AP Line Search-based Joint Estimation of Carrier Frequency Offset and Channel Response in Uplink OFDMA

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1. Introduction

Orthogonal frequency-division multiple access (OFDMA) is a promising technique for the 4th generation (4G) systems. To decrease the computing time of grid search in carrier frequency offset (CFO) estimation, several different optimization approaches have been applied, such as direct line search [5] and ad hoc [1], which are highly related to the characteristics of the target problem. In this work, one pilot preamble symbol is transmitted as a training sequence. The coarse frequency offset has been adjusted by assuming that the subscriber stations (SSs) have synchronized with the base station (BS) by down-link signals. The maximum-likelihood estimation can be conducted in the preamble transmission. To increase the efficiency of optimization, the system model is analyzed through matrix products. Alternating projection method is used as an example, where line search method is adopted to simply the frequency offset search. The system model will be derived in section 2, the alternating projection method in section 3, simulation results and conclusion in section 4.

2. System Model

Consider an OFDMA system with N subcarriers which are assigned to K SSs. One preamable symbol is transmitted with equal-distance pilot tones. The DFT domain symbol of the k th SS is [1]

$$x_k\left[m\right]=\begin{cases} c_k[u], & m=uD+\Delta_k, \quad (u=0,\cdots,N_p-1) \\ 0, & \text{otherwise}, \end{cases}$$

where N_p is the number of pilot tones, $D=N/N_p$, $\Delta_k=\lfloor D/K \rfloor$ is the index offset between SSs, with $\lfloor \bullet \rfloor$ the floor operation, where $c_k\left[u\right]$ is a random BPSK sequence. The discrete-time domain output is

$$\bar{s}_k=1/\sqrt{N}\,\bar{\bar{F}}^H \bullet \bar{x}_k ,$$

where $\left\{\bar{\bar{F}}\right\}_{m,n}=1/\sqrt{N}\;\; e^{-j\frac{2\pi}{N}mn}$, the superscript H denotes Hermitian operation. The signal with cyclic prefix (CP) is $\bar{u}_k=\left[s_k\left[N-N_g\right]\cdots s_k\left[N-1\right]\;\; s_k\left[0\right]\cdots s_k\left[N-1\right]\right]^t$, N_g is the length of CP. Considering both effects of CFOs and timing errors of all SSs, the discrete-time received signal at the BS can be represented as [2]

$$r'\left[n\right]=\sum_{k=1}^{K} e^{j\omega_k n}\sum_{\ell=0}^{L_k-1} h'_k\left[\ell\right]u_k\left[n-\ell-\mu_k\right]+v\left[n\right],$$

where $\omega_k=2\pi\Delta f_k/N$ is the normalized frequency offset with respect to the subcarrier spacing for the k th SS, L_k is the number of channel taps, μ_k is the integer –valued timing error for the kth SS, the fractional part of timing error is absorbed into the channel impulse response (CIR) $\bar{h}'_k$ [2], and

$v(n)$ is the complex white Gaussian noise with zero mean and covariance matrix $\sigma_v^2 \bar{\bar{I}}_N$. To remove the inter-symbol interference, $N_g \geq \max\{\mu_k + L_k\}$ is assumed for all SSs. After removing the CP part from $\bar{r}'$, the received signal can be represented in matrix form as

$$\bar{r} = \sum_{k=1}^{K} \bar{\bar{\Gamma}}(\omega_k) \bullet \bar{\bar{A}}_k \bullet \bar{h}_k + \bar{v} = \sum_{k=1}^{k} \bar{r}_k + \bar{v}$$

$$= \begin{bmatrix} \bar{\bar{\Gamma}}(\omega_1) \bullet \bar{\bar{A}}_1 & \cdots & \bar{\bar{\Gamma}}(\omega_K) \bullet \bar{\bar{A}}_K \end{bmatrix} \begin{bmatrix} \bar{h}_1 \\ \vdots \\ \bar{h}_K \end{bmatrix} + \bar{v}$$

