Wave Propagation in General Bi-isotropic Media

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1. Introduction

The wave phenomena associated with the class of bi-isotropic (BI) materials had attracted great interests for over two decades due to their extra medium parameter which gives additional freedom in designing various microwave devices [1]. Promising applications in antennas including the polarization rotating lenses and the compact microstrip antennas as well as microwave devices and radar engineering are all based on BI materials [1,2].

In the past, efforts have been focused on the development of equivalent parameters in order to treat a BI medium as an isotropic one [2]. Finite-Difference Time-Domain (BI-FDTD) technique had been applied for formulation of modelling wave interactions with BI media based on the decomposition of wavefields [3,4]. Additionally, time-domain modelling for electromagnetic wave propagation in BI media based on the Transmission Line Matrix (TLM) approach had also been proposed [5,6].

In this paper, we present a new approach to the propagation of uniform plane wave in BI media. While the case of normal incidence is the main focus in this paper, the same approach can be applied for the oblique incidence case as well. Firstly, we employ the technique of Scalarization to simplify the Maxwell equations in vector form to become a set of scalar ones, and then introduce the concept of perturbation analysis to exhibit the effect of the bi-isotropy on the wave characteristics in general. The method of analysis is based on the development of the dispersion relation of a BI medium, so that the result obtained are cast in a concise form and will appear as an extension of well known ones of isotropic media. In particular, the familiar technique of equivalent circuit is developed to enhance the understanding of the wave phenomena associated with BI media.

2. Scalarization of Maxwell equations

For the electromagnetic fields of the time variation, $e^{j\omega t}$, the source-free Maxwell equations for electromagnetic waves propagating in a bi-isotropic medium are written as:

$$\nabla \times \underline{E} = -j\omega \underline{B} \qquad (1a)$$

$$\nabla \times \underline{H} = j\omega \underline{D} \qquad (1b)$$

where $\underline{E}, \underline{D}, \underline{B}, \underline{H}$ are the electromagnetic field vectors. In a general BI medium, the constitutive relations can be expressed as:

$$\underline{D} = \varepsilon \underline{E} + \xi \underline{H} \qquad (2a)$$

$$\underline{B} = \zeta \underline{E} + \mu \underline{H} \qquad (2b)$$

with ε and μ being the permeability and permittivity of the medium, and ζ and ξ are the BI coefficients. Based on the law of energy conservation, it is required that the bi-isotropic coefficients be generally expressed by: $\zeta = \xi^* = \chi + j\kappa$, where χ is the Tellegen coefficient and κ is the chirality coefficient.

Without the loss of generality, consider a uniform plane wave that is propagating in a specific direction, say the z-direction, so that the Maxwell equations are reduced to a set of first-order coupled differential equations for the two-dimensional transverse field vectors, $\underline{E}_t$ and $\underline{H}_t$, and another set for the scalar longitudinal components, E_z and H_z, as given below:

$$\frac{d}{dz}\left[\underline{z}_o \times \underline{E}_t(z)\right] = -j\omega\mu \underline{H}_t(z) - j\omega\zeta \underline{E}_t(z) \tag{3a}$$

$$\frac{d}{dz}\underline{H}_t(z) = -j\omega\varepsilon\left[\underline{z}_o \times \underline{E}_t(z)\right] - j\omega\xi[\underline{z}_o \times \underline{H}_t(z)] \tag{3b}$$

$$0 = \omega\mu H_z(z) + \omega\zeta E_z(z) \tag{3c}$$

$$0 = \omega\varepsilon E_z(z) + \omega\xi H_z(z) \tag{3d}$$

Evidently, the two sets of equations for the transverse and longitudinal components are decoupled, and may be treated separately. First of all, (3c) and (3d) form a system of linear homogeneous equations which admits a non-trivial solution if and only if the $\mu\varepsilon = \xi\zeta$; otherwise, we have the trivial solution: $E_z = H_z = 0$, corresponding to a TEM mode with respect to the direction of propagation. Under the special condition, $\mu\varepsilon = \xi\zeta$, we may have a longitudinal wave with the two longitudinal components of the fields mutually related by: $H_z = -\sqrt{\varepsilon\zeta/\mu\xi}\, E_z$. The exploration of possible physical implications of such a longitudinal wave should be left to the advancement of technology in the future to develop an artificial material to satisfy the special condition: $\mu\varepsilon = \xi\zeta$; throughout this work, we shall assume that the medium parameters satisfy the condition: $\mu\varepsilon > \xi\zeta$.

