Asymptotic Analysis of Scattered Field by the Edge of a Cylindrically Curved Conducting Open Surface

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1. Introduction

When the high-frequency (HF) electromagnetic wave is incident on the edge of a cylindrically curved conducting surface, the scattered field in the concave portion of the curved surface may be analyzed asymptotically by dividing the observation region into two subregions. One of which is the subregion called the boundary layer which corresponds to the region bounded by the curved conducting surface and the modal caustic of the whispering gallery (WG) mode [1]. The other is the interior region surrounded by the boundary layer. The scattered field in the interior region may be analyzed by using only the geometrical ray (GO).

In this paper, we shall first study the analysis method for the scattered field in the boundary layer when the HF electromagnetic wave is incident on one of the edges of a curved surface. We shall derive the hybrid GO and modal ray solution for the scattered field in the boundary layer. Then, by extending the above hybrid solution, we shall derive the asymptotic solution for the scattered field by the curved open surface. We will confirm the validity of the asymptotic representation proposed in the present study by comparing with the reference solution obtained from the method of moment (MoM).

2. Formulation and Asymptotic Analysis

2.1 Asymptotic analysis for the scattered field in the boundary layer

Fig.1 shows the cylindrically curved conducting open surface of radius a and central angle ϕ_{B} (abbreviated as " the curved open surface") and the cylindrical coordinate system (ρ, ϕ, z). When a cylindrical wave radiated from the magnetic line source $\mathrm{Q}(\rho_0, \phi_0)$ directing the z direction is incident on the edge $\mathrm{A}(a, 0)$ of the curved open surface, the magnetic field $H_z(\mathrm{P})$ observed at the point $\mathrm{P}(\rho, \phi)$

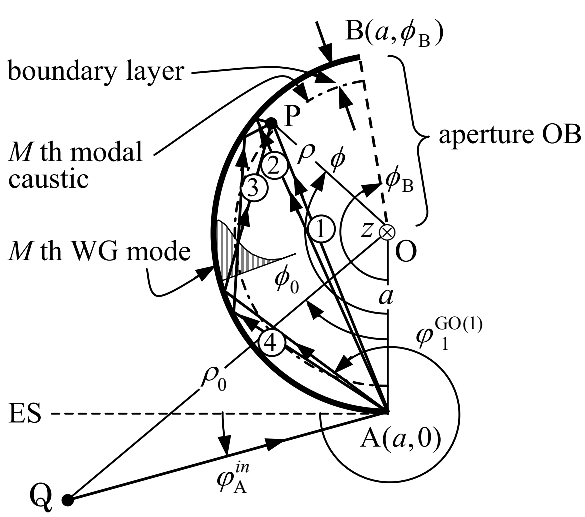

Fig.1 Cylindrically curved conducting open surface defined by radius a and central angle ϕ_{B} and cylindrical coordinate system (ρ,ϕ,z). $\mathrm{Q}(\rho_0,\phi_0)$: magnetic line source, $\mathrm{P}(\rho,\phi)$: observation point, ES: tangent at the edge A, boundary layer: the region bounded by the curved surface and the M th modal caustic (-·-·-). Also shown are the geometrical rays (GO) ①: $(j,n)=(1,0)$, ②: $(2,1)$, ③: $(1,1)$, and ④: $(2,2)$ emanated from the edge A and propagating in the concave portion of curved surface.

located in the boundary layer is given by

$$H_z(\mathrm{P}) = G(\mathrm{QA})\, H_d(\mathrm{P}) \tag{1a}$$

$$H_d(\mathrm{P}) \sim \sum_{j=1}^{2}\left[\sum_{n=0}^{N_j-1}\left\{(\ell_{j,n})\, D_{\mathrm{A}}(\varphi_n^{\mathrm{GO}(j)}, \varphi_{\mathrm{A}}^{in}) G_{j,n} + \frac{1}{2} D_{\mathrm{A}}(\varphi_{N_j}^{\mathrm{GO}(j)}, \varphi_{\mathrm{A}}^{in}) G_{j,N_j}\right\} + \sum_{m=1}^{M} D_{\mathrm{A}}(\varphi_m^{\mathrm{WG}}, \varphi_{\mathrm{A}}^{in}) \overline{G}_m^{(j)}\right] \tag{1b}$$