where $\bar{r}_k = \bar{\bar{\Gamma}}(\omega_k) \bullet \bar{\bar{A}}_k \bullet \bar{h}_k$, $\bar{\bar{\Gamma}}(\omega_k) = \operatorname{diag}\left\{1 \quad e^{j2\pi\omega_k/N} \quad \cdots \quad e^{j2\pi\omega_k(N-1)/N}\right\}$ and

$$\bar{\bar{A}}_k \bullet \bar{h}_k = \begin{bmatrix} u_k[-\mu_k] & u_k[-\mu_k - 1] & \cdots & u_k[-\mu_k - L_k + 1] \\ u_k[1-\mu_k] & u_k[-\mu_k] & \cdots & u_k[-\mu_k - L_k + 2] \\ \vdots & \vdots & \ddots & \vdots \\ u_k[N-1-\mu_k] & u_k[N-2-\mu_k] & \cdots & u_k[N-\mu_k - L_k] \end{bmatrix} \begin{bmatrix} h_k[0] \\ h_k[1] \\ \vdots \\ h_k[L_k - 1] \end{bmatrix}$$

To absorb the timing offset and channel length of each SS, the channel vector of the kth SS is extended to be $\bar{h}_k^e = \begin{bmatrix} \bar{0}_{\mu_k \times 1}^t & \bar{h}_k^t & \bar{0}_{(N_g - \mu_k - L_k) \times 1}^t \end{bmatrix}^t$, thus $\bar{\bar{A}}_k \bullet \bar{h}_k$ can be rewritten as

$$\begin{bmatrix} u_k[0] & \cdots & u_k[-\mu_k] & \cdots & u_k[-\mu_k - L_k + 1] & \cdots & u_k[1-N_g] \\ u_k[1] & \cdots & u_k[1-\mu_k] & \cdots & u_k[-\mu_k - L_k + 2] & \cdots & u_k[2-N_g] \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ u_k[N-1] & \cdots & u_k[N-1-\mu_k] & \cdots & u_k[N-\mu_k - L_k] & \cdots & u_k[N-N_g] \end{bmatrix} \begin{bmatrix} \bar{0}_{\mu_k \times 1} \\ \bar{h}_k \\ \bar{0}_{(N_g - \mu_k - L_k) \times 1} \end{bmatrix}$$

Since $u_k[-i]$ is in the cyclic part, which is the same as $u_k[N-i]$, $\bar{\bar{A}}_k \bullet \bar{h}_k$ can be further rewritten as $\left[\bar{\bar{A}}_k^{\text{circ}}\right]_{N_g} \bullet \bar{h}_k^e$, where $\bar{\bar{A}}_k^{\text{circ}}$ is a square circulant matrix of which the column vectors are the previous column vectors shifted down by one element and the top elements are the last element of the previous columns. The operation $[\bullet]_{N_g}$ selects the first N_g columns, it is equivalent to multiplying with $\bar{\bar{U}} = \left[\bar{\bar{I}}_{N_g \times N_g} \bullet \bar{\bar{O}}_{N \times N_g}^T\right]^t$. The matrix $\bar{\bar{A}}_k^{\text{circ}}$ can be decomposed into $\bar{\bar{F}}^H \bullet \bar{\bar{\Lambda}}_k \bullet \bar{\bar{F}}$, where $\bar{\bar{\Lambda}}_k = \operatorname{diag}\{\text{DFT}\{u_k[0] \cdots u_k[N-1]\}\}$. Hence, $\bar{\bar{A}}_k = \left[\bar{\bar{A}}_k^{\text{circ}}\right]_{N_g} = \left(\bar{\bar{F}}^H \bullet \bar{\bar{\Lambda}}_k \bullet \bar{\bar{F}}\right) \bullet \bar{\bar{U}}$, and the resulting signal becomes