(3a) and (3b) form a system of four coupled first-order differential equations, as shown below:

$$\frac{d}{dz}\left[\underline{z}_o \times \underline{E}_t(z)\right] = -j\omega\mu \underline{H}_t(z) + j\omega\zeta \underline{\underline{1}} \times \underline{z}_o \cdot \left[\underline{z}_o \times \underline{E}_t(z)\right] \tag{4a}$$

$$\frac{d}{dz}\underline{H}_t(z) = -j\omega\varepsilon\left[\underline{z}_o \times \underline{E}_t(z)\right] - j\omega\xi \underline{\underline{1}} \times \underline{z}_o \cdot \underline{H}_t(z) \tag{4b}$$

where $\underline{\underline{1}}$ is a two dimensional unit dyadic. The coefficient dyadics, $\underline{\underline{1}}$ and $\underline{\underline{1}} \times \underline{z}_o$, are commutative to each other and they share the same set of orthonormal eigenvectors on the transverse plane, as given by:

$$\underline{u}_1 = \frac{1}{\sqrt{2}}\left(\underline{x}_o - j\underline{y}_o\right) \tag{5a}$$

$$\underline{u}_2 = \frac{1}{\sqrt{2}}\left(\underline{x}_o + j\underline{y}_o\right) \tag{5b}$$

Physically, the two eigenvectors, $\underline{u}_1$ is recognized as the right-hand circularly polarized (RCP) mode, and $\underline{u}_2$ as the left-hand circularly polarized (LCP) mode, if they stands for the electric and magnetic field vectors.

In terms of the orthonormal set of eigenvectors, the transverse electric and magnetic fields may be represented generally as:

$$\underline{z}_o \times \underline{E}_t(z) = V_1(z)\underline{u}_1 + V_2(z)\underline{u}_2 \tag{6a}$$

$$\underline{H}_t(z) = I_1(z)\underline{u}_1 + I_2(z)\underline{u}_2 \tag{6b}$$

where the V's and I's describe longitudinal variations of the electric and magnetic fields. Substituting the last two expressions into the two equations in (4), we obtain the following coupled system of ordinary differential equations of the first order:

$$\frac{d}{dz}V_m(z) = -j\omega\mu I_m(z) + (-1)^m \omega\zeta V_m(z) \tag{7a}$$

$$\frac{d}{dz}I_m(z) = -j\omega\varepsilon V_m(z) - (-1)^m \omega\xi I_m(z) \tag{7b}$$

for $m = 1$ and 2. What we have achieved so far is the scalarization of the Maxwell equations in vector form to become two sets of scalar differential equations of the first order, one for the RCP and the other for LCP modes. Additionally, the general solutions of (7) yield the basic transmission-line equations for the forward travelling of RCP mode and the backward travelling of the LCP mode.

3. Dispersion Relation of BI Medium

The last term in (7a) is due to the effect of perturbation by the bi-isotropy, resulting in the equivalent voltage source and that in (7b) as the equivalent current source along the transmission line. Both equivalent sources depend on the local field strength; therefore, they belong to the class of dependent sources, as shown in Fig. 1. It is noted that the polarities of the dependent sources will affect the propagation of waves in the two opposite direction along the transmission line.

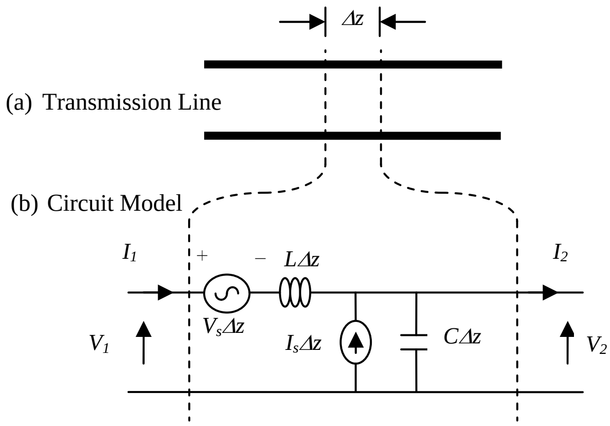

Figure 1: Equivalent Network for bi-isotropic medium: $L = \mu$, $C = \varepsilon$, $V_s = \omega\zeta V_1$ and $I_s = \omega\xi I_1$

Replacing d/dz by $-jk$ in (7) with proper algebraic operations, the following dispersion relation for the BI medium can be derived:

$$k^2 + (-1)^m j\delta k_p - k_p^2 = 0\ , \quad \text{for m = 1 and 2} \tag{8}$$

$$k_p^2 = \omega^2\left(\mu\varepsilon - \zeta\xi\right) \tag{9a}$$

$$\delta = \frac{\zeta - \xi}{\sqrt{\mu\varepsilon - \zeta\xi}} \tag{9b}$$

The dispersion relation is a quadratic equation from which two roots are expected. Once the dispersion roots are determined, the wave admittance and impedance are determined accordingly. The results are altogether summarized in Table 1 below.

