply Root-MUSIC algorithm for DOA estimation.

## 5. Simulation

In this section, the accuracy of DOA estimation is evaluated through some simulations by using RMSE (Root Mean Square Error) averaged for two waves:

$$\text{RMSE} = \frac{1}{2} \sum_{\ell=1}^2 \sqrt{\sum_{i=1}^K (\hat{\theta}_{\ell,i} - \theta_{\ell})^2}$$

where  $\hat{\theta}_{\ell,i}$  denotes the estimated DOA of  $\ell$ -th incident wave at  $i$ -th trial, and  $K$  is the number of trials. The normalized mapping errors used in figure 2 is

$$e(\theta_i) = \left\| \tilde{\mathbf{a}}(\theta_i) - \mathbf{B}\mathbf{a}(\theta_i) \right\|_F^2 / \left\| \tilde{\mathbf{a}}(\theta_i) \right\|_F$$

and the specifications of the simulations are summarized in table 1.

The dependence of virtual array elements  $\tilde{N}$  and normalized mapping error characteristic are shown in figure 2. Mapping error becomes remarkably large when  $\tilde{N}$  is set up more than the real array elements  $N$ . This is a natural property of the linear transformation. By the way, as shown in figure 3(a), although  $\tilde{N}$  is set upto 15 which much more than  $N$ , the accuracy of DOA estimation is still in acceptable range and this seem to be the best performance when  $N = 8$ .

The accuracy comparison between conventional FB-SSP method and the proposed conjugate approach is shown in figure 3. In case of FB-SSP, subarray number is set at  $(\tilde{N} + 1)/2$ , equal to the modified input signal which done by conjugate method. Both of characteristics in Fig. 3(a) and Fig. 3(b) are almost overlap, by this result, we found that the conjugate approach could achieve the same accuracy as FB-SSP.

In the case of mod C-ESPRIT, it is clear that after this process is implemented, the dimension of covariance matrix becomes  $(\tilde{N} + 1)/2 \times (\tilde{N} + 1)/2$  which is the same as in case of FB-SSP. By this point, we could say that although these two methods are performed in the different approach, but actually achieved the same accuracy of covariance matrix that finally bring out the same accuracy of DOA estimation.

Table 1: Specifications of Simulations

array configuration	UCA
# of elements	8
radius	$0.5\lambda$
# of incident waves	2 coherent waves
DOA in azimuth	$-50^\circ, 50^\circ$
DOA in elevation	$90^\circ$
carrier frequency	2.0 GHz
modulation type	QPSK
# of snapshots	256
# of trials	100

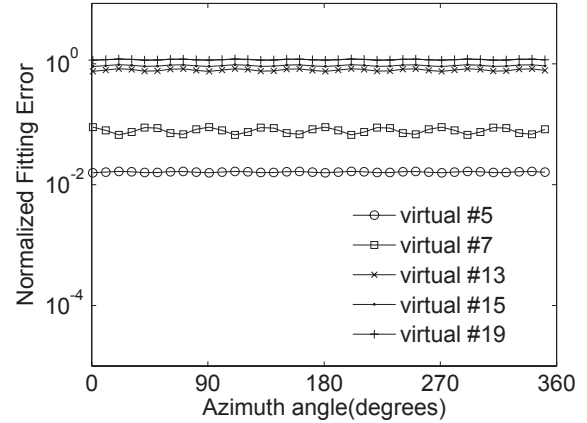
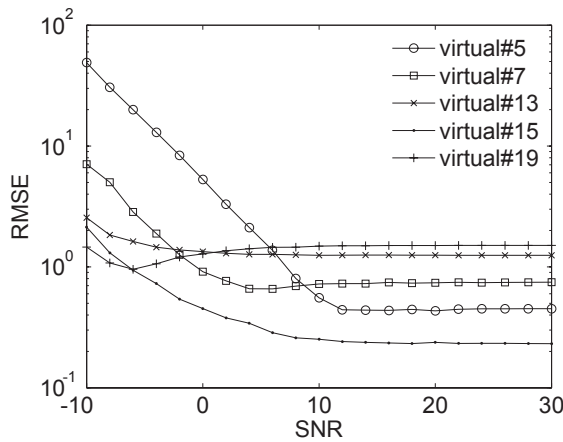
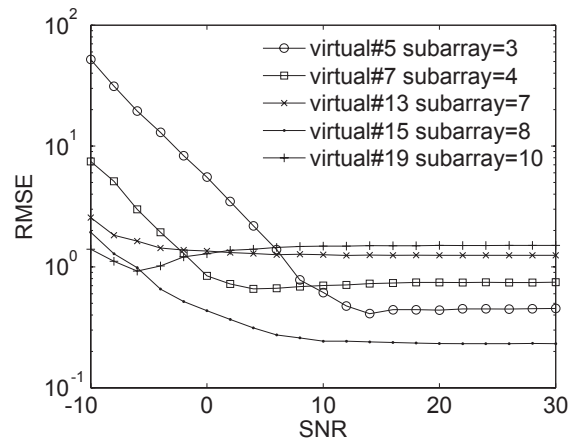


Figure 2: Normalized mapping error



(a) Array interpolation with Conjugate



(b) Array interpolation with SSP

Figure 3: Dependency of virtual array element with Conjugate(a) and Spatial Smoothing(b) processing

## 6. Concluding Remarks

This paper presented a well known mapping techniques, array interpolation with FB-SSP, and we applied an alternative correlativity suppression approach by using a mod C-ESPRIT algorithm. By the simulation result, we found that although the mapping error is comparatively large in case of virtual array is set more than the real array's, but the final evaluation, accuracy of DOA estimation, is still in acceptable range. The mod C-ESPRIT algorithm achieved the same accuracy as conventional FB-SSP when subarray elements is set at  $(\tilde{N} + 1)/2$ .

However, in the case of coherent signal, both mapping error and signal correlativity suppression is necessary to be analyzed in mathematical scheme so as to find an appropriate middle point to determine virtual array number, which will be considered as one of future studies.

## References

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