Here, $G(\mathrm{QA})$ denotes the incident cylindrical wave propagating from the source Q to the edge A, $H_d(\mathrm{P})$ the edge diffracted field observed in the boundary layer, and $D_{\mathrm{A}}(\varphi, \varphi_{\mathrm{A}}^{in})$ the diffraction coefficient at the edge A where $\varphi_{\mathrm{A}}^{in}$ is the incident angle and φ the diffraction angle (see Fig.1). $G_{j,n}$, $n \le N_j$ denotes the GO field with species j ($j = 1, 2$) (see Table 1) and n the number of reflection on the concave surface. N_j denotes the maximum number of reflection. The coefficient $\ell_{j,n}$ takes "0" for $j = 2, n = 0$, and "1" otherwise. Using the symbol (j, n), the GO ①~④ in Fig.1 can be expressed by $(j, n) =$ (1, 0), (2, 1), (1, 1), and (2, 2), respectively. The third term in (1b) denotes the waves reflected more than $N_j + 1$ times which can not be treated as the GO field. The third term can be treated by the first few lowest order modal ray $\overline{G}_m^{(j)}$, $j = 1, 2$, comprising the *m*th order WG mode (see Table 1) [1], [2], [3].

Table 1 Species j for geometrical ray (GO) and modal ray.

species j	P side
1	○
2	×

○ : has already passed through the turning point.
× : has not yet passed through the turning point.

2.2 Scattered field solution by curved open surface

The scattered field $H_d(\mathrm{P})$ in (1b) propagates toward the other edge B direction and is radiated from the aperture plane OB (see Fig.1). When the observation point $\mathrm{P}(\rho, \phi)$ or $\mathrm{P}(R, \psi)$ (see Fig.2) is located far away from the curved open surface as shown in Fig.2, only the GO and the GO converted from the modal ray [3], [4] belonging to the ray species $j = 1$ can reach the observation point $\mathrm{P}(R, \psi)$. The radiated ray solution can be derived by applying the ray tracing technique [4] and is given by

$$H_z(\mathrm{P}) = G(\mathrm{QA})\, H_d^{\mathrm{OB}}(\mathrm{P}) \tag{2a}$$

$$\begin{aligned} H_d^{\mathrm{OB}}(\mathrm{P}) \sim & \sum_{n=0}^{N_1-1} D_{\mathrm{A}}(\varphi_n^{\mathrm{GO}(j)}, \varphi_{\mathrm{A}}^{in})\, H_{d,n}^{\mathrm{GO}(1)}(\mathrm{P}) + \frac{1}{2} D_{\mathrm{A}}(\varphi_{N_j}^{\mathrm{GO}(j)}, \varphi_{\mathrm{A}}^{in})\, H_{d,N_j}^{\mathrm{GO}(1)}(\mathrm{P}) \\ & + \sum_{m=1}^{M} D_{\mathrm{A}}(\varphi_m^{\mathrm{WG}}, \varphi_{\mathrm{A}}^{in})\, H_{d,m}^{\mathrm{WG}(1)}(\mathrm{P}) \end{aligned} \tag{2b}$$

where $H_{d,n}^{\mathrm{GO}(1)}(\mathrm{P})$ and $H_{d,m}^{\mathrm{WG}(1)}(\mathrm{P})$ denote respectively the *n* times reflected GO solution with species $j = 1$ and the GO solution converted from the modal ray $\overline{G}_m^{(1)}$.

While, the incident wave on the edge B can be obtained directly from the scattered field solution in (1b) by locating the point P on the edge B and omitting the scattered field belonging to the species $j = 2$. Thus, the asymptotic solution for the double edge scattered field excited by the incident wave may be constructed by applying the idea of Keller's GTD [5].

3. Scattering Phenomena and Numerical Results

In order to interpret physically the scattering phenomena by the cylindrically curved conducting open surface with two edges A and B, we will show the propagation paths and the arrival range of the scattered wave comprising the HF scattered field when the plane wave illuminates the convex portion of the curved open surface. Then, we will show that the asymptotic solution derived in the present study agrees excellently with the reference solution calculated from the MoM.