Table 1 Characteristics of wave propagation in BI medium

	Mode 1: RCP		Mode 2: LCP	
Eigenvector	$\underline{u}_1 = \underline{x}_o - j\underline{y}_o$		$\underline{u}_2 = \underline{x}_o + j\underline{y}_o$	
Dispersion Relation	$k^2 + j\delta k_p - k_p^2 = 0$		$k^2 - j\delta k_p - k_p^2 = 0$	
Propagation Direction	Forward Travelling	Backward Travelling	Forward Travelling	Backward Travelling
Eigenvalues	$k = k_1 = k_p e^{-j\theta}$	$k = -k_2 = -k_p e^{j\theta}$	$k = k_2 = k_p e^{j\theta}$	$k = -k_1 = -k_p e^{-j\theta}$
Admittance	$Y_1 = \frac{1}{\omega\mu}\left(k_1 + j\omega\zeta\right)$	$Y_2 = \frac{1}{\omega\mu}\left(k_2 - j\omega\zeta\right)$	$Y_2 = \frac{1}{\omega\mu}\left(k_2 - j\omega\zeta\right)$	$Y_1 = \frac{1}{\omega\mu}\left(k_1 + j\omega\zeta\right)$
Impedance	$Z_1 = \frac{1}{\omega\varepsilon}\left(k_1 - j\omega\xi\right)$	$Z_2 = \frac{1}{\omega\varepsilon}\left(k_2 + j\omega\xi\right)$	$Z_2 = \frac{1}{\omega\varepsilon}\left(k_2 + j\omega\xi\right)$	$Z_1 = \frac{1}{\omega\varepsilon}\left(k_1 - j\omega\xi\right)$
Remarks	$\sin\theta = \frac{1}{2}\delta$			

We observe that the dispersion relations for RCP and LCP modes differ from each other only by the sign of δ; thus, the propagation characteristics of one mode can be obtained from those of the other mode by a simple change of the signs of ξ and ζ,. From the perturbation point of view, we may say that the bi-isotropy cause a positive (negative) effect on the RCP (LCP) mode.

4. Boundary-Value problem: scattering of plane wave by a single interface between isotropic and bi-isotropic half-spaces

Consider a uniform plane wave that is linearly polarized in the x-direction and is incident normally onto the interface between the air region (z < 0) and a bi-isotropic half space (z > 0). Since the eigenvectors, $\underline{u}_1$ and $\underline{u}_2$, are independent of the medium, they can be used for the representations of the electric and magnetic fields in the bi-isotropic as well as isotropic media. The incident, reflected, and transmitted fields can then be represented as:

$$\underline{z}_o \times \underline{E}_t^{(inc)}(z) = V_{inc}\,\underline{x}_o e^{-jk_o z} = \frac{1}{\sqrt{2}} V_{inc}\left(\underline{u}_1 + \underline{u}_2\right) e^{-jk_o z}; \underline{H}_t^{(inc)}(z) = Y_a V_{inc}\,\underline{x}_o e^{-jk_o z} = \frac{1}{\sqrt{2}} Y_a V_{inc}\left(\underline{u}_1 + \underline{u}_2\right) e^{-jk_o z} \quad (10a)$$

$$\underline{z}_o \times \underline{E}_t^{(refl)}(z) = \frac{1}{\sqrt{2}}\left(V_1^{(refl)}\underline{u}_1 + V_2^{(refl)}\underline{u}_2\right) e^{jk_o z}; \underline{H}_t^{(refl)}(z) = -\frac{1}{\sqrt{2}} Y_a \left(V_1^{(refl)}\underline{u}_1 + V_2^{(refl)}\underline{u}_2\right) e^{jk_o z} \quad (10b)$$

$$\underline{z}_o \times \underline{E}_t^{(tran)}(z) = V_1^{(tran)}\underline{u}_1 e^{jk_1 z} + V_2^{(tran)}\underline{u}_2 e^{jk_2 z}; \underline{H}_t^{(tran)}(z) = Y_o\left(V_1^{(tran)}\underline{a}_1 e^{jk_1 z} + V_2^{(tran)}\underline{a}_2 e^{jk_2 z}\right) \quad (10c)$$

Applying proper boundary conditions at the z = 0 interface, the reflection (Γ_m) and transmission (T_m) coefficients of the m^{th} mode can be readily derived as:

$$\Gamma_m = \frac{Y_o - Y_m^{(+)}}{Y_o + Y_m^{(+)}}; T_m = 1 + \Gamma_m \quad (11)$$

5. Conclusion

We have presented a unified approach to the analysis of wave propagation in bi-isotropic media. The concept of perturbation analysis is employed for a easy understanding of the physical processes involved, but the mathematical formulation is an exact one to establish a solid foundation for the ensuing study in every aspect. Through the development of dispersion relation and equivalent network, the results are cast in the form that is well known in the microwave community.

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