Dielectric resonator antenna design using FDTD topology optimization

#**Tsuyoshi Nomura**[1], **Kazuo Sato**[1], **Shintaro Yamasaki**[1], **and Shinji Nishiwaki**[2]

[1] Applied Electromagnetics Lab., Toyota Central Research and Development Laboratories, Inc.
Nagakute , Aichi 480-1192, Japan
[2] Dept. of Aeronautics and Astronautics, Kyoto University
Yoshida Honmachi, Sakyo-ku, Kyoto, 606-8501, Japan

1 Introduction

Topology optimization is the most flexible optimization method that can simultaneously deal with geometrical and topological configuration changes [1]. In this method, a fixed design domain is defined such that it is larger than the obtained designed domain. In the fixed domain, an arbitrary configuration can be expressed using a characteristic function, allowing large changes in geometrical and topological design during the optimization process. Topology optimization was originally developed for structural design problems, and has been recently adapted for many other types of design problems in various physics systems include electromagnetic wave propagation problems. Firstly, photonic crystal waveguides have been successfully designed by considering scalar wave propagation in optical waveguides using FEM [2, 3] as two-dimensional design problems. Kiziltas *et al.* [4] developed an optimum design method for substrate of patch antennas with the Finite Element-Boundary Integration method (FE-BI) with three dimensional vector wave formulation. In these works, however, several appropriate frequencies need to be selected in order to evaluate bandwidth performance using frequency domain analyses such as FEM or FE-BI. On the other hand, time domain analysis methods such as the Finite Difference–Time Domain method (FDTD) can be considered as attractive alternatives for numerical analysis methods since continuous frequency range can be analyzed at one calculation.

This paper introduces a topology optimization method for the structural design of multiband broadband Dielectric Resonator Antennas (DRAs)[5], based on time domain analysis using the FDTD method. Moreover, structural mechanics and fabrication constraints are simultaneously considered in order to obtain manufacturable and self supporting antenna design.

2 Optimization method

2.1 Topology Optimization

Consider the problem of determining the boundary of a design domain Ω_d by minimizing or maximizing an objective function. The key idea of topology optimization is the introduction of a fixed, extended design domain D that includes the original design domain Ω_d, *a priori*, and which utilizes the following characteristic function χ_Ω:

$$\chi_\Omega(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_d \\ 0 & \text{if } \mathbf{x} \in D \backslash \Omega_d \end{cases} \tag{1}$$

where $\mathbf{x}$ denotes a position in the extended design domain D. Topology optimization is defined as the problem of finding the optimal distribution of $\chi_\Omega(\mathbf{x})$, where an objective function is optimized, subject to constraint specifications including governing physical equations. In this study, we apply a method based on the density method[6] for representing the electric permittivity, and the homogenization method[1] for representing elastic tensor. Here, we assume that the design domain is composed of isotropic dielectric elastic materials, and that the electric property determining the antenna's performance is electric permittivity. Here we use a simple linear interpolation for the density function, since relative permittivity of isotropic material can be expressed with a scalar value. The isotropic relative permittivity at point $\mathbf{x}$ can be expressed as:

$$\varepsilon_r(\rho(\mathbf{x})) = \varepsilon_r^{\text{air}} + (\varepsilon_r^{\text{solid}} - \varepsilon_r^{\text{air}})\rho(\mathbf{x}), \quad (0 \leq \rho(\mathbf{x}) \leq 1) \tag{2}$$

where $\varepsilon_r^{\text{air}}$ is the relative permittivity in air, and $\varepsilon_r^{\text{solid}}$ is that in a solid. On the other hand, since the elasticity is expressed by elastic tensor value and it is difficult to express tensor value using linear interpolation even if it is isotropic, elastic tensor is, therefore, numerically calculated with homogenization method by introducing isotropic periodic unit cell structure[7].

2.2 Objective functions and multiobjective formulation

Fig. 1 shows the analysis model for an antenna system, used for the design of DRAs. As shown in this figure, the analysis domain consists of rectangular parallelepiped volume including the extended design domain D that is placed on the upper side of the metallic ground plane. The feed probe, a short length of bare wire protruding into the design domain, transmits electromagnetic waves from the shielded coaxial cable connecting the design domain to the electronic circuit driving the antenna, and electromagnetic waves radiated from the design domain are then analyzed in the analysis domain that includes it. Perfectly Matched Layers (PML)[8] that absorb electromagnetic waves cover five faces of the analysis domain, while the bottom surface is left bare.