In Fig.2, we have shown the scattering phenomena geometrically when the magnetic-type plane wave is incident on the convex portion of the curved surface. The observation point $\mathrm{P}(R,\psi)$ is located on the circle with the constant radius R measured from the vertex point O' of the curved open surface (this circle is called as "the observation circle"). The GO reflected on the convex surface is observed on the observation circle from the point A_1 to the point A_1' in clockwise direction. The range from A_1 to A_1' may be indicated by the symbol $(\mathrm{A}_1 \curvearrowright \mathrm{A}_1')$. The rays excited by the edge A and radiated from the concave portion are the GO reflected "once" and "twice" on the concave surface and the GO converted from the 1st order modal ray. Those GO can be observed in the ranges shown by $(\mathrm{A}_2 \curvearrowright \mathrm{A}_2')$, $(\mathrm{A}_3 \curvearrowright \mathrm{A}_3')$, and $(\mathrm{A}_4 \curvearrowright \mathrm{A}_4')$, respectively.

The scattered fields by the edge A in the convex side are the edge diffracted ray and the edge surface diffracted ray that can reach the regions $(\mathrm{A}_5 \curvearrowright \mathrm{A}_5')$ and $(\mathrm{A}_6 \curvearrowright \mathrm{A}_6')$, respectively. When the plane wave is incident on the other edge B, the scattering phenomena similar to those excited by the edge A are observed on the observation circle. The shaded regions Ⓡ, Ⓢ, and Ⓐ denote the transition regions near the geometrical boundaries, i.e. the reflection boundary (RB), the shadow boundary (SB), and the tangent (ES) at the edge A, respectively.

The scattered field by the curved open surface can be obtained asymptotically by the summation of the above mentioned scattered fields reaching the observation point $\mathrm{P}(R,\psi)$.

In Fig.3(a), the asymptotic solution (——: solid curve) is compared with the numerical solution (∘∘∘: open circles) calculated from the MoM. The scattered field magnitudes are calculated as the function of the observation angle ψ (see Fig.2). In the calculation of the asymptotic solution, we have considered the contribution up to the double edge scattered field. Note that the results in the transition region Ⓡ and Ⓢ

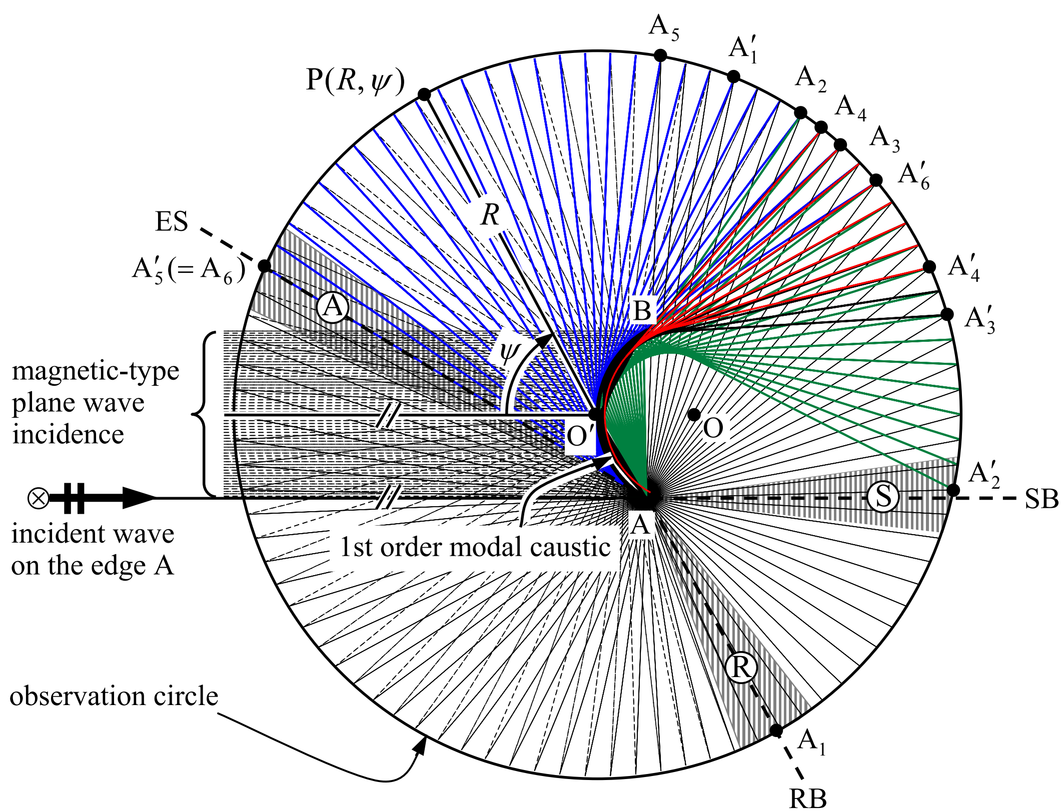

Fig.2 Geometrical representation of the scattered fields by the cylindrically curved conducting open surface $\widehat{\mathrm{AB}}$ excited by the magnetic-type plane wave incident on the edge A. The scattered fields are observed at $\mathrm{P}(R,\psi)$ located on the circle with the radius R. The shaded regions Ⓡ, Ⓢ, and Ⓐ denote the transition regions near the reflection boundary (RB), the shadow boundary (SB), and the tangent (ES) at the edge A, respectively.

