# A Study on Limit of Applying the Rough Sea Surface Model to the Electromagnetic Wave Propagation Analysis above Sea Level 

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## 1. Introduction

On developing a high efficiency radar system on a ship, the relationship between characteristics of the electromagnetic wave (hereafter we say "EM-wave" in distinction from waves on sea level) propagation and the wave height of the sea should be understood correctly. Ray tracing above rough sea surfaces model is often used for analyzing this problem, however the limit of application is not clear. In this paper, we show the characteristics of the EM-wave propagation above sea level calculated by ray tracing with rough sea surface model, and compare it with the result obtained by parabolic equation method.

## 2. EM-Wave Propagation Analysis using Rough Sea Surface Model

Electric field above sea level can be obtained by the summation of following two components; direct-reached field and reflected field by the sea surface[1]. Fig. 1 shows the two paths between transmission antenna and the observation point. If sea surface is not very rough, specular-reflection coefficient can be often used to obtain the reflected field.

The specular-reflection coefficient of rough surfaces $\Gamma$ is described as follows;

$$
\begin{equation*}
\Gamma=\rho_{\mathrm{s}} \mathrm{R} \tag{1}
\end{equation*}
$$

In this equation, R is a Flesnel reflection coefficient which is determined by grazing angle and the electric characteristics of sea water. For the smooth rough surfaces, the roughness foctor $\rho_{\mathrm{s}}$ is described as

$$
\begin{equation*}
\rho_{\mathrm{S}}=\exp \left(-g^{2} / 2\right) \tag{2}
\end{equation*}
$$

In this equation, $g$ is the Rayleigh roughness criterion described as follows[2];

$$
\begin{equation*}
g=4 \pi\left(\sigma_{h} / \lambda\right) \sin \alpha \tag{3}
\end{equation*}
$$

where $\lambda$ is a wave length, $\alpha$ is a grazing angle and $\sigma_{h}$ is a standard deviation of wave height. In [2], it is referred that the surface can be considered to be smooth if $g$ is smaller than 0.3.

## 3. Parabolic Equation Method for Obtaining Electromagnetic Fields over the Irregular Sea Surface

Fig. 2 shows the EM-wave propagation in the Cartesian coordinate. Now we consider the variation of EM-wave independent from $y$-direction, and we assume that strong variations of the refractive index does not occur. Now our interest is to obtain fields where energy propagates at small angles from the boresight of transmission antenna. Maxwell's equations are transformed into the next form[3][4];

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial z^{2}}(x, z)-2 j k \frac{\partial u}{\partial x}(x, z)+k^{2}\left(m^{2}-1\right) u(x, z)=0 \tag{4}
\end{equation*}
$$

In this equation, $u$ can be written as $u(x, z)=\exp (j k x) \psi(x, z)$, where $\psi(x, z)$ is a solution of the wave
equation, and $m$ is a modified refraction index of the atmosphere, described as

$$
\begin{equation*}
m=n+z / a \tag{5}
\end{equation*}
$$

where $n$ is the refraction index and $a$ is the Earth's radius. For given height pattern of electric field, the height pattern at arbitrary distance from the first given point can be obtained step by step[4].

The effect of irregular sea surface on the electromagnetic field is considered as follows. At first, the height pattern of the field at $x=x_{i+1}$ is derived from the given height pattern at $x=x_{i}$ as a flat sea surface. Next, the fields under sea surface at $x=x_{i+1}$ is set to zero. Here, irregular sea surface is represented as the stair, shown in Fig. 3. We show the image of EM-wave propagation in Fig. 4.

## 4. Calculation Results

Irregular waveheight data are obtained from normal distributed random numbers, whose standard deviation is set to be that of wave height. Fig. 5 denotes range profiles of the wave height. Fig. 5 (a) is the $\sigma_{h}=0.5 \lambda$ case and Fig. 5 (b) is the $\sigma_{h}=3.0 \lambda$ case.

Table 1 denotes conditions of the calculation. We calculated 9 cases of the standard deviation of wave height from $0.5 \lambda$ to $4.5 \lambda$. The height distribution of the modified refractivity $M=(m-1) \times 10^{6}$ of standard atmosphere is shown in Fig. 6.

Fig. 7 shows range profiles of the received power at a height of 100 m from sea level. The black line and grey line denote the calculated results by parabolic equation method and ray trace respectively. In the case of $\sigma_{h}=0.5 \lambda$, both results agree well. On the other hands, in the case of $\sigma_{h}=3.0 \lambda$, the difference between both results can be seen.

Fig. 8 denotes a plot of difference between received power by parabolic equation method and that by ray trace in terms of the parameter $g$ of Eq.(3) at the point of range $=15 \mathrm{~km}$ and height $=100 \mathrm{~m}$. In the region $g<0.3$, the difference is smaller than 0.6 . This result shows that the application limit of the rough sea surface, $g<0.3$, is valid.

## 5. Conclusions

The authors compare the calculation results by rough sea surface and ray tracing with the results by the parabolic equation method with irregular wave height data, and verify accuracy of EM-wave propagation analysis by rough sea surface model. We show the validity of the application limit referred in [2].

## References

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[3] M. Levy, "Horizontal parabolic equation solution of radiowave propagation problems on large domains," IEEE Trans. Antennas and Propagat, vol.43, no.2, pp.137-144, Feb. 1995.
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Fig. 1: Two paths between transmit antenna and observation point.


Fig. 2: EM-wave propagation in the Cartesian coordinate.


Fig. 3: Stair shape modeled wave height.


Fig. 4: Image of the EM-wave propagation.


Fig. 5: Range profile of the wave height.

Table. 1: Analysis condition.

| Antenna Height | 10 m |
| :--- | :--- |
| Polarization of Electic Field | H-pol |
| Beam width | $2.0^{\circ}$ |
| Atmosphere | Standard <br> Atmosphere |
| Standard deviation of <br> Wave Height | $0.5-4.5$ |
| Transmitted Power | 1 W |



Fig. 6: Modified refractivity profiles of the standard atmosphere.

(a) $\sigma_{h}=0.5 \lambda$

(b) $\sigma_{h}=3.0 \lambda$

Fig. 7: Range profile of the received power from 100 m over the sea level.


Fig. 8: Parameter $g$ in the equation (3) and the difference between two calculation methods