The electromagnetic wave propagation problem is described by Maxwell's equations. Maxwell's equations for lossless media can be written as:

$$\mu\frac{\partial \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E}, \quad \varepsilon\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} \tag{3}$$

where $\mathbf{E}$, $\mathbf{H}$, ϵ, μ and t are electric field, magnetic field, electric permittivity, magnetic permeability and time, respectively. The time domain analysis is used for electromagnetic fields calculation and the entire reflected electric energy which is accumulated through all the analysis time is calculated in order to optimize whole frequency range given by input pulse spectrum. Therefore the objective function of electromagnetic problem is formulated as:

$$W_{\text{ref}} = \int_0^T \int_{\Omega_{\text{obs}}} \frac{1}{2} \left|\mathbf{E}_{\text{obs}}(t) - \mathbf{E}_{\text{in}}(t)\right|^2 \mathrm{d}\Omega \mathrm{d}t \tag{4}$$

where W_{ref} is the electric energy returned into the cable point and Ω_{obs}, $\mathbf{E}_{\text{obs}}(t)$ $\mathbf{E}_{\text{in}}(t)$ are observation surface inside of the cable, observed electric field and the electric field at the observation plane excited by the input pulse alone, without reflection. The frequency range and its weighting to frequencies are determined by the spectrum of given input pulse in this objective function[9].

The linear material without body force is assumed in structural mechanics analysis, and the governing equation is written as:

$$\int_D \mathbf{u}^* : \mathbf{C} : \mathbf{u}\mathrm{d}\Omega = \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{u}^* \mathrm{d}\Gamma \tag{5}$$

where $\mathbf{u}^*$, $\mathbf{C}$, $\mathbf{u}$ and $\mathbf{t}$ are virtual displacement, stiffness tensor, displacement and traction force, respectively. The mechanical objective is to maximize the stiffness under load, and the mean compliance minimization problem is formulated.

$$l(\mathbf{u}) = \int_{\Gamma_t} \mathbf{t} \cdot \mathbf{u}\mathrm{d}\Gamma \tag{6}$$

The ϵ constraint approach[10] is utilized for formulation of multiobjective optimization problem. The electromagnetic objective is chosen to be transformed into a constraint, since the electromagnetic objective function can be considered as a highly multimodal function while the mean compliance has convex characteristics. Considering other fabrication constraints, the optimization problem can be formulated as follows:

$$\begin{array}{lll} \underset{\rho}{\text{Minimize}} & F = \frac{l(\mathbf{u})}{l(\mathbf{u})^{\text{init}}} & \\ \text{subject to:} & 0 \leq \rho \leq 1 & \\ & \bar{F} = \frac{W_{\text{ref}}}{W_{\text{ref}}^{\text{init}}} < \frac{W^U}{W^{\text{init}}} & S = \int_\Omega H(\rho(\mathbf{x}))\mathrm{d}\Omega - \int_\Omega \rho(\mathbf{x})\mathrm{d}\Omega = 0 \\ & R_1 = \rho(\mathbf{x}) < 0 & \text{if} \quad \phi_o < |\mathbf{x} - \mathbf{x}_c| \\ & R_2 = \rho(\mathbf{x}) > 1 & \text{if} \quad \phi_o - \Delta r < |\mathbf{x} - \mathbf{x}_c| < \phi_o \\ & T_1 = \rho(\mathbf{x} - \Delta\mathbf{z}) - \rho(\mathbf{x}) < 0 & \text{if} \quad |\mathbf{x} - \mathbf{x}_c| < \phi_o - \Delta\phi \\ & \text{Maxwell's equations in Eq. (3)} & \text{Elastic equilibrium equation in Eq. (5)} \end{array} \tag{7}$$

where F is the objective function for structural mechanics, $\bar{F}$ is that for the electromagnetics and S is a constraint to ensure single homogeneous material design. R_1 and R_2 are constraints for outer appearance, and T_1 is a constraint for fabrication with molding. Superscript $^{\text{init}}$ stands for values of initial design, superscript ^{U} stands for upper limit values, $H(x)$ is the Heaviside function, ϕ_o is outer radius of spherical shell which DRA should be fit into, and $\mathbf{x}_c$ is the center of the spherical shell.

3 Numerical examples

We designed the structure of the multiband wideband self supporting DRA in order to confirm the usefulness of the proposed method. The DRA with dimension $D_x = 30$mm, $D_y = 30$mm, $D_z = 8$mm and relative dielectric constant $\varepsilon_r^{\text{solid}} = 12.0$ is fed at the center by the coaxial probe that extents $P_z = 5\text{mm}$ into the DRA. We choose the calculated area with 216000 $(60 \times 60 \times 60)$ cubic cells of 1mm. The structural support areas are defined on the bottom side of the design domain. Displacements of areas within 2mm radius from four corners are constrained. The load is added on a area within 2mm from the center on the top side of the design domain. These conditions are also shown in Fig. 1. In order to design multiple operational band antenna, two 0.5GHz bandwidth Gaussian pulse, whose peak frequency are respectively set to 5.0GHz and 7.0GHz, are used for input pulse. These pulses are mixed and feed simultaneously.