have been calculated by using the uniform edge diffraction coefficient and the results in the transition region A have been evaluated by the extended UTD solution in the illuminated side and the modified UTD solution in the shadow side, respectively [6]. It is clear from Fig.3(a) that the asymptotic solution agrees excellently with the reference solution calculated by using the MoM in the entire region. In Fig.3(a), the numerical result calculated by using only the GO reflected on the convex portion is shown by the dashed curve (----). It is observed that the dominant contribution to the total scattered fields arises from the GO reflected on the convex surface in the range $(\mathrm{A}_1 \leftrightarrow \mathrm{A}'_1)$ shown in Fig.3(a) (see Fig.2). In Fig.3(b), the asymptotic solution calculated from the summation of the single and double edge scattered fields excited by the plane wave incidence on the edge A is compared with the reference solution obtained by the MoM. In the region $(\mathrm{A}_4 \leftrightarrow \mathrm{A}'_4)$ of the GO converted from 1st order modal ray and in the vicinities of the RB and SB (see Fig.2), the single and double edge scattered field solution (----) contributes strongly to the total scattered field.

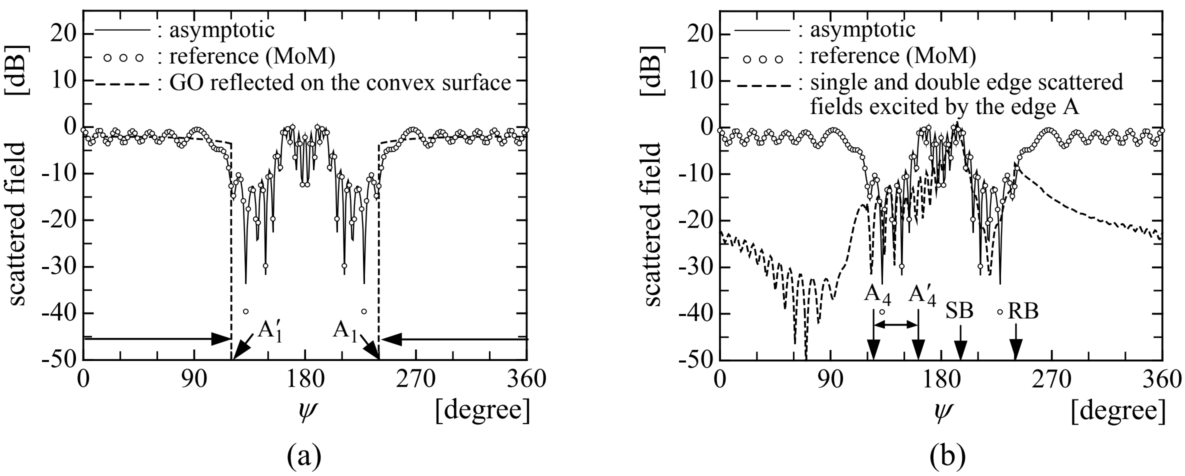

Fig.3 Numerical results for scattered field excited by the magnetic-type plane wave incidence.
The numerical parameters: $a = 5.07\times10^{-2}\,m$, $\phi_{\mathrm{B}} = 120°$, 35 GHz, $\psi_i = 0°$, $R = 0.19\,m$.

4. Conclusion

In this paper, we have derived first the hybrid ray and modal ray solution for the scattered field in the boundary layer. Then by extending the hybrid solution, we have derived the asymptotic solution for the scattered field by the concave portion of the cylindrically curved conducting open surface. By using the geometrical representation of the scattered field, we have shown the scattering phenomena and the arrival range of each scattered wave comprising the total scattered fields. We have confirmed the validity of the asymptotic representation proposed in the present paper by comparing with the reference solution calculated from the method of moment.

References

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