$$\bar{r} = \sum_{k=1}^{K} \bar{\bar{\Gamma}}(\omega_k) \bullet \bar{\bar{A}}_k \bullet \bar{h}_k^e + \bar{v} = \left[\bar{\bar{\Gamma}}(\omega_1) \bullet \bar{\bar{A}}_1 \cdots \bar{\bar{\Gamma}}(\omega_K) \bullet \bar{\bar{A}}_K\right] \bullet \bar{h}^e + \bar{v} = \bar{\bar{Q}} \bullet \bar{h}^e + \bar{v}, \tag{1}$$

where $\bar{h}^e = \left[\left[\bar{h}_1^e\right]^t \cdots \left[\bar{h}_K^e\right]^t\right]^t$ and $\bar{\bar{Q}} = \left[\bar{\bar{\Gamma}}(\omega_1) \bullet \bar{\bar{A}}_1 \cdots \bar{\bar{\Gamma}}(\omega_K) \bullet \bar{\bar{A}}_K\right]$.

3. Alternating Projection Method

The procedure of alternating projection (AP) consist of cycles and steps where each cycle is divided into K steps [6]. Only the CFO of one SS is updated in each step, while other CFOs are kept unchanged. Without lose of generality, the step order is chosen the same as the order of SSs. Using the least square (LS) error method, the estimate of $\bar{h}^e$ in (1) is

$$\hat{\bar{h}}^e = \left(\bar{\bar{Q}}^H \bullet \bar{\bar{Q}}\right)^{-1} \bullet \bar{\bar{Q}}^H \bullet \bar{r} . \tag{2}$$

The maximum likelihood estimate of CFO of the kth SS can be derived as

$$\left\{\hat{\omega}_k^i\right\} = \arg \min_{\tilde{\omega}_k} \left\{\left\| \bar{r} - \bar{\bar{Q}}\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right) \bullet \bar{h}^e \right\|\right\}, \tag{3}$$

where $\hat{\omega}_k^i$ denotes the estimate of ω_k in the kth step of cycle i and all the previous updated CFOs are denoted as $\hat{\bar{\omega}}_k^i = \left[\hat{\omega}_1^i \cdots \omega_{k-1}^i \;\; \hat{\omega}_{k+1}^{i-1} \cdots \omega_K^{i-1}\right]$. For example, in case of k=1,

$$\bar{\bar{Q}}\left(\tilde{\omega}_1, \hat{\bar{\omega}}_1^i\right) = \left[\overbrace{\bar{\bar{\Gamma}}\left(\tilde{\omega}_1\right) \bullet \bar{\bar{A}}_1}^{\bar{\bar{C}}\left(\tilde{\omega}_1\right)} \;\; \overbrace{\bullet \bar{\bar{\Gamma}}\left(\hat{\omega}_2^i\right) \bullet \bar{\bar{A}}_2 \cdots \bar{\bar{\Gamma}}\left(\omega_K^i\right) \bullet \bar{\bar{A}}_K}^{\bar{\bar{B}}\left(\hat{\bar{\omega}}_1^i\right)} \right] \tag{6}$$

The projection $\bar{\bar{P}}_Q\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right) = \bar{\bar{P}}_B\left(\hat{\bar{\omega}}_k^i\right) + \bar{\bar{P}}_{C_B}\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right)$, where $\bar{\bar{P}}_B\left(\hat{\bar{\omega}}_k^i\right) = \bar{\bar{B}} \bullet \left[\bar{\bar{B}}^H \bullet \bar{\bar{B}}\right]^{-1} \bullet \bar{\bar{B}}^H$ represents the projection onto the column space of $\bar{\bar{B}}$, $\bar{\bar{P}}_{C_B}\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right) = \bar{\bar{C}}_B \bullet \left[\bar{\bar{C}}_B^H \bullet \bar{\bar{C}}_B\right]^{-1} \bullet \bar{\bar{C}}_B^H$ and $\bar{\bar{C}}_B = \left(\bar{\bar{I}}_N - \bar{\bar{P}}_B\right) \bullet \bar{\bar{C}}$, $\bar{\bar{P}}_{C_B}\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right)$ represents the projection onto the column space of $\bar{\bar{C}}$ but not in the column space of $\bar{\bar{B}}$. It is similar to the Gram-Schmidt procedure. Since $\bar{\bar{P}}_B\left(\hat{\bar{\omega}}_k^i\right)$ is independent of $\tilde{\omega}_k$, (5) can be reduced to