Fig. 2 and 3 show frequency characteristics of reflection coefficient $|S_{11}|$ for the final configuration and the top view and the bottom view of the designed material structure, respectively. The obtained configuration has clear manufacturable and self supporting shape while achieved return loss in the target range was less than -10dB.

4 Conclusion

We proposed a time-domain topology optimization method for designing the optimal structure of RF components using the FDTD method. A DRA design example was provided to examine the usefulness of the proposed method. We confirmed that the proposed method enables multiband broadband dielectric resonator antenna design while considering structural mechanics for assuring self supporting structure and fabrication constraints.

References

[1] M. P. Bendsøe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," *Computer Methods In Applied Mechanics And Engineering*, vol. 71, pp. 197–224, 1988.

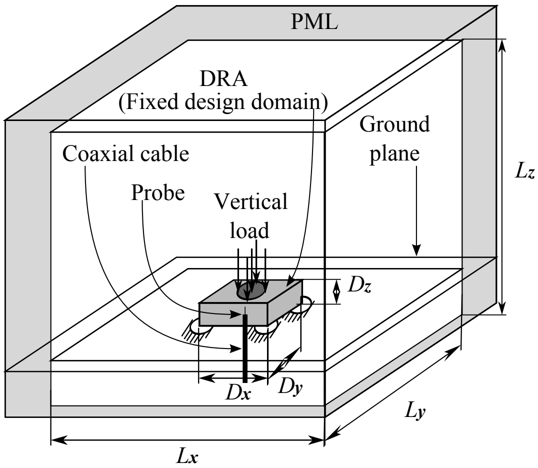

Figure 1: DRA analysis model

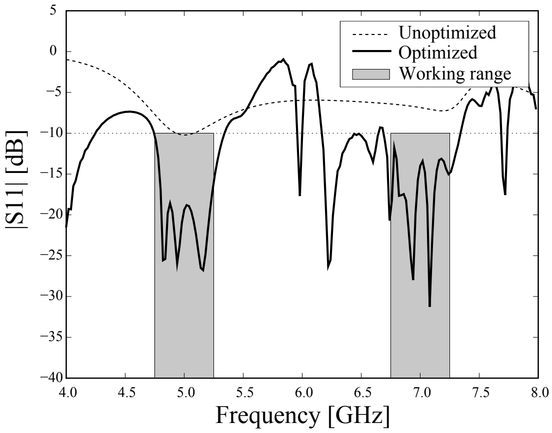

Figure 2: Frequency characteristics of reflection coefficients of the optimal configuration.

Figure 3: Top view and bottom view the optimal configuration.

[2] J. S. Jensen and O. Sigmund, "Systematic design of photonic crystal structures using topology optimization: Low-loss waveguide bends," *Applied physics letters*, vol. 84, pp. 2022–2024, Mar. 2004.

[3] Y. Tsuji, K. Hirayama, T. Nomura, K. Sato, and S. Nishiwaki, "Design of optical circuit devices based on topology optimization," *IEEE Photonics Technology Letters*, vol. 18, pp. 850 – 852, Apr. 2006.

[4] G. Kiziltas, D. Psychoudakis, J. L. Volakis, and N. Kikuchi, "Topology design optimization of dielectric substrates for bandwidth improvement of a patch antenna," *IEEE Transactions On Antennas And Propagation*, vol. 51, pp. 2732 – 2743, Oct. 2003.

[5] S. A. Long, G. L. Conway, and L. Shen, "The resonant cylindrical dielectric cavity antenna," *IEEE Transactions on Antennas and Propagation*, vol. 31, pp. 406–412, May 1983.

[6] R. Yang and C. Chuang, "Optimal topology design using linear programming," *Computers and Structures*, vol. 52, no. 2, pp. 265–275, 1994.

[7] K. Matsui and K. Terada, "Continuous approximation of material distribution for topology optimization," *International Journal for Numerical Methods in Engineering*, vol. 59, pp. 1925–1944, Apr. 2004.

[8] J. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Computat. Phys.*, vol. 114, pp. 185–200, Oct. 1994.

[9] T. Nomura, K. Sato, K. Taguchi, T. Kashiwa, and S. Nishiwaki, "Structural topology optimization for the design of broadband dielectric resonator antennas using the finite difference time domain technique," *International Journal for Numerical Methods in Engineering*, vol. 71, pp. 1261–1296, Sept. 2007.

[10] Y. Haimes, L. Lasdon, and D. Wismer, "On a bicriterion formulation of the problems of integrated system identification and system optimization," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 1, no. 3, pp. 296–297, 1971.