$$\left\{\hat{\omega}_k^i\right\} = \arg \max_{\tilde{\omega}_k} \left\{\left\| \bar{\bar{P}}_{C_B}\left(\tilde{\omega}_k, \hat{\bar{\omega}}_k^i\right) \bullet \bar{r} \right\|\right\}. \tag{7}$$

Compared to $\bar{\bar{P}}_B$, significantly smaller matrix inversion is required for $\bar{\bar{P}}_{C_B}$. After all CFOs converge, $\hat{\bar{h}}^e$ can be obtained by using (2).

4. Simulation Results and Conclusion

The simulation parameters are selected from the scalable OFDMA system in [3]. The system bandwidth is 2.5 MHz. The sampling frequency is approximately equal to $F_s = \lfloor 8/7 \times 2.5 \text{ MHz} / 0.008 \rfloor \times 0.008 \simeq 2.857 \text{ MHz}$ with an over-sampling factor of 8/7, thus the sampling period is $T_s = 1/F_s = 350$ ns. Assume N = 256, and the subcarrier frequency spacing is 11.16071429 kHz, the usable symbol time will be $T_\text{b} = 1/11.16071429 \text{ kHz} \simeq 89.6\ \mu\text{s}$. The length of guard time is chosen to be $T_\text{b}/16 = 5.6\ \mu\text{s}$, which implies $N_g = 16$. The integer timing error μ_k is uniformly distributed over the integer set $\{0 \cdots 7\}$. The normalized CFO, Δf_k, is uniformly distributed over [-0.5, 0.5]. The SUI-3 model is used where three taps ($L_k = 3$) are allocated for the CIR of each SS [4]. The number of pilots N_p is the same as N_g. The signal-to-noise ratio (SNR) for the k th SS is defined as SNR $k = \mathrm{E}\left[\bar{s}_k^H \bullet \bar{s}\right] / \left(N\sigma_v^2\right)$, where $\mathrm{E}[\bullet]$ stands for the expectation value of the enclosed parameter. The signal-to-noise-and-interference ratio (SINR) is defined as in (15) of [1]. Two hundred samples are used in each simulation. The CFO estimation is obtained using the instruction 'fmincon' of Matlab$^{\text{TM}}$ [7]. The result will be acquired by the line search method.

Figure 1 shows the convergence rate of CFO1 estimation with SNR1=10 dB and variable K. It is shown that all the mean square errors (MSE) of CFO1 converge after two cycles in spite of different K. Figure 2 shows the SINR1 with different SS number two hundred. The SINR1 converges in about two cycles, and the SINR1 drops as K is increased. Figure 3 shows the MSE of Δf_1 under the most stringent case of K=8. The MSE of CFO1 becomes smaller as SNR1 increases. Figure 4 shows the monotonic trend of SINR with SNR. The SINR1 becomes nearly saturated at large SNR due to the error of channel estimation using the LS.

In summary, the system model proposed in this paper can effectively resolve the CFO and CIR with AP and line search optimization method instead of grid search.

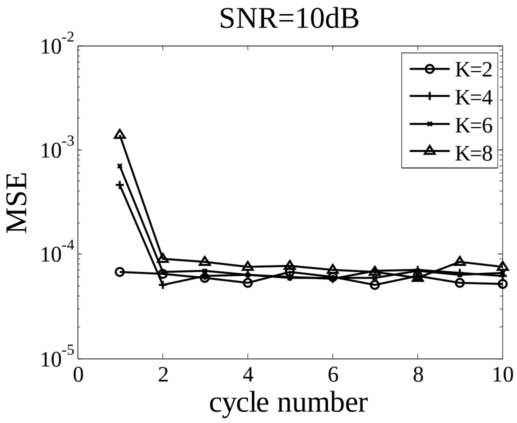

Figure 1: MSE of Δf_1 after several cycles.

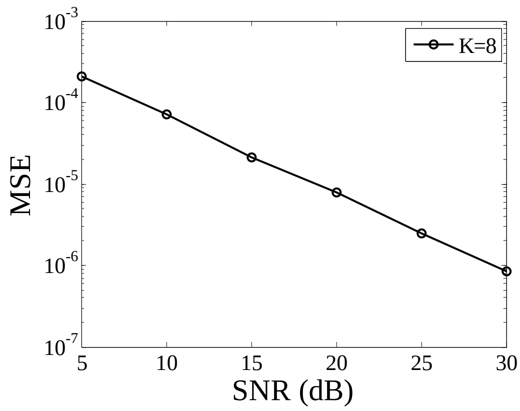

Figure 3: MSE of Δf_1 related to SNR.

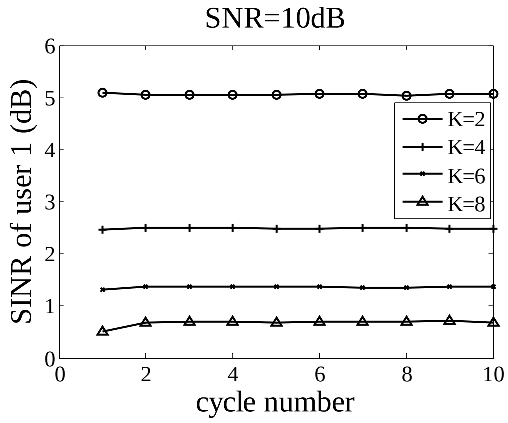

Figure 2: SINR1 after several cycles.

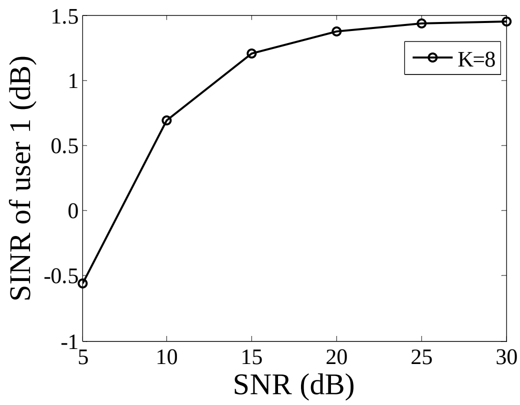

Figure 4: SINR1 related to SNR.

References

[1] X. Fu, H. Minn, and C. D. Cantrell, "Two novel iterative joint frequency-offset and channel estimation methods for OFDMA uplink," *IEEE Trans. Commun.*, vol. 56, no. 3, pp. 474-484, 2008.

[2] M.-O. Pun, S.-H. Tsai, and C.-C. J. Kuo, "Joint maximum likelihood estimation of carrier frequency offset and channel in uplink OFDMA systems," *IEEE Globecom*, pp. 3748-3752, 2004.

[3] H. Yaghoobi, "Scalable OFDMA physical layer in IEEE 802.16 WirelessMAN," Intel Technol. J., vol. 8, no. 3, pp. 201-212, 2004.

[4] IEEE LAN/MAN Standards Committee, "Channel models for fixed wireless applications," IEEE.802.16.3c-01/29r4.

[5] Y. Na and H. Minn, "Line search based iterative joint estimation of channels and frequency offsets for uplink OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4374-4382, 2007.

[6] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection ," *IEEE Trans. Acou., Speech Signal Process.*, vol. 36, no. 10, pp. 1553-1560, 1988.

[7] http://www.mathworks.com/access/helpdesk/help/toolbox/optim/index.html